Understanding Stolt stretch

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In SEP-35 both the author and Jon Claerbout gave simple (and related) derivations for the Stolt stretch function and generalizations. We did not consider the effect of the modified dispersion relation on the accuracy of the migration. Nor will I attempt to here. In this note I'm simply committing to paper the formula for the implied the summation path, this being about the fifth time I've rederived it.

The modified Stolt dispersion relation is given by

$$k_\tau = \left(1 - \frac{1}{s} \right) \omega + \frac{1}{s} \sqrt{\omega^2 - s v^2 k_x^2} \tag{1}$$

where s is a fixed constant parameter.

We can use stationary phase to calculate the summation path implied by (1). Given an output location (x, t) the phase function to be extremized is

$$\Phi = \left(\tau - \frac{\tau}{s} - t \right) \omega + \frac{\tau}{s} \sqrt{\omega^2 - s v^2 k_x^2} - x k_x \quad . \tag{2}$$

Setting partial derivatives to zero:

$$\begin{aligned} 0 &= \frac{\partial \Phi}{\partial \omega} = \left(\tau - \frac{\tau}{s} - t \right) + \frac{\tau}{s} \frac{\omega}{\sqrt{\omega^2 - s v^2 k_x^2}} \\ 0 &= \frac{\partial \Phi}{\partial k_x} = - \frac{\tau v^2 k_x}{\sqrt{\omega^2 - s v^2 k_x^2}} - x \end{aligned} \tag{3}$$

we can eliminate both ω and k_x in this case to obtain the summation path

$$\left(\frac{st + \tau - s\tau}{\tau} \right)^2 - \left(\frac{x\sqrt{s}}{v\tau} \right)^2 = 1 \quad . \tag{4}$$

This can be rewritten in the more illuminating form

$$t = \left(1 - \frac{1}{s} \right) \tau + \frac{1}{s} \sqrt{\tau^2 + s\,(x/v)^2} \tag{5}$$

as a weighted average of a hyperbola with a modified velocity and a flat line. As expected, $t = \tau$ at $x = 0$ and so the apex is still in the right place. Taking derivatives

$$\frac{dt}{dx} = \frac{x}{v^2}\left[\tau^2 + s\,(x/v)^2\right]^{-1/2}$$
$$\frac{d^2t}{dx^2} = \frac{1}{v^2}\left[\tau^2 + s\,(x/v)^2\right]^{-1/2} - \frac{s\,x^2}{v^4}\left[\tau^2 + s\,(x/v)^2\right]^{-3/2} \qquad (6)$$

we find that the second derivative at the apex $x=0$ is $1/v^2\tau$ independent of s. This means that the apex has the same curvature as the constant velocity hyperbolas to which the Stolt stretch function tries to maps diffractions.

The slopes of the asymptotes, given by

$$\left.\frac{dt}{dx}\right|_{x\to\pm\infty} = \frac{\pm 1}{v\sqrt{s}} \qquad (7)$$

are different. It is in this way that the flanks are adjustable up or down to better fit the stretched diffraction patterns.

Footnote

The formula I gave in SEP-35 for Stolt stretch should read:

$$\frac{d}{d\tau}\left[f^2(\tau)\right] = 2\tau\frac{v^2}{v_o^2} \qquad \text{(SEP-35.13)}$$

The velocity ratio was upside down.

I remark that this formula holds just as well if v_o is not constant. This implies that lateral velocity variations can be handled or mishandled in phase-shift migration by using a residual Stolt stretch with v_o the downward continuation velocity function. Because v_o is the desired RMS migration velocity, it is a function of f, the stretched coordinate, rather than τ, the unstretched coordinate. This means that (SEP-35.13) is now really an implicit definition of Stolt stretch because f appears on both sides of the equation. However, because RMS velocity changes relatively slowly as one moves down the trace, one can still integrate for f step by step and incur negligible error using the known value $v_o(f-df)$ instead of the unknown value $v_o(f)$.

REFERENCES

Claerbout, J.F., 1983, 4.5 Stretching Tricks: SEP-35, p. 191-194.

Levin, S.A., 1983, Remarks on two-pass 3-D migration error: SEP-35, p.195-200.