# Sampling theory for velocity space dip-moveout and migration

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## INTRODUCTION

In a previous paper, I discussed how to implement dip-moveout and migration on constant velocity stacks, so as to image data prior to velocity analysis (Fowler, 1984). The economics of implementing this algorithm depend directly on the number of constant velocity stacks which must be created and processed. In this paper I will discuss attempts to obtain estimates of the number of stacks required.

## SAMPLING STRATEGY FOR CONSTANT VELOCITY STACKS

For a specified zero-offset time $\tau$ and a range of velocities $v$, normal moveout defines a suite of hyperbolic summation trajectories in offset-time space $(h, t)$. Figure 1a shows such a set of curves, in this case for a zero-offset time of 0.4 second and an offset range up to 3 km. The velocities illustrated range from 1.5 km/s up to 6 km/s in increments of 0.5 km/s. As can be seen, the curves bunch together for higher velocities, unevenly sampling the time axis for a given offset. Since velocity discrimination is much poorer for high velocities than for low, we wish to sample low velocities at a denser rate than we need for high velocities. We also want to be able to stack in steeply dipping events, so we need to cover a velocity range which extends effectively up to an infinite velocity. For this latter reason, we will henceforth consider only sampling in terms of functions of inverse velocity, or slowness.

To determine an adequate slowness sampling, we need to relate the slowness sampling rate to the time axis sampling rate. What function of slowness would give us a family of moveout curves which sampled the time axis at a specified offset evenly? Unfortunately, there is no simple answer to this question, since any velocity (or slowness) function that samples time evenly at one offset and one zero-offset time would not do so

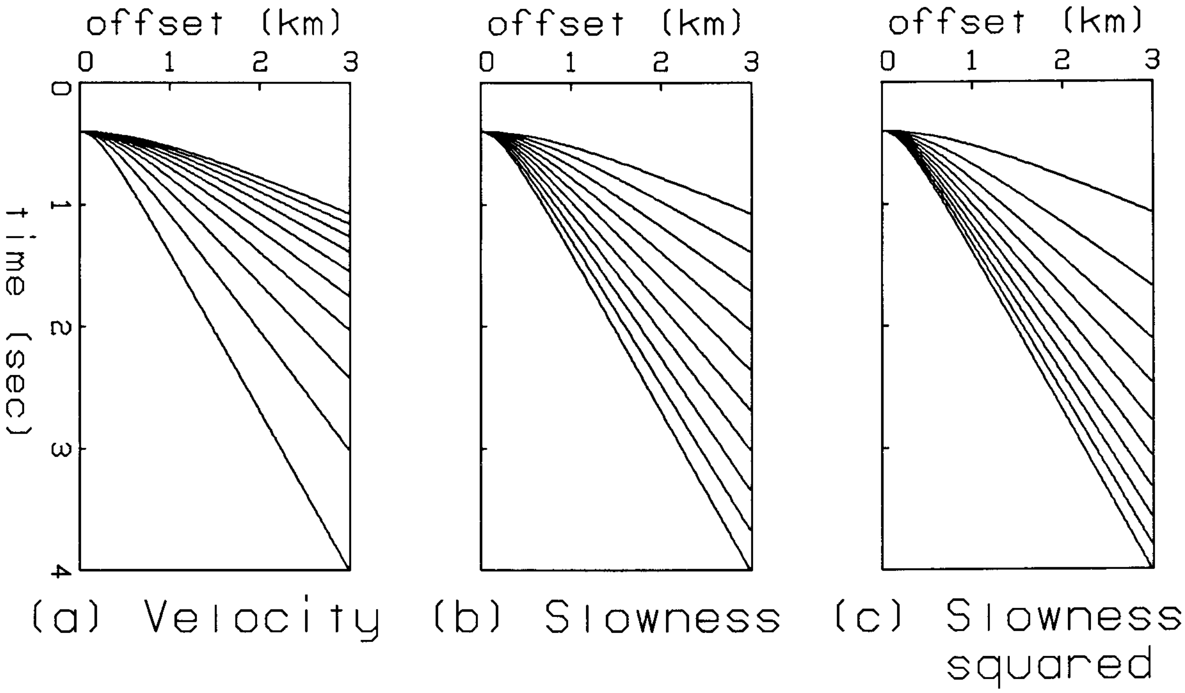



FIG. 1. Sampling a given range of moveout velocities with different sampling schemes. All examples cover a range from 1.5 km/s to 6 km/s. (a) Equal velocity intervals (b) Equal slowness intervals (c) Equal slowness squared intervals

for others. We can write moveout at a slowness $s$ as:

$$t = (\tau^2 + s^2 h^2)^{1/2} \tag{1}$$

where $h$ is offset, $\tau$ is zero-offset time, and $t$ is time. Offset here is full offset, and all times are two-way travel times. If we consider two moveout curves with the same zero-offset time $\tau$ but different slownesses $s_1$ and $s_2$, we have

$$\begin{aligned} \Delta t &= t_2 - t_1 \\ &= (\tau^2 + s_2^2 h^2)^{1/2} - (\tau^2 + s_1^2 h^2)^{1/2} \end{aligned} \tag{2}$$

