Erratum/solution to the asymmetric ellipse paradox

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ERRATUM

Offset extrapolation (p. 157 of SEP-38)

Consider the equation

$$P_0(\omega_0,k_x,h) = \int dt_n \; A^{-\alpha} e^{-i\omega_0 A t_n} p_n \; (t_n,k_x,h) \tag{1}$$

where $P_0(\omega_0,k_x,h)$ is the the Fourier-transform of the extrapolated zero-offset section, $P_n(t_n,k_x,h)$ is a constant-offset section (after NMO and Fourier Transform over x), A is $1+h^2k_x^2/\omega_0^2t_n^2$, and α is unknown because the derivation of equation (1) is kinematic. If the extrapolated zero-offset section P_0 is independent of the offset then for all ω_0 and k_x we have

$$0 = \frac{\partial}{\partial h} P_0(\omega_0,k_x,h) = \frac{\partial}{\partial h} \int dt_n \; A^{-\alpha} e^{i\omega_0 A t_n} \; P_n(t_n,k_x,h) \tag{2}$$

For all t_n, k_x and ω_0 we have,

$$0 = \frac{\partial}{\partial h} \left[A^{-\alpha} \; e^{i\omega_0 A t_n} \; P_n(t_n,k_x,h) \right] \tag{3}$$

Differentiation of equation (3) gives

$$\frac{\partial P_n}{\partial h} = \left(-\alpha A^{-1} + i\omega_0 t_n \right) \frac{\partial A}{\partial h} \; P_n \tag{4}$$

which can be solved by

$$P_n = C \; A^{-\alpha} e^{i\omega_0 A t_n} \; .$$

With a particular choice of the constant C and rearranging we have

$$P_0(\omega_0,k_x;t_n) = A^{\alpha} \; e^{-i\omega_0 [A \; - 1] t_n} P_n(t_n,k,h) \; , \tag{5}$$

that † closes a circle back to equation (1).

This shows that the DMO produces the same zero-offset section from any constant offset section (ignoring noise and aliasing). Therefore, there is no hope that data from other offsets will correct the asymmetry and produce a symmetric stack (Levin 1984).

Sine factor (p. 154 of SEP-38)

The result (2-4) is correct only for two dimensions. For three dimensions we should have for the pseudo reflection coefficient:

$$R(\theta) = \frac{\sin\phi d\,\phi}{\sin\theta d\,\theta}$$

This error has no importance to the paradox because the reflector will still be asymmetric for two and three dimensions.

THE PARADOXICAL ELLIPSE

The asymmetry of the elliptical reflector (Ronen 1984, equation (2-4)) was derived by requiring an isotropic (circularly symmetric) wave diverging from the shot, to be reflected by a certain earth model, *Model*−1, as an isotropic wave converging at the receiver. This is not the same as requiring that a certain earth model, *Model*−2, will produce a constant-offset section which has only one spike. *Model*−1 was found by Stew and myself to be asymmetric. It is easy to see that *Model*−2 is symmetric: we record a spike when the shot and receiver are in the foci of the ellipse, shifting the shot-receiver pair (adjacent traces on a constant offset section) infinitesimal distance to either side we record the same only if *Model*−2 is symmetric. Asymmetry is excluded although the reflection coefficient of *Model*−2 will not be uniform.

CONCLUSION

There is no reason for an asymmetric DMO.

