Linearly transformed wave equation modeling

Zhiming Li

ABSTRACT

A new wavefield extrapolation algorithm, the Linearly Transformed Wave Equation (LITWEQ) method, was developed in 1984 (Li, 1984). The method was originally developed to migrate seismic reflection data. This paper will show how to use the LITWEQ in the modeling of (1) zero-offset sections of primary reflections, and (2) nonzero-offset sections (including common-shot profile, common-midpoint gather and VSP) of all seismic waves, such as direct arrival, primary reflections, refractions, and multiples.

INTRODUCTION

The LITWEQ method has the advantage that it extrapolates wavefields in all possible directions without approximations. Both the large-angle and the small-angle events are handled properly throughout the extrapolation. Imposing certain boundary and initial conditions will make the algorithm suitable for the modeling of a seismogram for a given earth model.

Seismic modeling is usually calculated from a half-space elastic medium with a free surface as the upper boundary. The assumption that no waves are transmitted into the space above the free surface establishes the upper boundary condition: the wave field is zero above the free surface. Certain boundary conditions, such as zero slope or absorbing (Engquist and Majda, 1977) boundary conditions, must be imposed on the side boundaries of the calculation grids. The initial boundary conditions are given by the source distribution on the (x, z) plane. The causality of modeling, i.e. that no waves are generated before the source starts to explode, gives us another initial condition. These conditions together with the LITWEQ equation form a well-posed problem of wave propagation.

A zero-offset section can be calculated by a reversal of the steps of the LITWEQ migration, with new boundary conditions for modeling.

Because the LITWEQ equation describes wave propagation in a two-dimensional ((x, z)) elastic medium, the equation can be used to model all possible seismic events, including direct arrivals, reflections, diffractions, refractions and multiples, etc.. After certain boundary and initial conditions are imposed, the LITWEQ wavefield extrapolation method can be applied to the modeling of all possible seismic waves. The source and the receivers can be fixed at any points on the (x, z) plane. The full-wave modeling region (t, x, z) needs to be at least twice as large as that used for the zero offset modeling of primary reflections, because waves in the full-wave modeling are going in both the up and down directions.

The LITWEQ full-wave modeling algorithm described in the paper also gives us a full wavefield extrapolation method that may be used in the wave equation inversion and the prestack migration.

THE LITWEQ EQUATION

The LITWEQ method is based on the following equation (Li, 1984):

$$\frac{\partial^2 P}{\partial x'^{\,2}} + \frac{2}{v} \frac{\partial^2 P}{\partial t' \, \partial z'} \tag{1}$$

where P is wavefield.

Equation (1) is obtained by the transformation of the two-dimensional full-wave equation,

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial z^2} - \frac{1}{v^2} \frac{\partial^2 P}{\partial t^2} = 0, \tag{2}$$

along the characteristic lines of one-dimension wave propagation: $z - vt$ and $z + vt$.

Certain boundary conditions must be imposed if equation (1) is used for modeling seismograms.

ZERO-OFFSET MODELING OF REFLECTIONS

Using the concept of exploding reflector, the zero-offset modeling can be done with the following boundary conditions: (1) No waves will be recorded before time $t = 0$ (casuality in time); (2) No waves will be recorded above the surface (perfect reflecting surface at depth $z = 0$); (3) Zero-value, or zero-slope, or absorbing, boundary conditions exist at both ends of the x axis.

An implicit scheme is used along x' axis, though an explicit scheme can be used if the stability condition is satisfied. The finite-differencing grids on the (t', z') plane for zero-offset modeling are shown in Figure 1. The boundary conditions on the (t, z) plane are:

$$\begin{cases} P(t=-\Delta t, z) = 0 \\ P(t, z=-\Delta z) = 0 . \end{cases} \tag{3}$$

The calculation starts from the plane $t=0$ where the structure of the model is specified, and finishes at the plane $z=0$ where the zero-offset record is received.