Then

$$t_2^2 - t_1^2 = h^2 (s_2^2 - s_1^2) \tag{3}$$

$$\Delta t \approx \frac{h^2 (s_2^2 - s_1^2)}{2t} \tag{4}$$

$$\approx \frac{h^2 s}{t} \Delta s \tag{5}$$

These equations can be interpreted to suggest two different sampling schemes: evenly in slowness $w$ or in slowness squared $\sigma \equiv w^2$. First, if $hs << \tau$ then $t \approx \tau$, and equation 4 becomes

$$\Delta t \approx \frac{h^2}{2\tau} \Delta\sigma \tag{6}$$

So for small offsets, large velocities, or late times, sampling evenly in $\sigma$ will approximately evenly sample the time axis for a given $\tau$ and $h$. If, instead, we have $hs >> \tau$, then $t \approx hs$ and equation 5 becomes

$$\Delta t \approx h\,\Delta s \tag{7}$$

So for early times, large offsets, or low velocities, the time axis sampling rate is approximately proportional to the slowness sampling rate. These two different approximations may be interpreted as approximating the moveout hyperbolas in the slowness squared case by parabolas at the inner offsets where curvature is large, and in the slowness case by the straight lines which form their asymptotes.

For an example comparing sampling in increments of velocity, slowness, and slowness squared, see Figure 1. In this particular example, even sampling in slowness can be seen to sample the time axis very evenly at wide offsets. However, as we shall see, slowness squared sampling makes DMO particularly simple, and may require fewer stacks in typical cases. We will find approximate bounds on sampling rates for both schemes. Note that

$$\Delta\sigma \approx 2s\,\Delta s \tag{8}$$

so it is generally easy to go from a sampling estimate for one scheme to one for the other.

For those more used to sampling in velocity, or in even increments of $\Delta v / v$, we can approximate slowness squared sampling as:

$$\begin{aligned}\Delta\sigma &= \frac{1}{v_1^2} - \frac{1}{v_2^2} \\ &= \frac{v_2^2 - v_1^2}{v_1^2 v_2^2} \\ &= \frac{\Delta v\,(2v_1 + \Delta v)}{(v_1 + \Delta v)^2 v_1^2} \\ &\approx \frac{2\Delta v}{v^3}\end{aligned} \tag{9}$$

Similarly for even slowness sampling,

$$\Delta s = \frac{1}{v_1} - \frac{1}{v_2}$$
$$\approx \frac{\Delta v}{v^2} \qquad (10)$$

So sampling in slowness is approximately equivalent to increasing the velocity sampling rate proportional to the square of velocity, and slowness squared is equivalent to increasing velocity sampling proportional to the cube of velocity.

## SAMPLING FOR DIP-MOVEOUT

Dip moveout can be implemented by dip decomposing constant velocity stacks by double Fourier transforming each stack, and then shifting events along the velocity axis according to:

$$v = v_{stack} \left( 1 + \frac{v_{stack}^2 \, k_y^{\,2}}{4\omega^2} \right)^{-1/2} \qquad (11)$$

where $v_{stack}$ is the stacking velocity, and $v$ is the DMO corrected, or zero-dip, velocity. (Fowler, 1984) Because DMO expressed this way only shifts data along the velocity axis, if we have sampled velocity adequately in stacking, DMO should require no higher sampling rate. Note that in terms of slowness squared $\sigma$ the DMO equation 11 becomes

$$\sigma = \sigma_{stack} + \frac{k_y^{\,2}}{4\omega^2} \qquad (12)$$

The DMO correction thus becomes a pure shift along the $\sigma$ axis, with no stretching or shrinking, in which form it is particularly easy to implement.

## ESTIMATING SAMPLING DENSITY

We now turn to the question of how small an increment of $s$ or $\sigma$ must be used. Our basic goals are to be able to estimate the moveout velocity of hyperbolas as well as possible, and to have all significant events in the data appear identifiably on at least one stack. The two goals are, of course, intertwined. We can characterize the difference between two hyperbolas with the same zero-offset time $\tau$ but different moveouts by the time difference between them at a given offset. We wish, therefore, to ascertain how small $\Delta s$ or $\Delta\sigma$ must be to keep the corresponding $\Delta t$ below a given size. We will look at $\Delta\sigma$ first.

From equation 6 we can see that $\Delta t$ will be largest at small $t$ and large $h$. We can rewrite equation 6 as

$$\Delta t \approx \frac{\partial t}{\partial \sigma}\Delta\sigma = \frac{h^2}{2t}\Delta\sigma \tag{13}$$

For a specified $\epsilon$, if we want to ensure that $\Delta t < \epsilon$, we therefore need to ensure that $\Delta\sigma < \frac{2t}{h^2}\epsilon$. Hence we need to put a lower bound on the size of $\frac{2t}{h^2}$. This quantity will have a lower bound, even though $t$ can approach 0, since $h$ will also go to zero in such an event. For any non-zero offset $h$, the data are zero before some minimum time, so there is alway a minimum value for $t/h$. In practice, a mute is usually applied to data with $t/h$ less than some specified value to mute out direct arrivals and head waves. We will call this mute cutoff value $s_{mute}$. We now have the bound:

$$\frac{2t}{h^2} \geq \frac{2s_{mute}}{h} \tag{14}$$

If we let $h_{\max}$ represent the largest offset in our survey, we then get the bound

$$\frac{2t}{h^2} \geq \frac{2s_{mute}}{h_{\max}} \tag{15}$$

We thus require

$$\Delta\sigma \leq \frac{2s_{mute}}{h_{\max}}\epsilon \tag{16}$$

as an estimated bound on how finely we must sample $\sigma$ to ensure that neighboring moveout hyperbolas intercept the furthest offset at time spacing no greater than a specified amount $\epsilon$.

