Suppressing wraparound in $\omega - x$ migration

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INTRODUCTION

While preparing an abstract on reducing wraparound artifacts in phase-shift migration for the upcoming SIAM-SEG-SPE conference in Houston, I started thinking about a comparison with wraparound suppression techniques in ω–x finite-difference migration. Aside from zero-padding, the only method that I know to have been successfully employed was introduced in SEP-15 by Kjartansson. It was implemented in his monochromatic migration program by subtracting the reflector imaged at $t = 0$ from the wavefield after each downward continuation step.

WHY IT SHOULD WORK

There are two rationales for removing $t = 0$. The first honors the exploding reflectors model; reflectors turn on at $t = 0$ during upward extrapolation so one turns them off at $t = 0$ during downward extrapolation. Recognizing that the discrete Fourier transform (DFT) has wrapped the original time axis around a cylinder which is rotated backwards in time during downward extrapolation, the second view is to erase the section as it passes into history $\equiv$ the future. Both are implemented by subtracting the $t = 0$ slice from the wavefield after each depth step. Are these reasons valid?

The former makes physical sense regardless of the extrapolation domain or method. If the wavefield really did arise from exploding reflectors then it could be modeled thusly: start with a time section of zeros below the deepest reflector in your model. As you upward continue towards the surface, add the reflector at each z level to the corresponding $t = 0$ time slice of the wavefield. When you reach the surface, you're done. Kjartansson's scheme (ignoring attenuation and dispersion) simply reverses this procedure to accomplish migration. While one may cavil that the time section is not identical to an exploding reflectors section, this argument at the very least indicates that

Kjartansson's method shouldn't do any harm. The other argument is germane to DFT wraparound. You don't want data to move across time zero and so you place a vacuum cleaner at $t = 0$.

Something is not quite right. Suppose we downward continue one 10 meter step and compare it to downward continuing 10000 steps of one millimeter. Leaving aside numerical dispersion, the two wavefields must be quite different. In the former case we've subtracted out a single $t = 0$ slice; in the latter we've subtracted 9999 of them. When the seismic wavelength is much greater than 10 meters, these 9999 time slices will add in phase. We see that at the very least we are missing some normalization factors.

Reexamining this argument, we can picture the same difficulty even more clearly if we model upwards with 10 meter steps or one millimeter steps. The amplitudes on the latter time section would be about 10000 times stronger than those on the former. Ouch.

WHAT IS REALLY GOING ON

To take a depth step in finite-difference migration one applies some linear operator to the data. The response of this operator is a Kirchhoff-like weighted summation of the values in the time section we are extrapolating. The amount energy can move during an extrapolation step is determined by the size of the summation operator. Whenever this operator spans at least two time samples we can expect to see some energy moving from $t = \Delta t$ to $t = -\Delta t$ in one step. Zeroing the $t = 0$ time slice will not eliminate it, the energy has already skipped past $t = 0$ and wrapped around. If you think about it a little, you'll realize that only flat dip events are truly suppressed by subtracting $t = 0$ in this situation. Flat dips are, however, no problem. It's the nonzero dips that move "too fast", i.e. migrate fast enough to overlay unwrapped-around data. Roughly, 60 degree dip moves two samples per time step and 90 degree dip moves all the way to the surface in one time step. This may seem puzzling to those steeped in phase-shift migration - according to that viewpoint steep dips move *less* than flat dips, specifically $dt = d\tau \cos\theta$. This reasoning is deceptive: it is based on phase velocity rather than the group velocity at which events actually migrate.