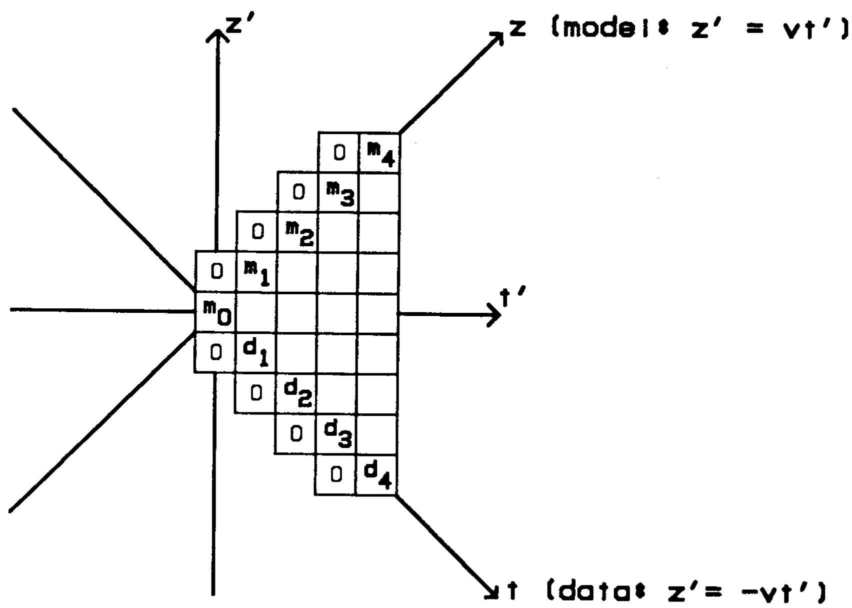

FIG. 1. Finite-differencing grids used in the zero-offset LITWEQ modeling. The structure of the model is given on the $(t' = z/v\sqrt{2}, x', z' = z/\sqrt{2})$ plane, and the data is recorded on the $(t' = t/\sqrt{2}, x', z' = -vt/\sqrt{2})$ plane. The boundary and the initial conditions are denoted by 0's along the grid boundary. Velocity used in the zero-offset modeling is half of the rock velocity.

Figure 2(a) shows a model of a section that has both vertical and lateral velocity variations. Figure 2(b) is the corresponding zero offset seismogram calculated by the LITWEQ zero-offset modeling algorithm. The reflections and the diffractions have been calculated and shown in the figure. However, direct arrivals, multiples, and refractions are not modeled by the LITWEQ zero-offset modeling algorithm. In order to model all seismic waves for any recording geometry, we have to use the full-wave modeling described in the later sections.

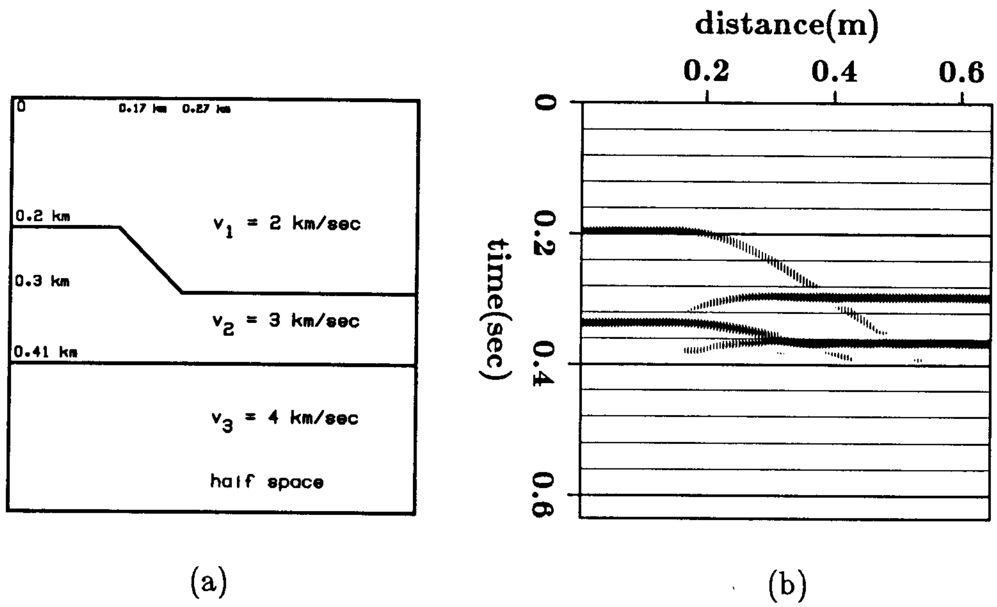

FIG. 2. Zero-offset modeling by LITWEQ method. (a) Model with vertical and lateral changes in velocity. (b) Calculated zero-offset seismogram.