We have used $s_{mute}$ to indicate the direct arrival mute. The effective mute applied is often more extensive than this, since it is common to mute out events for which NMO stretch becomes excessive. The amount of NMO stretch will be given by $\Delta\tau/\Delta t$. We will approximate this by the derivative

$$\frac{\Delta\tau}{\Delta t} \approx \frac{\partial\tau}{\partial t} = t\left(t^2 - \sigma h^2\right)^{-1/2} = \frac{t}{\tau} \tag{17}$$

So if we require that $t/\tau < k$ for some constant $k > 1$, we find that the data not muted must satisfy

$$\frac{t^2}{h^2} \geq \frac{k^2\sigma}{k^2 - 1} \tag{18}$$

Then, as for the direct arrival mute, we get

$$\Delta\sigma \leq \frac{2\epsilon}{h_{\max}} \left( \frac{k^2\sigma}{k^2-1} \right)^{1/2} \tag{19}$$

This stretch mute, because it depends on $\sigma$, will only be significant for large $\sigma$, as we might expect.

Looking now at a bound for $\Delta s$ , we begin with

$$\Delta t \approx \frac{\partial t}{\partial s}\Delta s = \frac{h^2 s}{t}\Delta s \tag{20}$$

Using reasoning similar to that above, we find that if we use the bound

$$\Delta s \leq \frac{\epsilon}{h_{\max}} \tag{21}$$

then

$$\begin{aligned} \Delta t &\approx \frac{h^2 s}{t}\Delta s \\ &\leq \frac{h^2 s\,\epsilon}{t h_{\max}} \\ &\leq \frac{hs\,\epsilon}{t} \\ &\leq \frac{s\,\epsilon}{s_{mute}} \\ &\leq \epsilon \end{aligned}$$

Because we are using a wide offset approximation for which constant $\Delta s$ produces constant $\Delta t$ , we do not need to consider the actual values of the direct wave or stretch mutes in this case.

## SAMPLING FOR POST-STACK MIGRATION

The last factor we need to consider is the movement of events during migration. Successive DMO corrected stacks will be migrated at the corresponding velocities, and we want to be sure that energy does not move too far in either the lateral direction $x$ or in time-depth $\tau$. We need to express the movement of energy $\Delta x$ and $\Delta\tau$ as a function of the sampling interval $\Delta s$ or $\Delta\sigma$.

The desired relations may be derived either by a simple ray picture in the time-space domain, or by a stationary phase calculation based on the wave equation in the frequency-wavenumber domain. We will use the first approach here, and describe the second in the appendix. Suppose in our stacked data we have a small segment at $(x_0, t)$ with dip $\frac{dt}{dx}$ which we wish to migrate at the earth velocity $v$. We can "hand migrate" using the picture in figure 2 as a guide.

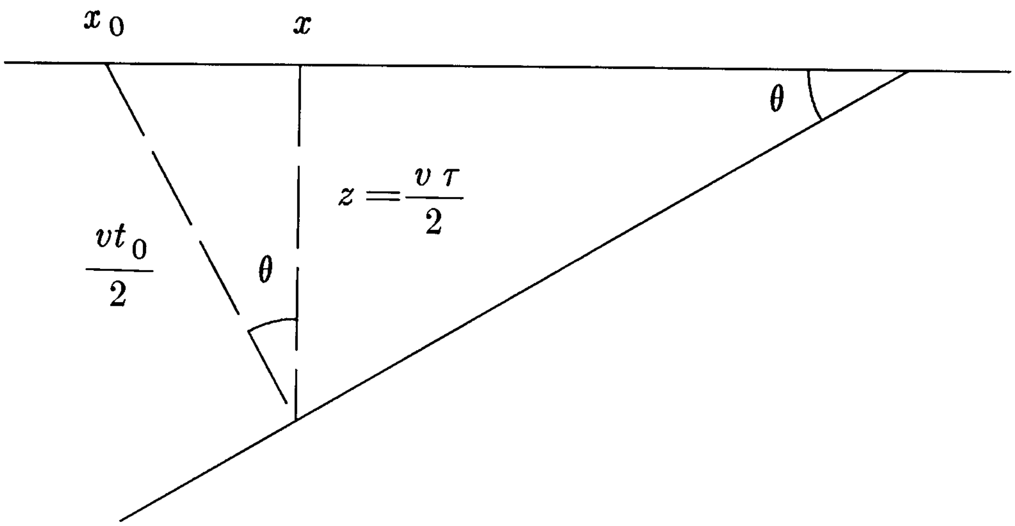



FIG. 2. Geometry of zero-offset migration in a constant velocity medium. A point at $(x, z)$ on a bed with dip $\theta$ appears on a zero-offset survey at $(x_0, t_0)$. Migration will reverse this process.