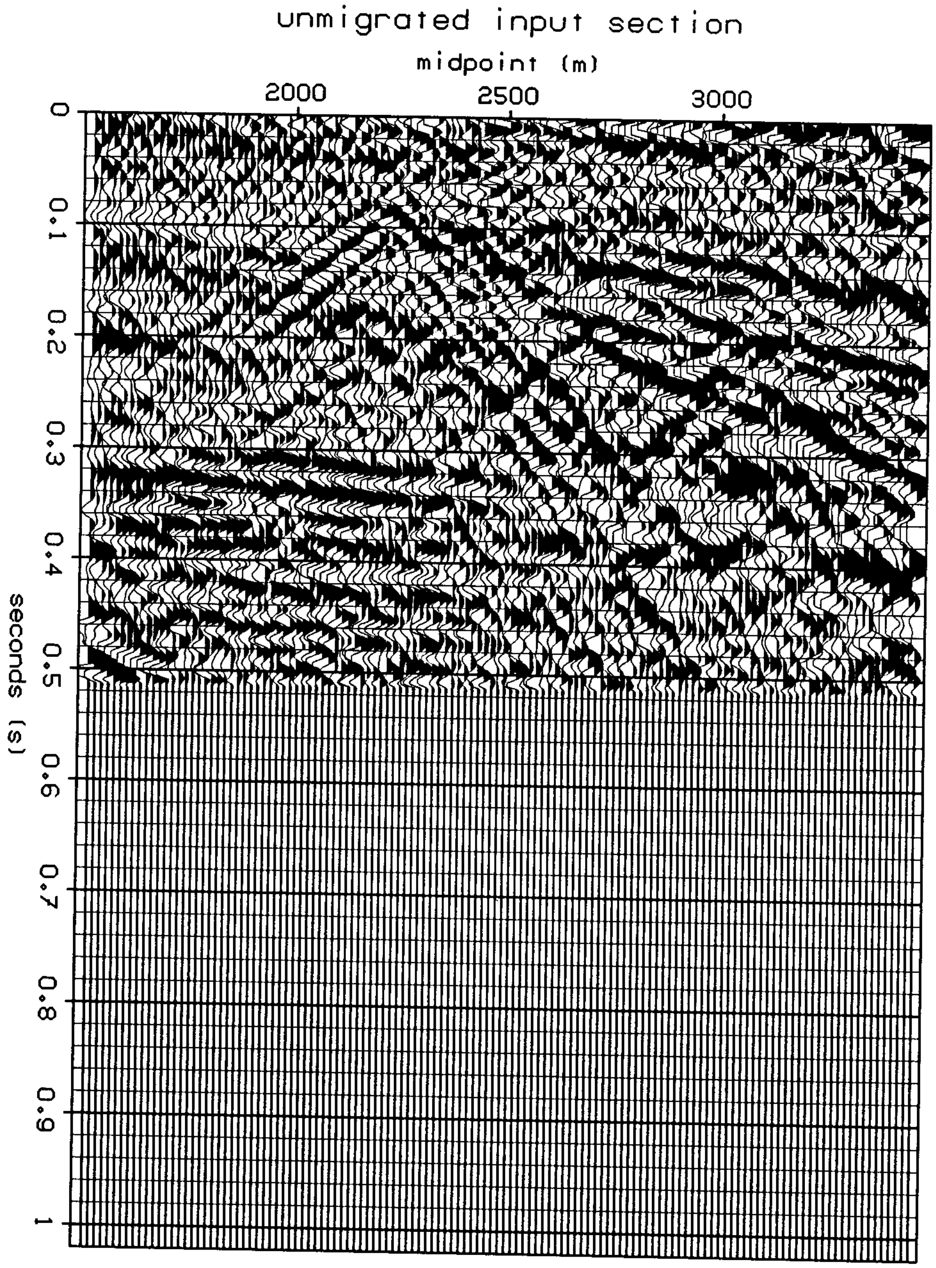


FIG. 1. Zero-padded stacked data used for finite-difference wraparound tests. Sampling interval is .004 seconds and trace spacing is 15.244 meters. The migration velocity used for all tests was 2000 m/s. This was chosen for convenience and is not intended to represent the best migration velocity for these data.

EXAMPLES

To test the degree of wraparound problem and the effectiveness of Einar's suppression method, I took a small window of stacked data (courtesy of Jill McCarthy) and zero padded each trace to twice its length. Shown in Figure 1, this served as input to my experiments. I used the bulletproof 45 degree migration code Dave Hale wrote here at SEP for all migrations. Migration velocity was set to a constant 2000 m/s and the input time section had $dt = 4$ ms and $dx = 15.244$ m. With these parameters $dz = 4$ m corresponds to $d\tau = 4$ ms. Six tests were done: $dz = 2$, 4, and 8 m both with and without $t = 0$ subtraction. These outputs are displayed at a common gain level in Figures 2 through 4.

From these results, I make a number of observations. First, for $d\tau = dt$ Einar's method is effective if not perfect. Second, the dominant effect of not subtracting $t = 0$ is to add very low frequency biases randomly to traces by superposition with the nearly vertical sides of wrapped-around smiles. This also explains the occurrence of similar low frequency artifacts I have seen in phase-shift migrations. Also note that these are low frequencies appearing after migration and would not be substantially affected by low-cut frequency filtering of the input time section. They should however be reduced by dip filtering before or during migration because this would remove the energy that would migrate into the sides of a smile. Third, we see that Einar's method with a depth step of $d\tau = 2dt$ loses a lot of its effectiveness. Fourth, subtracting $t = 0$ when $d\tau = dt/2$ has noticeably reduced that amplitudes of the unwrapped-around migrated image. This supports the small depth step argument I raised above.

