Velocity space imaging: formalism, methods, and prospects

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INTRODUCTION

In previous papers I have discussed methods of imaging seismic reflection data after transformation to a velocity parametrized space (Fowler, 1984a, 1984b). Related techniques have also been suggested by others (Shurtleff, 1984; Gray, 1985; Chun and Jacewitz, 1978; Tieman, 1984). The purpose of such methods is to draw more closely together the steps of velocity analysis and reflector imaging. In this paper I want to discuss a formalism for understanding these efforts as approaches to approximate inversion. I hope to contrast these velocity space methods with other inversion methods, and to suggest ways in which to extend their range of applicability. The reader should be forewarned that I will introduce a certain amount of formal notation, but that many points deliberately will be left fuzzy in definition; the following is in not at all a rigorous theory, but more a heuristic framework for understanding a perhaps unconventional approach to extracting useful images from seismic data.

MODELS AND DATA

Seismic data as it is commonly collected is a function of midpoint x, offset h, and time t. I will restrict the discussion here to two dimensional line surveys; most of what I will say carries over to three dimensions readily (until one goes to implement it!) The data may represent pressure, the vertical component of velocity, or some similar function; in any case we may consider the set of all possible data sets which could be recorded by a fixed survey geometry recording over any possible subsurface geology as some subset $\mathbf{D}$ of a space $\mathbf{F}(R^3)$ of functions defined on three real variables. If we were mathematical pedants instead of geophysicists, we might want to limit our function space by insisting on continuity or some such restriction. Throughout this paper I will ignore many such technicalities.

Our model, that is, the set of earth parameters we choose for characterizing the earth over which we run our seismic survey, could contain many variables. For the purposes of this paper, I will restrict attention to only two, and will use velocity v and reflectivity r as the two parameters conventionally extractable from normal seismic data. I will assume that what we really want from the data is first and foremost a somewhat vaguely defined quantity called "the position of the reflectors"; I will not deal further with just how this is really related to amplitudes of reflected seismic waves or to the acoustic impedance or reflectivity of the earth, except to say that I tacitly (and admittedly circularly) assume that the source waveform has been removed, ignore band limiting, and treat reflectivity operationally as "that which migration solves for". The model space $\mathbf{M}$ then comprises functions of the form $m(x,z)=[v(x,z),r(x,z)]$ where x and z are the midpoint and depth, respectively. The model space $\mathbf{M}$ is thus a subset of a function space $\mathbf{F}(R^2)\times\mathbf{F}(R^2)$

SEISMIC EXPERIMENTS AS MAPPINGS BETWEEN FUNCTION SPACES

Performing a seismic survey with a given geometry of midpoints and offsets will associate with any particular subsurface geology a data set $d \in \mathbf{D}$. We can't pretend to know all the details on all scales of the physics that was involved in producing these data, but we hope that we have adequately chosen a good set of model parameters so that this real data set is somehow "close" to a data set which could be produced by a good enough theoretically simulated experiment with some model $m \in \mathbf{M}$ as input. That is, in reality there exists a "mapping" which takes each real subsurface geology onto the collected data; in our theory and programs there will exist a forward modeling scheme which takes models onto data. Our ability to extract physically meaningful or useful earth parameters from the data clearly can be no better than the degree with which our modeling theory emulates all the details of the physical processes.

Let us assume then that we have settled on the best and most appropriate forward modeling equation or algorithm, and trust it to adequately describe the principal features of our seismic experiment. Designate this mapping from the function space $\mathbf{M}$ to the function space $\mathbf{D}$ by Ψ. In order to discuss inverting the action of Ψ we have to know that the operator is formally invertible, which requires that it take each model to a distinct data set, and that every data set is the image of some model. The latter may well not be true. (Does every pattern represent the outcome of a possible seismic experiment? Would you get suspicious if the words "DRILL HERE" appeared as a pattern of reflectors on your brute stack?) This problem is easily fixed; we simply restrict attention to the range $\Psi(\mathbf{M}) \subset \mathbf{D}$. Real seismic data may well not fall in this space if our chosen model parameters or modeling theory are inadequate, and we must trust that any field data set we wish to invert will be "close" to some $d \in \Psi(\mathbf{M})$. Our Ψ operator should not map two dissimilar models onto the same data (except possibly for trivial cases, such as two different velocities but no reflectors at all, which would both give zero data). With real, band limited, noisy data there will of course be problems of poor resolution and null spaces, but we will take our operator still to be formally one-to-one, or redefine our model space to exclude unresolvable models.

We then take Ψ^{-1} to be formally well defined, and ask what properties such an operator should have. We certainly want to believe that two nearly identical data sets correspond to very similar models, that is, we want Ψ^{-1} to be a continuous operator. The continuity of the forward operator is also desirable, but we recognize that at some points such as critical angles or caustics, we will probably lose at least uniform continuity; small perturbations in the model might give rise to large changes in the data.