The apparent dip on the section $\frac{dt}{dx}$ will be related to the real earth dip $\theta$ by

$$\sin\theta = \frac{v}{2}\frac{dt}{dx} \tag{22}$$

The migrated depth $z$ of the event will be

$$z = \frac{vt}{2}\cos\theta \tag{23}$$

or, converted to time-depth $\tau = 2z/v$,

$$\tau = t\cos\theta \tag{24}$$

The lateral position $x$ of the migrated event will be given by

$$x = x_0 + \frac{v}{2} t \sin\theta \tag{25}$$

If instead we had migrated with a different velocity $v'$ we would have

$$(\tau', x') = \left( t\cos\theta',\ x + \frac{v'}{2} t \sin\theta' \right) \tag{26}$$

where

$$\sin\theta' = \frac{v'}{2}\frac{dt}{dx} \tag{27}$$

The difference between the positions of the event on the two migrated sections would then be given by

$$(\Delta\tau, \Delta x) = \left[ t\,(\cos\theta - \cos\theta'),\ \frac{t}{2}(v\sin\theta - v'\sin\theta') \right] \tag{28}$$

Substituting

$$\sin\theta' = \frac{v'}{v}\sin\theta \tag{29}$$

we get

$$\Delta\tau = t\left[ \cos\theta - \left( 1 - (\frac{v'}{v})^2 \sin^2\theta \right)^{1/2} \right] \tag{30}$$

$$\Delta x = \frac{vt}{2}\sin\theta \left( 1 - (\frac{v'}{v})^2 \right) \tag{31}$$

These relations may be written in terms of $\sigma$ and $\sigma'$ or $s$ and $s'$ and solved after extensive algebra for $\Delta\sigma$ or $\Delta s$ in terms of $\Delta\tau$ and $\Delta x$, but the exact expressions are too complicated to be of much use in obtaining bounds. For our purposes, it is adequate to drop all higher order terms in $\Delta\sigma$ or $\Delta s$ and use linearized approximations.

Working from either equations 24 and 25, or 30 and 31, we get for $\Delta\sigma$

$$\Delta\tau \approx \frac{\partial\tau}{\partial\sigma}\Delta\sigma = -\frac{t\sin\theta\tan\theta}{2\sigma}\Delta\sigma \tag{32}$$

$$\Delta x \approx \frac{\partial x}{\partial\sigma}\Delta\sigma = -\frac{t\sin\theta}{2\sigma^{3/2}}\Delta\sigma \tag{33}$$

From these, we can get the estimated bounds

$$\Delta\sigma \le \frac{2\sigma_{\min}}{t_{\max}\sin\theta\tan\theta}\Delta\tau \tag{34}$$

$$\Delta\sigma \le \frac{2\sigma_{\min}^{3/2}}{t_{\max}\sin\theta}\Delta x \tag{35}$$

where $\Delta\tau$ and $\Delta x$ are the largest intervals in $\tau$ and $x$ one can allow compatible with the temporal and spatial spectra of the data.

Similarly, for $\Delta s$ we have

$$\Delta \tau \approx \frac{\partial \tau}{\partial s}\, \Delta s \; = -\frac{t\, \sin\theta \tan\theta}{s}\, \Delta s \tag{36}$$

$$\Delta x \;\approx \frac{\partial x}{\partial s}\, \Delta s \; = -\frac{t\, \sin\theta}{s\, 2}\, \Delta s \tag{37}$$

From these, we can get the estimated bounds

$$\Delta s \;\leq\; \frac{s_{\,min}}{t_{\,max}\, \sin\theta \tan\theta} \Delta \tau \tag{38}$$

$$\Delta s \;\leq\; \frac{s_{\,min}^{\,2}}{t_{\,max}\, \sin\theta} \Delta x \tag{39}$$

There will be no minimum $\sigma$ or $s$ for stacking, but after DMO there will be a minimum over which we need to migrate accurately. Note that $x$ movement is nicely bounded at steep dips, but no $\sigma$ sampling can be fine enough to contain movement of energy in $\tau$ at near vertical dips.

## EXAMPLES AND DISCUSSION

Previously I demonstrated the velocity space DMO and migration algorithm using as an example a portion of a data set from the U.S. Gulf Coast (Fowler, 1984). The image was extracted from a series of stacks using a constant velocity sampling interval of 0.03 km/s to cover a range from 1.5 to 2.7 km/s. Let us apply our estimated bounds and see how many stacks we would calculate are needed to properly handle this data set. The relevant parameters are: maximum offset $h_{\,max}$ is 3.55 km, maximum time used here is 3 seconds, maximum slowness used is water slowness 0.67 s/km, which is also the mute slowness, and the NMO stretch mute constant $k$ is 2. The data is sampled at 4 msec, and midpoint spacing is 33.5 m. If we set the maximum allowable time shift $\Delta t$ to the sampling interval, then data below 62 Hz will be shifted less than a quarter wavelength and will effectively still add in phase. These data contain little useful information with frequency content that high. Applying the inequality estimate 16 with $\epsilon$=0.004 we find that we need $\Delta\sigma \leq 0.0015$. If we were to cover the slowness range from 0.0 to 0.67 using slowness squared sampling this implies that we need 296 stacks. We can improve on this a little by allowing for the effects of the NMO stretch muting. Using $\sigma$=0.44 and $k$ =2 in the inequality 19, we get the estimate $\Delta\sigma \leq 0.00174$, which reduces the number of stacks to 256. If we now apply this last sampling rate in the bounds given by 34 and 35 with minimum migration $\sigma$=0.189, which corresponds to a velocity of 2.3

km/s, we find that $\Delta x$ is less than the midpoint spacing for all dips, but $\Delta \tau$ is held under the limit 0.004 only for dips up to about 30 degrees.