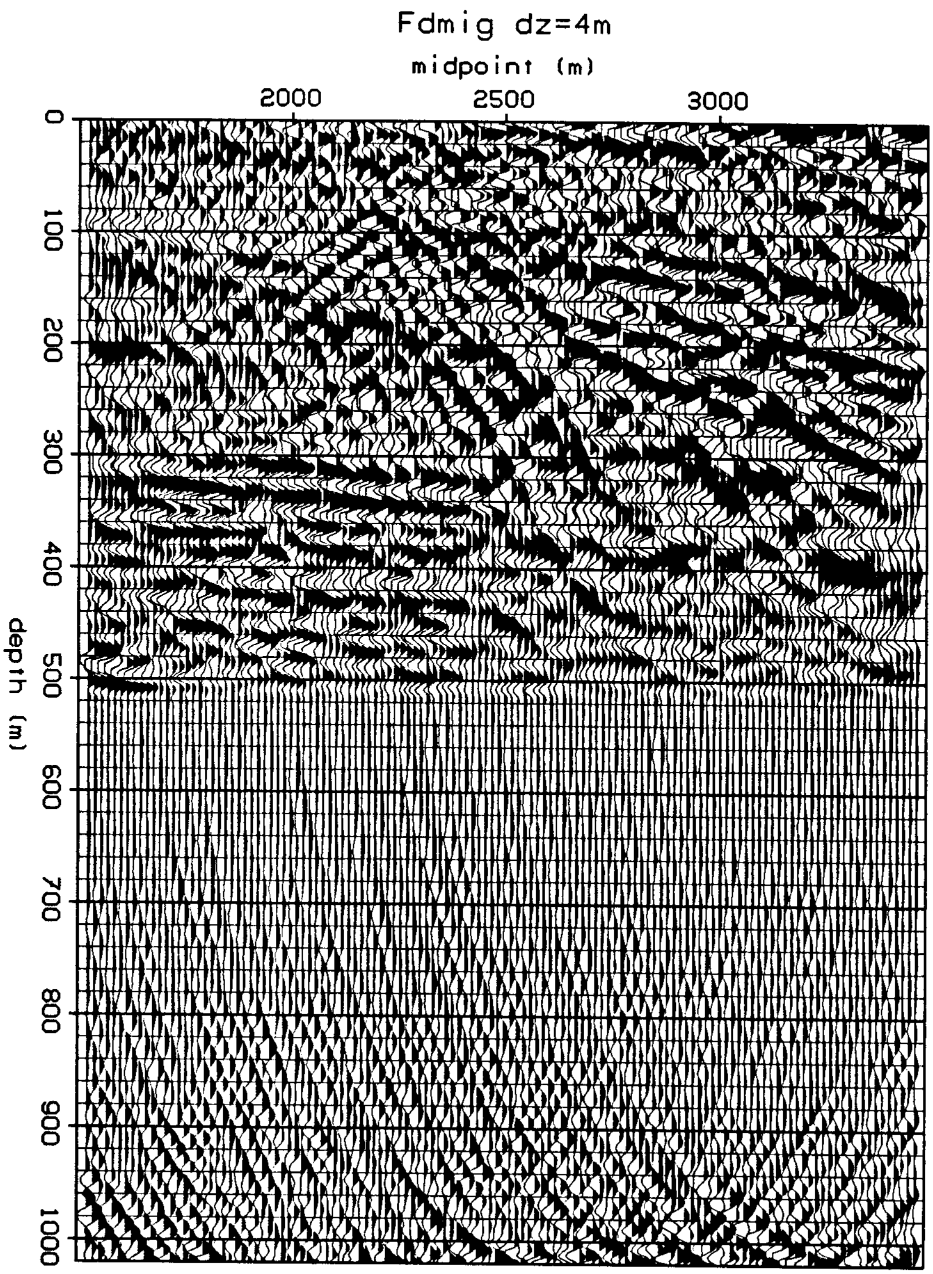
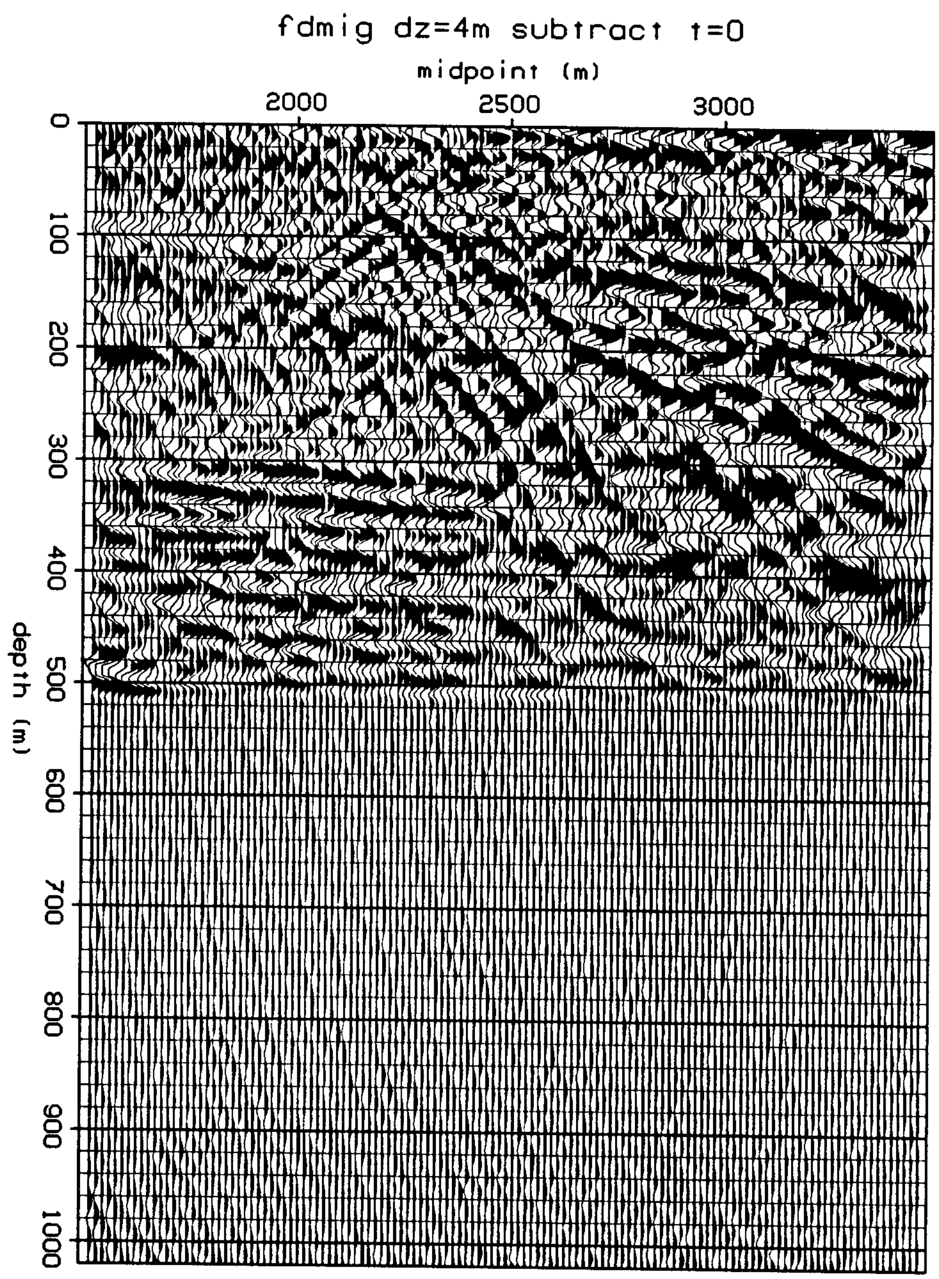