THE FULL WAVE LITWEQ MODELING

Expanded grids for modeling full waves

The calculation of zero-offset modeling is carried out in the triangular region $0<(t+z/v)<t_{\max}$ on (z,t) plane, as shown in Figure 1. In order to calculate all possible seismic waves characterized by the wave equation for any source location (x_s, z_s) and any receiver location (x_g, z_g) at any time t, we must extend our calculation to a rectangular region $0<t<t_{\max}$ and $0<z<z_{\max}=(z_s+vt_{\max})$, where $t_{\max}$ is the maximum time of observation and z_s is the depth of the source.

The boundary conditions $P(t<0,x,z)=0$ and $P(t,x,z<0)=0$ are still used in the full-wave modeling. Another condition $P(t<t_{\max},x,z>z_{\max})=0$, i.e. no waves can reach the depth of $z_{\max}$ when $t<t_{\max}$, is necessary for the full-wave modeling. The expanded finite-difference grids and boundary conditions are shown in Figure 3.

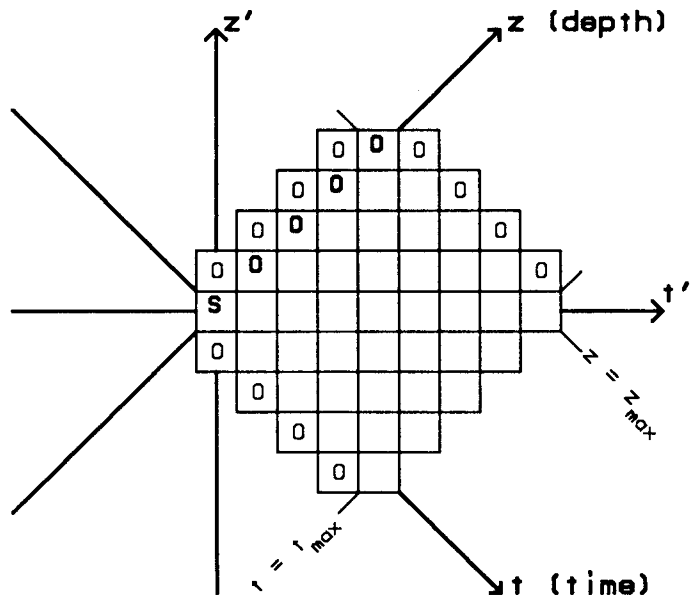

FIG. 3. Finite-differencing grids used in the full-wave modeling by the LITWEQ method. The seismic source is denoted by S on the $(t' = z/v\sqrt{2}, x', z' = z/\sqrt{2})$ plane. An impulse is placed at S at $t=0$. The initial values of the wave field at non-source positions on this plane are zero. The zero-value boundary conditions are specified by 0's along the boundary. The wave field is calculated for any position on the (t', x', z') plane.

The source locations are specified on the plane $(t=0, x, z)$. An impulse is placed on each shot position. The interfaces, or the structures, are specified by the velocity discontinuities in the velocity profile. The result of the LITWEQ full-wave modeling is a two-dimensional wave field $P(t, x, z)$.

Full-wave modeling

Figure 4 shows a cube where the wavefield $P(t, x, z)$ is calculated and displayed. The source is located at the origin $(t=0, x=0, z=0)$. There are two flat reflectors in the model. The wavefield is calculated for every point in the (x, z) plane at any instant time t. The front frame shows a snapshot of the wavefield, i.e. wavefronts on the (x, z) plane at time t. The top frame shows a common-shot profile, i.e. wavefield observed along the x axis. The side frame shows a VSP (vertical seismic profile), i.e. the wavefield observed along a vertical line $x = x_g$.