Considering sampling evenly in slowness $s$ instead of its square $\sigma$, we can apply relation 21 to get the estimate $\Delta s \leq 0.00113$ which implies we should use about 596 stacks. Looking at the migration estimates, equations 38 and 39, we find that this sampling rate is no problem for $\Delta x$, but limits dips to about 39 degrees. The higher number of stacks implied suggests that we are better off using $\sigma$ sampling.

The actual sampling used in that paper was $\Delta v = 0.03$ km/s, which corresponds to a range of $\Delta\sigma$ from 0.0031 to 0.0173. This is up to an order of magnitude larger than what we estimate as acceptable. The results shown in the aforementioned paper, however, appear quite acceptable, and make us suspect that our estimates are much too harsh. In particular, the extracted stack was compared with a conventional variable velocity NMO and stack, with only very minor differences visible, so the sampling rate could not have been too unreasonable. A larger portion of the same data set is shown in Fig 3. It was extracted from a suite of 88 stacks after DMO and migration. The sampling was at an even rate of $\Delta\sigma$=0.005, about 3 times larger than the $\Delta\sigma$ size calculated above. The steeply dipping fault plane reflections are handled well, and the image on the whole is fairly good. The obvious loss of signal at early times is not a sampling problem; it occurred because no normalization was applied at early time to the stacks to account for the fewer number of traces being summed.

Our estimates are of course worst case upper bounds; we have not shown that using a coarser sampling rate will introduce unacceptable errors. In fact, there is good reason to believe that these estimates are too harsh by a large margin; this author's experience has been that for several data sets, somewhere between 50 and 150 stacks has usually appeared to be adequate. A more careful look reveals one place we have overestimated: our criterion for stacking rate may be much too limiting. When shifts larger than $\Delta t$ are allowed, the errors are not usually major, and can be corrected by proper interpolation. Even with a large shift at the outermost offset, the principal effect is a decrease in amplitude in the stacked event.

As a test I generated a synthetic model comprising an event with velocity 4 km/s, zero-offset time of 0.47 seconds, and a maximum offset of 4.7 km. The wavelet used was a 10 to 60 Hz Butterworth bandpass filter response. Applying our stacking estimates suggests that we need a sampling rate of $\Delta\sigma \leq 0.00042$. The data of figure 4 represent 200 $\sigma$ samples at this sampling rate; the plot shows the stack amplitude at the zero-offset time 0.47 seconds against the $\sigma$ used for stacking. Obviously, our estimates not withstanding, we have grossly oversampled in $\sigma$. Figure 5 shows the same data