Defining continuity of course requires definition of metrics in both spaces. For our purposes we will assume that the metric induced by a standard norm such as L_2 is adequate in the data space. It is not obvious that such a norm is very appropriate in the model space, since the difference of two simple models which appear to differ only slightly can have the same norm as two models which differ radically. (Think of two compact scatterers, or two simple planar reflectors. The norm of their difference is independent of the distance between them provided they do not overlap.) Perhaps it is better to define the norm in the model space by using Ψ^{-1} to pull back the norm in the data space, thus making Ψ^{-1} continuous by definition.

RESTRICTING THE OPERATOR TO SIMPLE VELOCITY FUNCTIONS

In general a good Ψ is often either not well known or is very expensive to compute even approximately, and Ψ^{-1} may not be not known at all. One approach to inversion is to assume that we know a model m_0 fairly close to the desired $\Psi^{-1}(d)$, and try by iterative forward modeling to find the m close to m_0 for which $\Psi(m)$ is closest to d. Another approach is to try to find an approximate Ψ^{-1} and apply it directly to the data; usually such efforts have to assume that $v(x,z)$ is a perturbation around a known, simple function. For a further discussion of inversion methods, see Al-Yahya (1984) and the references therein. The approach we will take here is to select a whole family of approximations to Ψ^{-1} and recompose the image after applying each of the approximate inverse operators to d. We hope than that the recomposition provides us either with a good inversion itself, or at least a with a better starting point than might otherwise be available for one of the other types of inversion methods which require a good first guess to begin with.

Suppose we define some class of simple velocity functions $\mathbf{V}$. Then for each $v \in \mathbf{V}$, we can designate the restriction of Ψ to $v \times \mathbf{F}(R^2)$ by Ψ_v. That is, Ψ_v is the forward modeling operator for a specific velocity function. We can now define an inverse operator Ψ_v^{-1} which takes the subset of the data space data $\Psi_v(v \times \mathbf{F}(R^2))$ back to the the subset of model space $v \times \mathbf{F}(R^2)$ characterized by the particular velocity function v. If we have chosen $\mathbf{V}$ well, we may be able to specify Ψ_v^{-1} analytically or algorithmically. We can then extend the inverse operator Ψ_v^{-1} to an operator Φ_v defined on the entire data space $\mathbf{D}$ by formally applying that same analytic or algorithmic solution to any data set. We cannot in general expect that $\Phi_v(d)$ will be close to $\Psi^{-1}(d)$ for an arbitrary d unless d is close to $\Psi_v(m)$ for some $m \in v \times \mathbf{F}(R^2)$. However, by exploiting linear properties of the operators, we can reconstruct a good approximation to $\Psi^{-1}(d)$ from the various $\Phi_v(d)$.

RECOMPOSITION OF AN IMAGE

The forward modeling operator Ψ is in general not linear in velocity. To see this, consider a single point diffracter modeled at two different velocities. The superposition of the diffraction hyperbolas is not the same as forward modeling at the sum of the velocities. However, Ψ will be linear in reflectivity, and hence the restricted operators Ψ_v will be linear, since they are not functions of velocity. Therefore we can take the inverse operators Ψ_v^{-1} and their extensions Φ_v to be linear. We can then think of decomposing the space (x, z) into small subsets α, forward modeling the reflectivity function defined on each subset, and then adding together the images to reconstruct the complete data set. Our approach to inversion then is to consider each subset α and try to find the velocity function and reflectivity function which best match the data on that subset, then piece together these partial models to create a whole one.

Applying each of the Φ_v in turn produces a space which has the spatial axes x, z, and the velocity function v as coordinates. For the spatial axes practical application, we will assume that the reference velocity functions used can be described by one or more simple parameters. This transformation produces what I call generically a velocity space. Our problems now are to specify how to extract an image from the velocity space given a general velocity function, and how to decide what velocity function to use. Suppose we specify some general velocity function v_0 which is not necessarily in our special set $\mathbf{V}$. For each subset α, select the velocity function $v \in \mathbf{V}$ which is "closest" to v_0 on α. For any particular reflectivity function r, let $d_r = \Psi(v_0, r)$. If v_0 is not in $\mathbf{V}$, $\Phi_v(d_r)$

may not look much like the reflectivity function $r(x,z)$. However, if we restrict our attention solely to the subset α on which v is close to v_0, by the continuity of all the functions involved, we can approximate Ψ^{-1} by Φ_v, so $\Phi_v(d)$ will be an approximation to the reflectivity on α. We can repeat this for each subset α and build up a reflectivity function defined everywhere.