The LITWEQ calculation of the wave field at the (t, x, z) space can provide seismograms for any arrangement of sources and receivers. For example, defining a

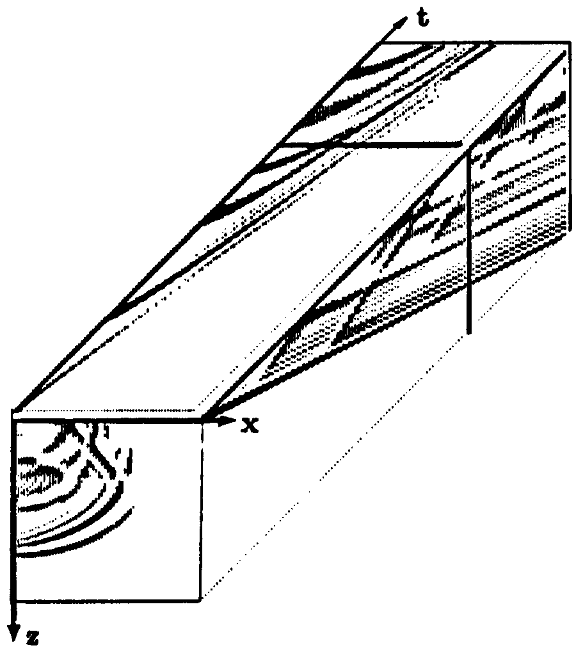

FIG. 4. A cubic display of the two-dimensional wavefield $P(t, x, z)$. t is the time axis. x is the horizontal axis. z is the depth axis.

source s at (x_s =0, z_s =0) and receivers g 's at (x_g, z_g =0) will give us a common-shot profile. Defining source positions and receiver positions symmetrically about one point on the surface will give us a common-midpoint gather. Specifying the receiver's positions along a vertical line and the source's position on the surface will give us a VSP.

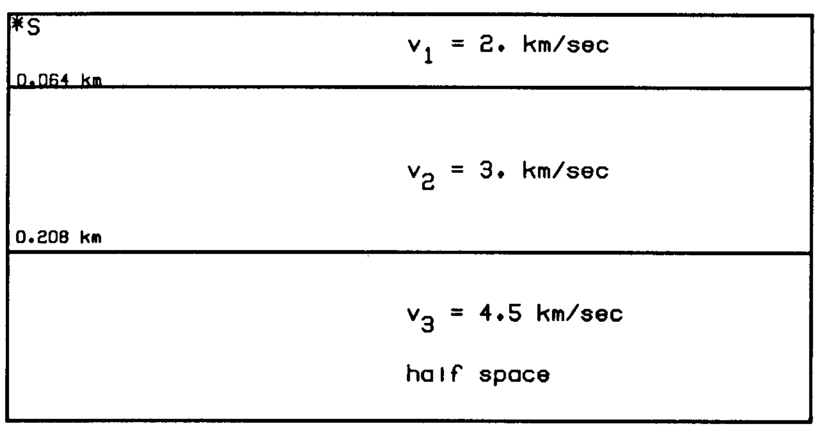

FIG. 5. A three-layer model.

Figure 5 shows a model having 3 layers. Figure 6(a) is the common-shot profile of the model. Notice that the direct arrival, refraction, reflections, and multiples are all present on this common-shot profile. The first arrival at near traces is direct arrival, while the first arrival at the far offset traces is the refraction from the first interface. At the near traces, the second event is the reflection from the first interface, the third and the fifth events are the multiples of the first reflection, the fourth event is the reflection from the second interface, and the last event is the peg leg multiple of the reflection of the second interface. Figure 6(b) is the common-midpoint gather from the model.