FIG. 2. 45 degree finite difference migration of the data in Figure 1 plotted at the same scale. On the facing page t=0 has been subtracted from the data before each depth step. Migration depth step was 4 meters which gives, at the selected 2000 m/s migration velocity, a two-way vertical traveltime equal to the input time sampling interval of .004 seconds. Wraparound artifacts are clearly visible crossing the zero padding. We see they continue almost vertically into the data and appear as biases added randomly to the traces. $t=0$ wraparound subtraction has clearly helped in this example.


fdmig dz=4m subtract t=0
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2000
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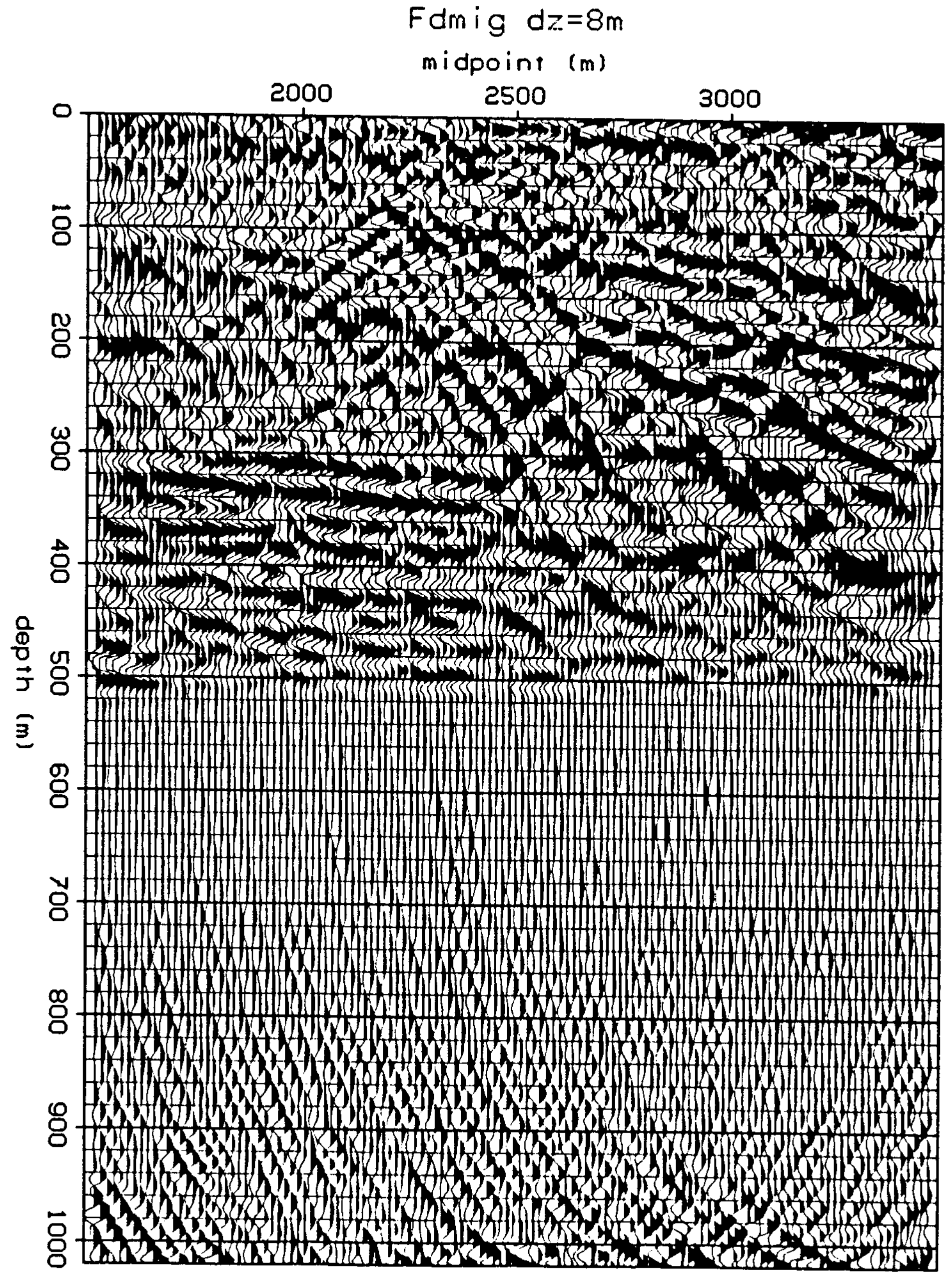
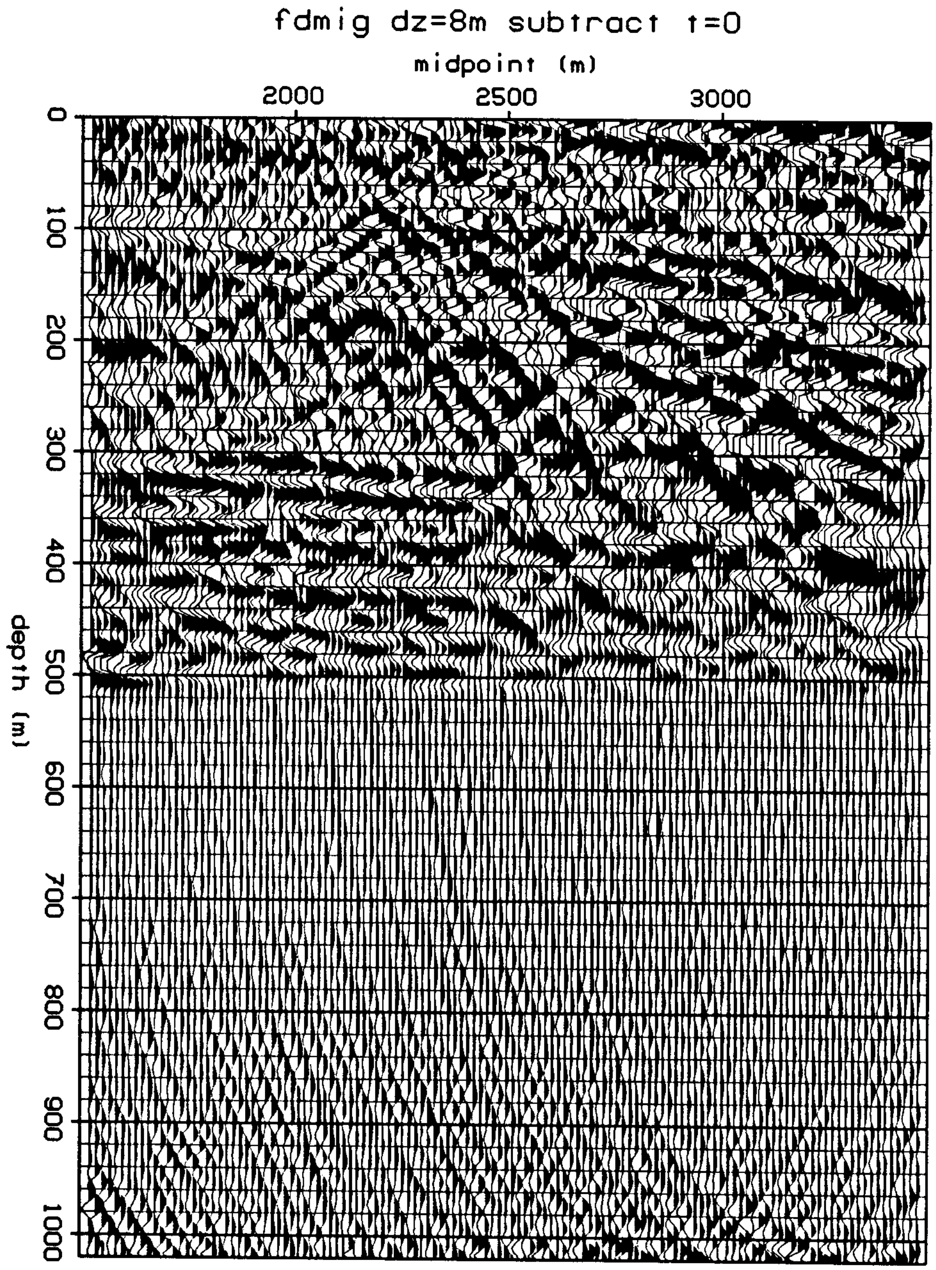