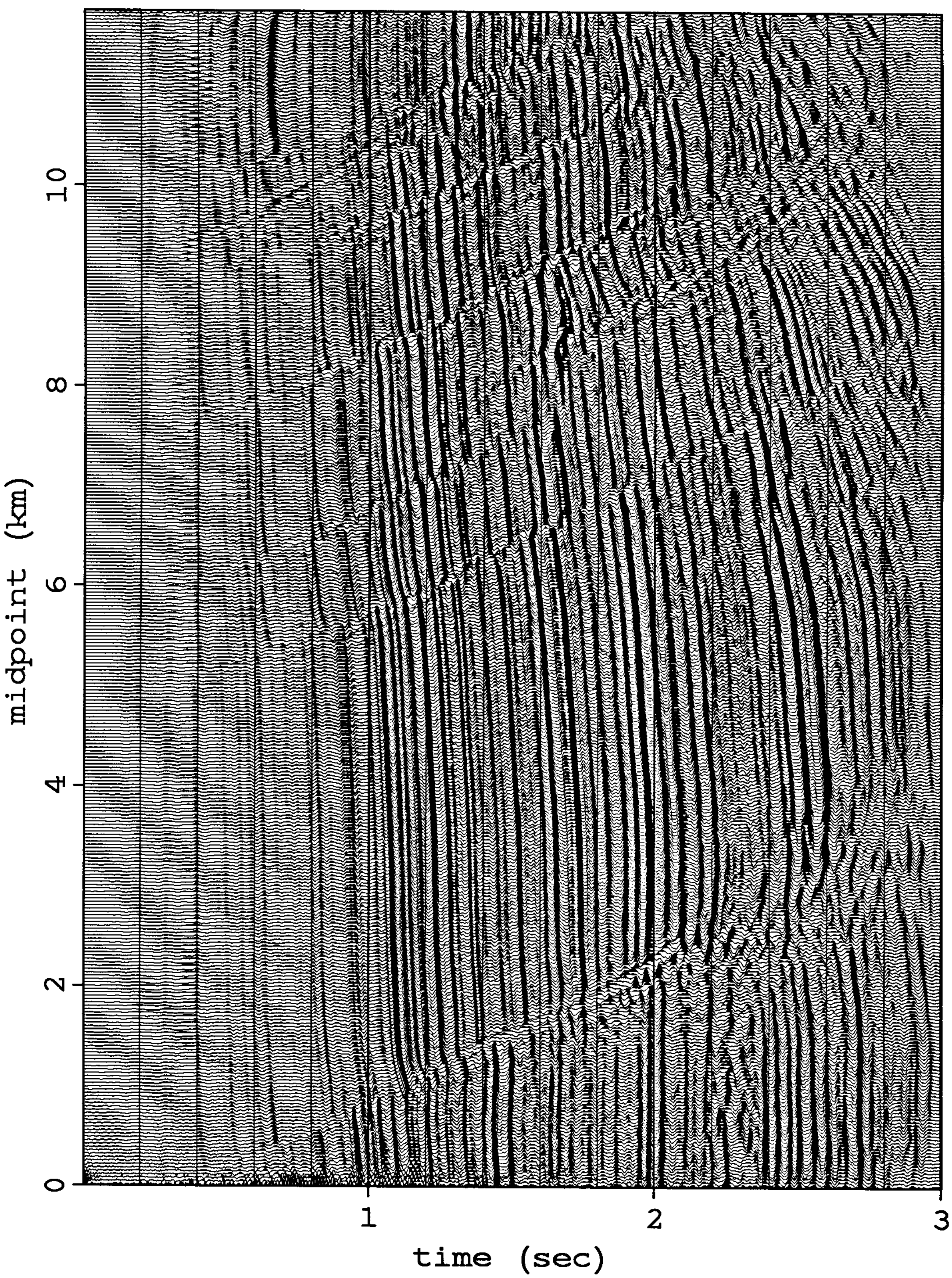



FIG. 3. Portion of a data set from the U.S. Gulf coast, extracted from a suite of DMO corrected and migrated stacks sampled with $\Delta\sigma$=0.005. Weakness of signal at early time is caused by lack of normalization of stacks to account for fewer traces.

undersampled by a factor of 8, without noticeable loss of definition of the peak. The peak is down in amplitude, but can still be recovered readily by good interpolation.

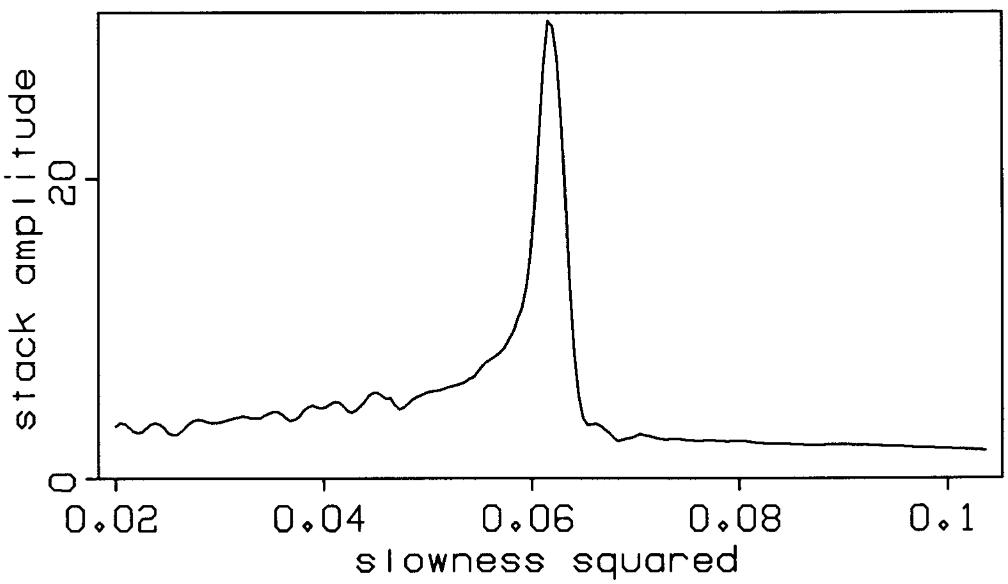



FIG. 4. Amplitude against stacking $\sigma$ for a synthetic model event.

Clearly, these examples do not tell the whole story, but they do suggest that in practice our proposed criteria for estimating the number of stacks needed may be too strict. This paper must end, then, on a somewhat unsatisfactory note. We have derived estimates of the number of stacks which we can be pretty sure will enable us to reconstruct an image well. The answers these estimates provide seem significantly larger than prior experience would suggest, large enough, in fact, to make the method economically suspect. What we have not answered yet is whether we have been undersampling and not recognizing the resultant errors, or whether we have overestimated the number of stacks needed due either to overstrict criteria, or to mathematical error.

## ACKNOWLEDGMENTS

I thank Francis Muir, who first suggested to me the efficacy of sampling in slowness squared, and whose ongoing suggestions, support and criticism have been invaluable. I