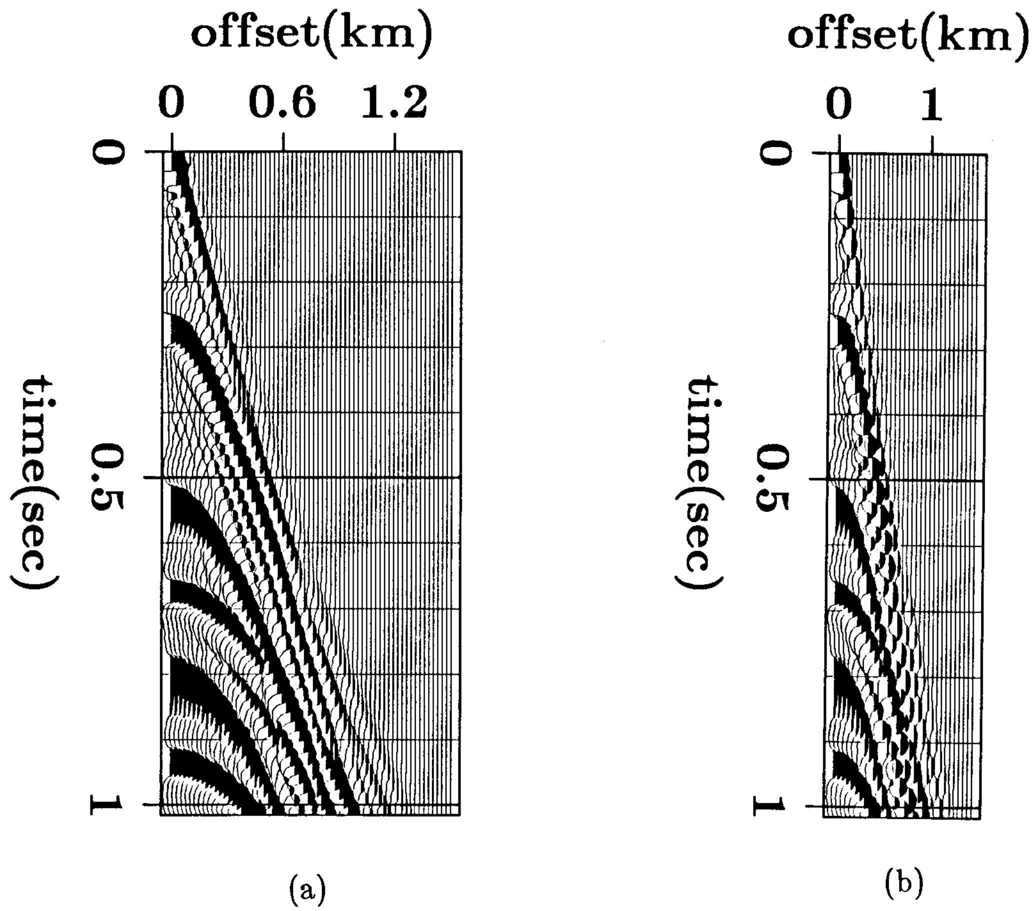

FIG. 6. (a) Common-shot profile recorded on the surface; the source is located at (x_x =0,z_s =0). (b) Common-midpoint gather recorded on the surface; the sources and the receivers are located symmetrically about one point on the surface.

Dispersion and aliasing effects can be seen in these synthetic seismograms, especially in the far offsets, because large step sizes are used in this finite differencing calculation of the seismograms. Reducing the step sizes can suppress these effects.

Figure 7 is the VSP with the source offset 120 m from the well (in which the receivers are placed). Again we can see the reflections, direct arrivals, and multiples in the VSP.