We have shown how to generate the best combination of Φ's for a specified velocity function v_0. However, we have again been very cavalier with the word "close", this time with reference to comparing velocities. What we really mean by two velocity functions v and v_0 being "close" on a subset α is that, for a given reflectivity function r, $\Psi[v(\alpha),r(\alpha)]$ and $\Psi[v_0(\alpha),r(\alpha)]$ are close in the data space metric. The relation between the actual wave propagation velocity and the best v_0 to use may be quite complicated; in solving for depth variable velocity functions using constant velocity migrations for the Φ_v operators, the relation is known to be approximately a root-mean-square averaging of interval velocities in overlying layers. In fact, our goal is not really to be able to go from a specified velocity function to a reflectivity image, since we have fairly good migration techniques to do this already. Rather, we want to be able to use our suite of reference velocity function images, $\Phi_v(d)$, to find a good estimate of velocity as well as reflectivity. To do this we need to appeal to the fact that our seismic experiment contains multiple offsets, the information from which will be combined by stacking together (over offset or offset wavenumber) at some point. Imaging at the correct velocity should maximize the redundancy, or agreement between the information recovered from the various offsets, and so should maximize the power present in the final image. This is basically same idea as underlies most conventional velocity analysis. In fact, if we take our modeling scheme Ψ to be ray tracing in stratified media, and our approximate inverse operators Φ_v to be normal-moveout and stack at a constant velocity v, our scheme essentially reduces to standard velocity analysis.

Most of our velocity information comes from the optimizing the power in the resulting image. Beyond this, however, there are other criteria, such as focusing of diffractions and insuring continuity of reflecting horizons, which could be incorporated. We may well want to include a priori constraints on the allowable velocity functions, too, based perhaps on our knowledge of the limitations of our migration operators. Finally, we may want to include some (subjective) judgements on the geological feasibility of the results.

into a modified velocity stack algorithm except by the (expensive) sequence: NMO, DMO, stack, phase shift migrate, each over a range over specified depth variable velocity functions. Note, however, that if a reasonable guess is known to the real depth variable velocity function, a rather limited range of velocity functions around the first guess might be all that was needed. Moreover, the velocity functions could "fan out", with the velocity sampling interval increasing with depth where velocity resolution decreases. Alternatively, one could perform pre-stack phase shift migration with a similar fan of velocities. The result of either method could be expected to be more accurate than using only constant reference velocities, since some allowance is made for the non-hyperbolicity of diffractions in media with vertical gradients. Also, it might be possible to use substantially fewer velocity functions, provided one makes any sort of reasonable guess about the increase of velocity with depth. This decrease in number of velocities needed might, in the case of pre-stack phase shift migration, partly or fully compensate for the increased cost of the phase shift over the Stolt algorithm. In using either of these algorithms over a range of velocity functions, of course, the forward three dimensional Fourier transform of the data need only be done once.

A bigger challenge is to move beyond assumptions of lateral velocity invariance. All of the methods discussed or proposed so far have been time migrations only, and will weaken and misposition events if velocity varies laterally substantially. To attack this problem at all, laterally varying reference velocity functions would be required. It is not at all clear to me at this point, despite a fair amount of thought, just what the best set of such velocity functions might be. Perhaps the simplest idea is to use velocity functions with a constant lateral gradient in velocity or slowness. The velocity function then is specified at each point by two parameters, magnitude and lateral gradient, rather than only one. For a specified velocity function one would have to read off not only the value of the velocity at a point, but also estimate from its neighbors the local value of the lateral derivative. Implementation would require then, either deriving and applying a pre-stack migration operator which explicitly was designed for known laterally variable media, or figuring out the effects of simple lateral variation on stacking, DMO, and post-stack migration.

Reference function imaging methods can probably be automated in part or whole. This basically requires designing designing a good algorithm for automatic picking of a maximum power velocity function. This is an optimization problem, probably including reasonable constraints on the allowable nature of the velocity function such as limiting the rapidity of the allowable lateral velocity change. For the case of the constant reference functions, this is just automating what a person does in picking velocities. For laterally varying velocities with more than one parameter describing each reference function at a point, an automatic picking scheme would be almost mandatory, since few people are good at scanning through spaces of dimension of four or more dimensions, e.g. $(x, \tau, v, dv/dx)$. Any perturbation of such an automatically picked function could, of course, be tested by the interpreter by rapid re-interpolation of the reference images.

The real question, however, is not whether extending existing velocity space methods is possible (at least approximately), but whether the results would justify the extra effort and cost. I do not believe that one could in general obtain a satisfactory depth migration this way in the presence of arbitrary lateral velocity variation. What might be hoped for is a significant improvement in the strength and positioning of the events in the stack, that is, a better starting point for further inversion efforts. Specifically, iterative forward modeling methods normally require identification and digitization of a limited set of major reflectors which can be used for ray tracing, as do tomographic velocity analysis methods. Events that are never identified because of poor prior imaging cannot be accounted for this way, and may possibly degrade or destabilize the quality of the result. The point here is that reference function methods perhaps can be nearly, or fully, automated, to provide a good starting image for expensive or time consuming iterative methods. Additionally, for cases in which the lateral velocity variation is not too large, these methods can provide both a good velocity analysis and a good pre-stack time migration simultaneously.

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