FIG. 3. 45 degree finite difference migration of the data in Figure 1, this time migrated with a depth step of 8 meters. This is a two-way vertical traveltime of .008 seconds, twice the input sampling interval. Again we show t=0 subtraction on the facing page. For this display, the result was resampled at .004 sec with twelve point tapered sinc interpolation. Here wrapround subtraction was ineffective.


fdmig dz=8m subtract t=0
midpoint (m)
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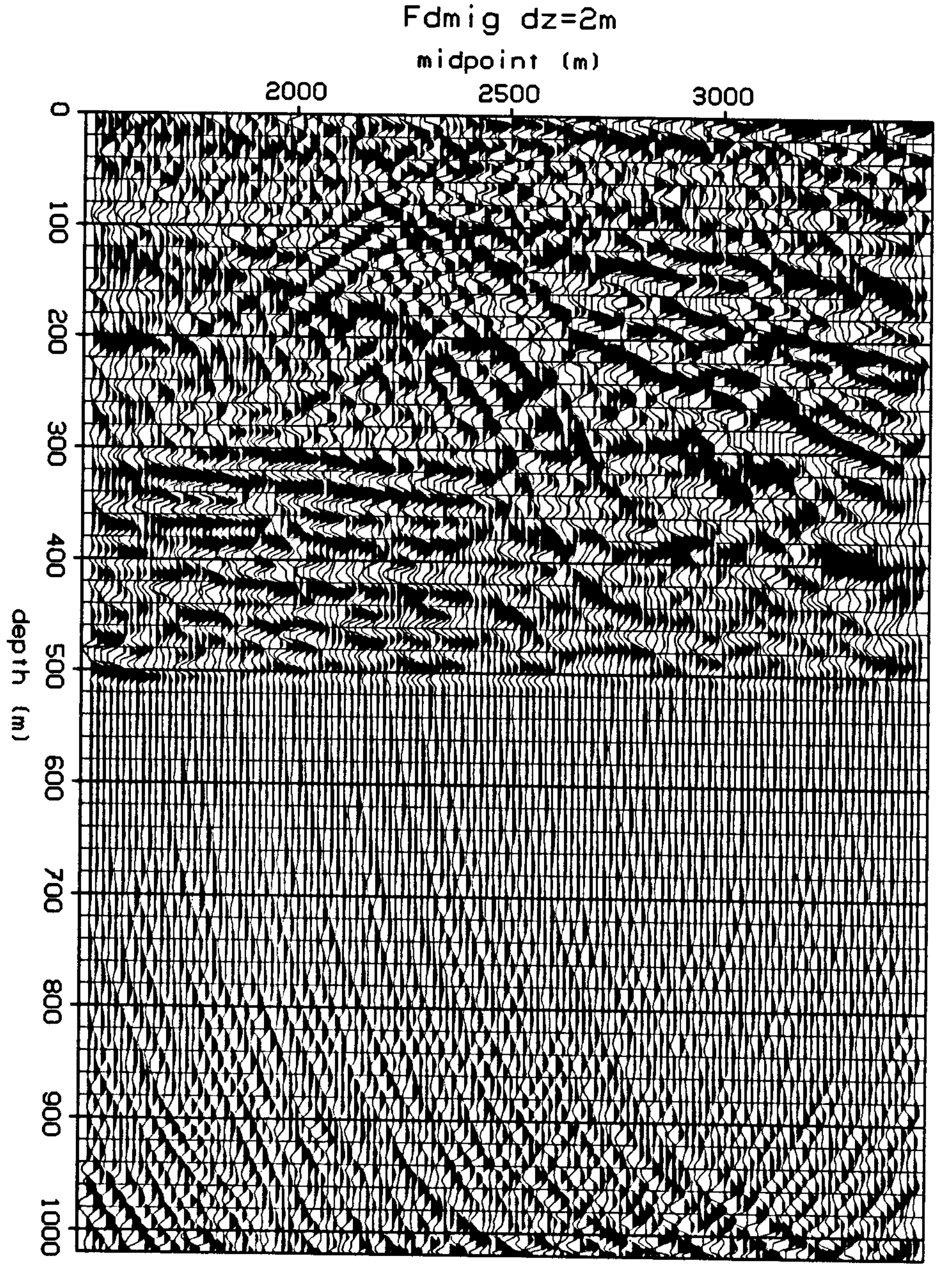
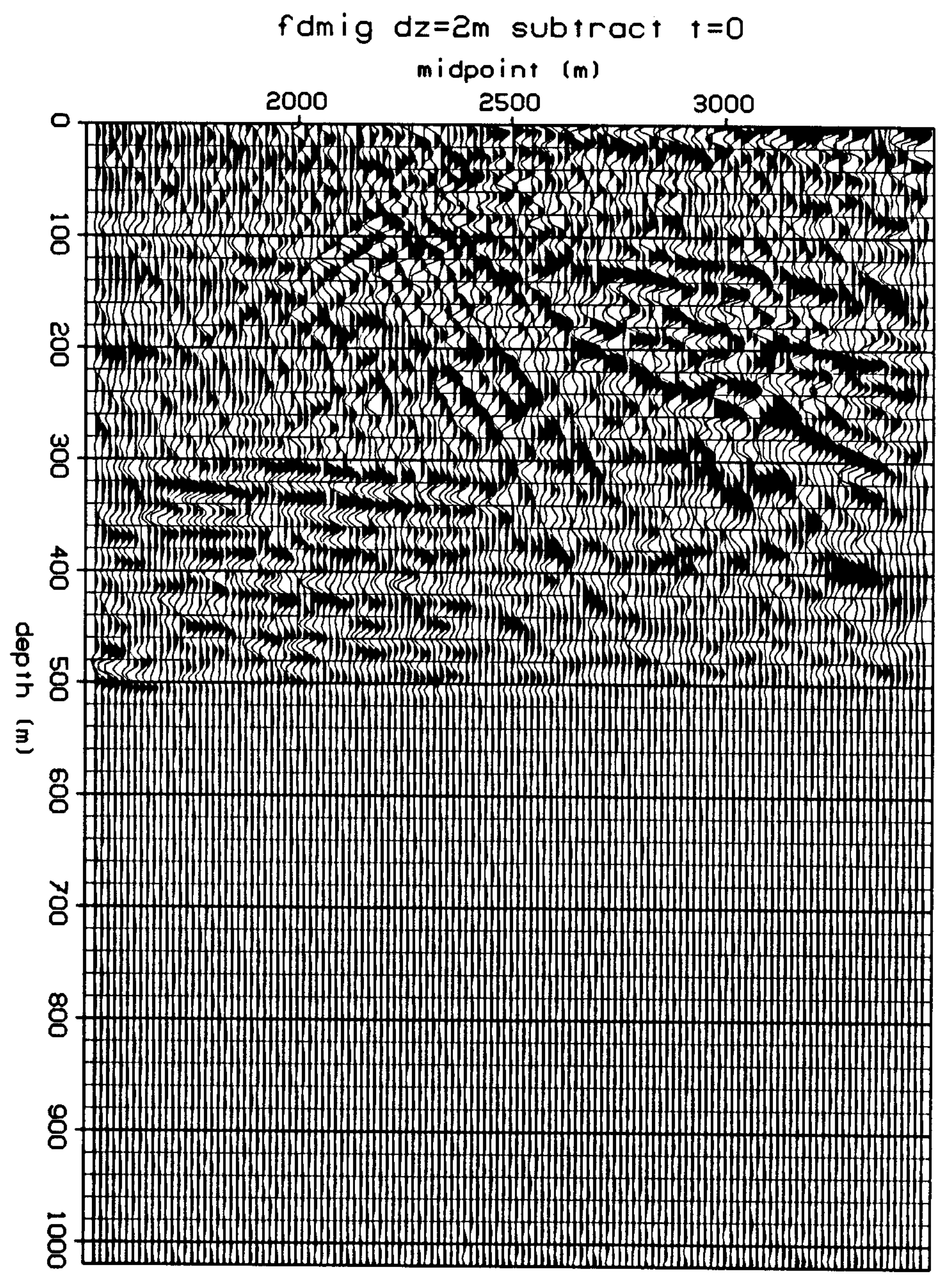