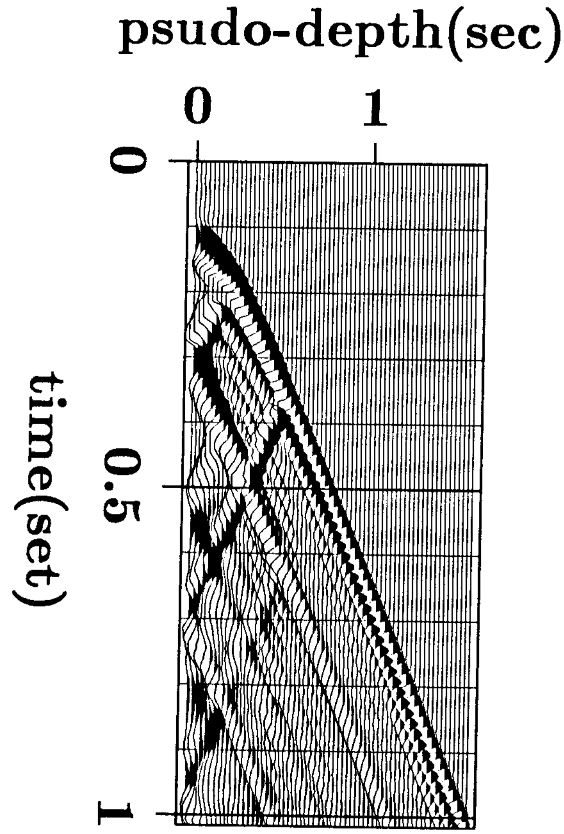

FIG. 7. VSP recorded in a vertical well located 120 m away from the source.

The seismogram of a given source waveform can be obtained by the convolutions of impulse response $P(t, x, z)$ with source waveform $S(t)$ along time axis.

Other boundary conditions

The zero-value boundary condition at $z = -\Delta z$ can be changed to a zero-slope boundary condition, $P(t, x, z = -\Delta z) = P(t, x, z = 0)$, or an absorbing, or a partially transmitting boundary condition, depending on the reflectivity of the surface.

The partially transmitting boundary condition (waves are partially transmitted into the space above the upper boundary) is obtained on the upper boundary $z=0$, when the transmission coefficient of the surface is given.

Imposing the absorbing side boundary conditions at $x=x_{\min}$ and $x=x_{\max}$, the artifacts of side boundary reflections generated by the zero-slope, or zero-value, boundary conditions can be suppressed.

Applications

The LITWEQ method can be applied to full-wave modeling of the seismic response of a given earth model. The algorithm calculates not only reflections but also direct arrivals, refractions, diffractions, and multiples, etc.. Therefore, it can be used to analog the field data for any given recording geometry, after the amplitude correction. The amplitude correction is necessary because the modeling of seismogram is two-dimensional but the earth is three-dimensional. The two-dimensional LITWEQ modeling is easy to be implemented by the finite-differencing method, though three dimensional modeling by the LITWEQ method is possible.

The full-wave modeling can be used in seismic inversion problems where the calculation of seismograms for a given model of velocity, or impedance, is needed.

The LITWEQ full-wave modeling method is actually a full-wave extrapolation algorithm. The modification of the boundary conditions and the use of the LITWEQ full-wave extrapolation algorithm may give us a prestack migration algorithm.

CONCLUSION

The LITWEQ modeling algorithm can be used in not only the calculations of zero-offset seismograms but also, and more importantly, the full-wave modelings. Because of the high accuracy of preserving all events in all propagating directions, the LITWEQ full-wave modeling algorithm can analog the field data more precisely than the lower-order (e.g. 15 degree equation, or 45 degree equation) approximation methods, especially at far-offsets where the propagating angles of seismic reflections become large. The flexibility of handling both vertical and lateral velocity changes in the LITWEQ method gives us an advantage of the LITWEQ method over the Fourier wavefield extrapolation methods (e.g. the phase-shift method and Stolt's method), when the velocities of the media change both vertically and laterally, because interpolation or stretch of the velocity profile is necessary in the Fourier methods (Gazdag and Sguazzero, 1984).

The LITWEQ full-wave modeling method also give us an full-wave extrapolation algorithm that may be used to the wave equation inversion and the prestack migration.

ACKNOWLEDGMENTS

I thank Jon Claerbout for his advice on the research and his teaching of finite difference solutions to wave equations which are the bases of the LITWEQ method.

REFERENCES

Engquist, B. and Majda, A., 1977, Absorbing boundary conditions for the numerical simulation of waves: Mathematics of Computation, V. 31, Number 139, July, 1977, pp 629-651.

Gazdag, J. and Sguazzero, P., 1984, Migration of seismic data by phase shift plus interpolation: Geophysics, vol. 49, February, 1984, pp 124-131.

Li, Zhiming, 1984, Wave field extrapolation by the linearly transformed wave equation operator: SEP-41, pp 167-189.