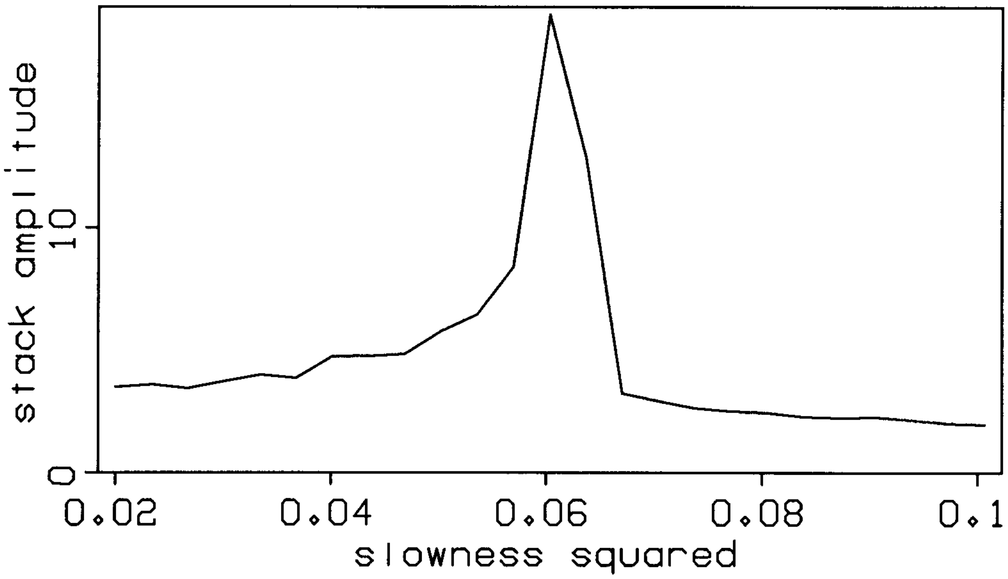



FIG. 5. Amplitude against stacking $\sigma$ for a synthetic model event. This plot is the same as figure 3, but the sampling rate for $\sigma$ is one eighth that used in the previous figure.

also thank Dan Rothman for critical reading of parts of this paper.

## REFERENCES

Claerbout, J.F., 1985, Imaging the earth's interior: Blackwell, London

Fowler, P.J., 1984, Incorporating dip corrections in velocity analysis using constant velocity stacks: SEP-39

## APPENDIX

Post stack migration can be written as

$$p(x,t=0,z) = \int d\omega \int dk_x\, e^{ik_x x + ik_z(\omega,k_x)z}\, p(k_x,\omega,z=0) \tag{A1}$$

Suppose we consider a spike at $(x_0,t_0)$ in our recorded data, so

$$p(x,t,z=0) = \delta(x-x_0,t-t_0) \tag{A2}$$

But

$$p(k_x,\omega,z) = \int dt \int dx e^{i\omega t - ik_x x}\, \delta(x-x_0,t-t_0) \tag{A3}$$

$$= e^{i(\omega t_0 - k_x x_0)} \tag{A4}$$

So the migration equation A1 becomes

$$p(x,t=0,z) = \int d\omega \int dk_x \, e^{ik_x(x-x_0)+ik_z(\omega,k_x)z+i\omega t_0} \tag{A5}$$

The principal contribution to this integral will come from the stationary phase path, and we may regard this curve as describing (in a high frequency asymptotic sense) the location of the energy in $(x,z)$ after migration (cf. Claerbout, 1985). We can write the phase function as

$$\Psi(\omega,k_x) = k_x(x-x_0)+k_z(\omega,k_x)z+\omega t_0 \tag{A6}$$

Then we find the stationary phase curve by setting the derivatives of $\Psi$ to zero.

$$0 = \frac{\partial \Psi}{\partial \omega} = z\frac{\partial k_z}{\partial \omega}+t_0 \tag{A7}$$

$$0 = \frac{\partial \Psi}{\partial k_x} = z\frac{\partial k_z}{\partial k_x}+x-x_0 \tag{A8}$$