ACKNOWLEDGMENTS

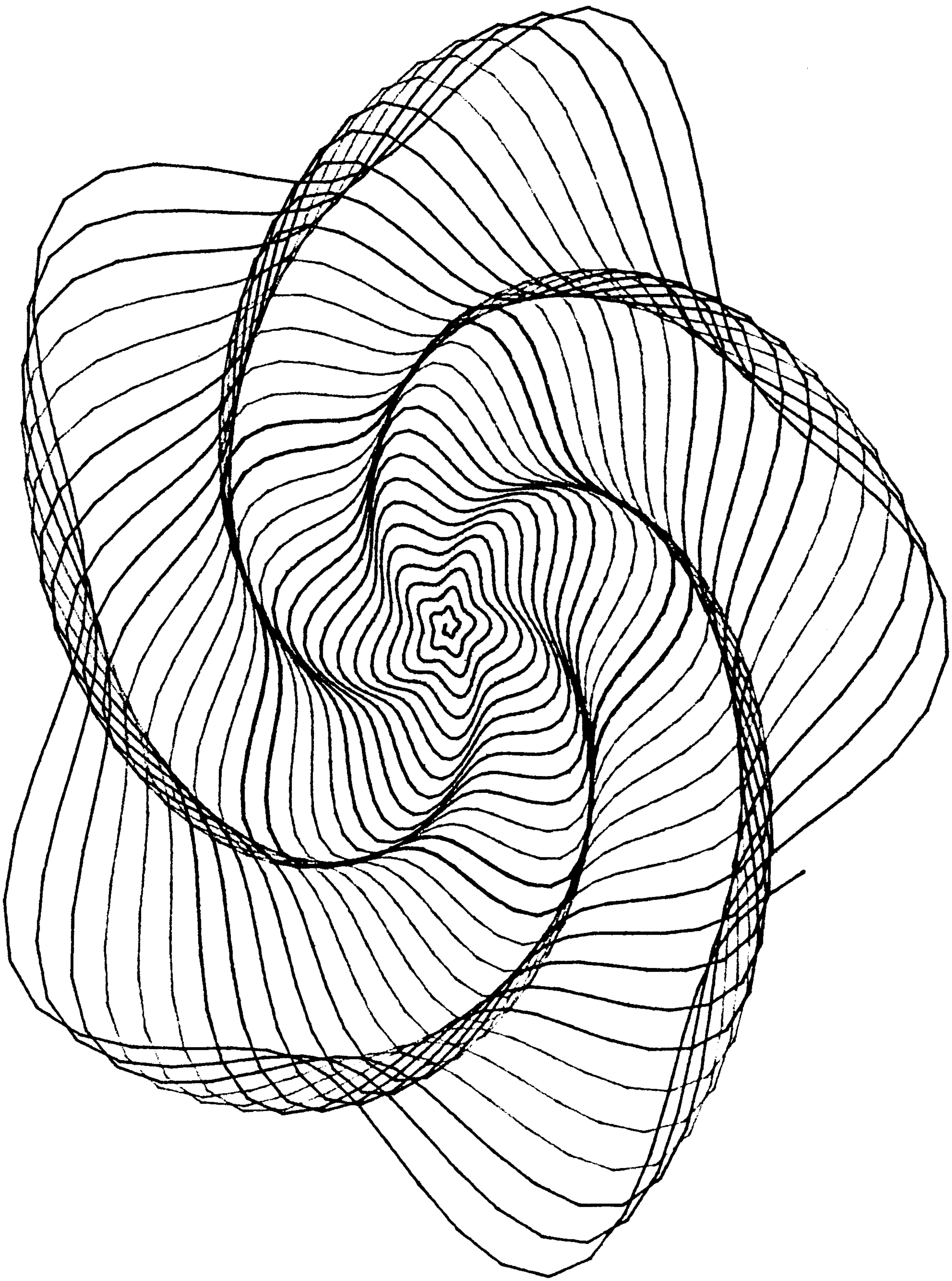
The solution to the DMO asymmetry problem came from Fabio Rocca. Helmut Jakubowicz found the algebraic error regarding the offset extrapolation.

† t_n appears on the left side because the operator is time dependent and should be applied by equation (1):

$$P_0(\omega_0,k_x) = \int dt_n\, P_0(\omega_0,k_x\,;t_n)$$

REFERENCES

Levin, S. 1984, The paradoxical elliptical reflector, SEP-38, p. 361-367.
Ronen, S. 1984, Kinematics and dynamics of DMO, SEP-38, p. 151-158.



Erratum:

SEP-38, page 318, second paragraph.

For each iterative re-estimate of the signal we shall choose to linearize the nonlinear transformation $\overline{f}$ assuming the new estimate to be a small perturbation of the previous one, $\overline{s}_0$. We shall see that his linearization greatly simplifies the transformed statistics and their estimation. Because of the central-limit theorem, components with gaussian m.p.f.'s yield gaussian m.p.f.'s after linear transformation. Thus, gaussian signal and noise remain indistinguishable as a third useless component, hereafter called gaussian noise. In the previous section we found that, for gaussian noise, and for a linear, or linearized, $\overline{f}$, the MAP inverse becomes the l.s. inverse, made as a series of linear **transformations, (7) and (8).** If we iteratively extract (subtract) both nongaussian signal and noise from the data, then the l.s. inverse will approach the optimum MAP inverse. A linearized $\overline{f}$ will become increasingly accurate as signal perturbations decrease in magnitude.