Now

$$k_z = -\left(\frac{4\omega^2}{v^2}-k_x^2\right)^{1/2} \tag{A9}$$

where $v$ is the migration velocity. Then

$$\frac{\partial k_z}{\partial \omega} = \frac{-4\omega}{v^2 k_z} \tag{A10}$$

$$\frac{\partial k_z}{\partial k_x} = \frac{-k_x}{k_z} \tag{A11}$$

So we get a pair of equations for $x$ and $z$

$$0 = t_0-\frac{4z\omega}{v^2 k_z} \tag{A12}$$

$$0 = x-x_0-\frac{zk_x}{k_z} \tag{A13}$$

Let

$$\sin\theta = \frac{vk_x}{2\omega} \tag{A14}$$

$$\cos\theta = \frac{vk_z}{2\omega} \tag{A15}$$

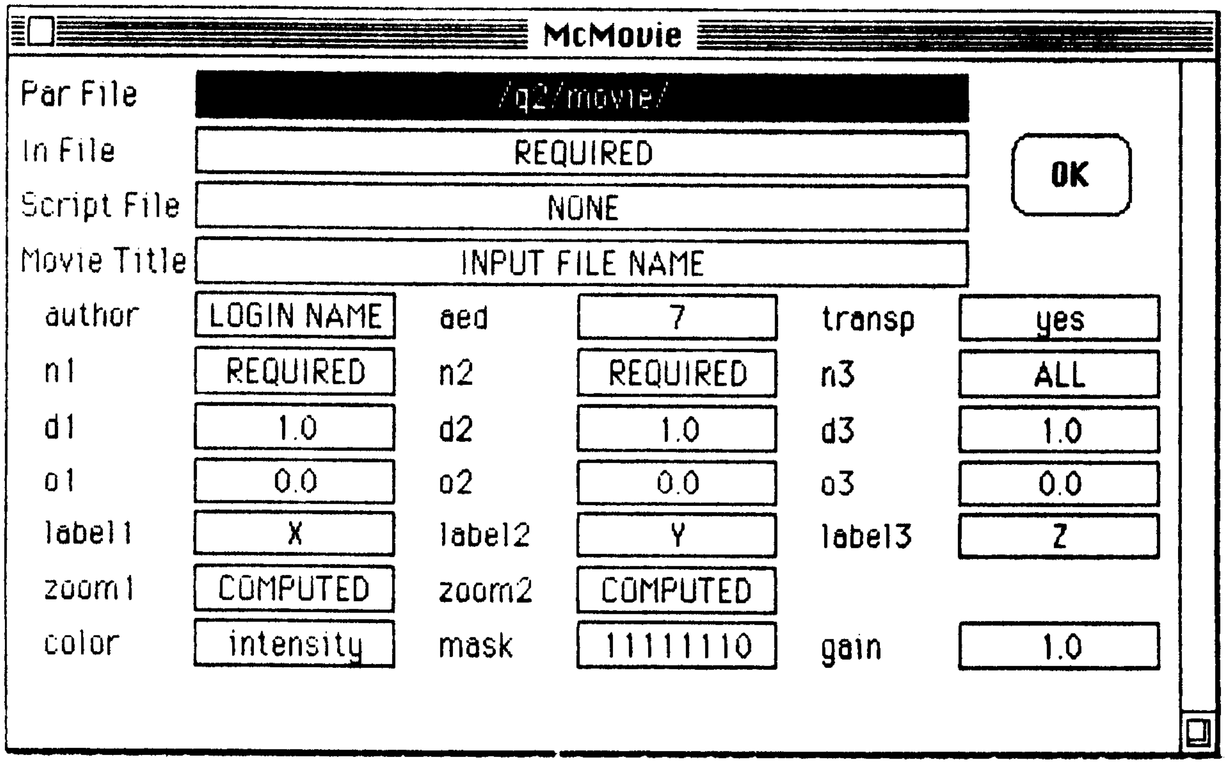
$$\tan\theta = \frac{k_x}{k_z} \tag{A16}$$

Then equations A12 and A13 for $(x,z)$ can be parametrized by $\theta$ as

$$z = \frac{vt_0 \cos\theta}{2} \tag{A17}$$

$$x = x_0 + \frac{vt_0 \sin\theta}{2} \tag{A18}$$

These agree with the geometric results of equations 23 and 25 in the main text.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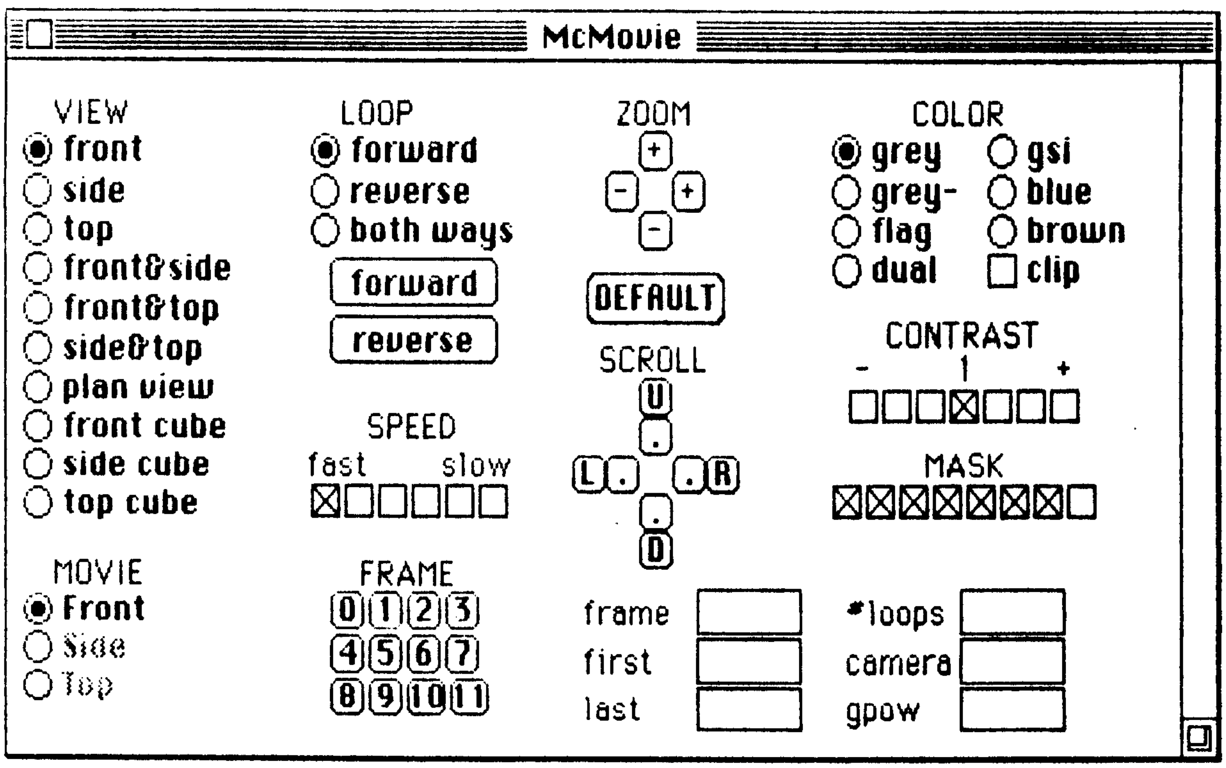
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