FIG. 4. 45 degree finite difference migration of the data in Figure 1, this time migrated with a depth step of 2 meters. This is a two-way vertical traveltime of .002 seconds, half the input sampling interval. For this display, the result was high-cut filtered to half Nyquist then resampled at .004 sec. Comparing this with the two previous figures shows the amount of wraparound is little influenced by the size of the migration depth step but the amount removed by t=0 subtraction is. For this depth step wraparound subtraction has been too effective. Events that weren't wrapped around have been noticeably attenuated by the subtraction process.


fdmig dz=2m subtract t=0
midpoint (m)
2000
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JON'S PUNCHLINE *or* WHAT TO DO ABOUT IT

> "Einar and Stew subtracted the mean (over ω) of $P(\omega)$. i.e. they used a **very** low quefrency lifter on $P(\omega)$. What they should try is a low quefrency lifter with a bit more bandwidth."
>
> – Claerbout on ω–x wraparound

A number of options are available to deal with these shortcomings.

1) Simply ignore them. Most sections pass through a bandpass filter before display that would suppress the primarily low frequency wraparound. Further, high frequencies and steep dips tend to be dispersed by finite differences. In migration examples that Jon Claerbout has run he found empirically that the causal temporal finite difference approximation

$$\frac{1 - \rho Z}{1 + \rho Z}$$

with $\rho = 1 - \epsilon$ acted as a high-cut dip filter.

2) Explicitly couple zero-padding with low-cut filtering of the migrated output to suppress the near D.C. wraparound. From my experience this works effectively although the cost of migration goes up proportional to the amount of zero padding. Figure 5 shows the result of applying this suggestion to the sample dataset.

3) Follow Jon's suggestion and widen the swath of the vacuum cleaner* to encompass more time levels. This needn't be applied at every downward extrapolation step. Some preliminary experiments of his indicate you can get away with remarkably short quefrency lifters. Figure 6 is one result using a one zero, one zero notch for the purpose.

4) Use dip filtering, either in or accompanying migration, to attenuate the high dips most subject to being wrapped around. This is the method I proposed in SEP-37 to handle the equivalent wraparound problem in phase-shift migration. Jacobs and Muir outline in SEP-26 a method of building dip filtering directly into migration operators. Figure 7 shows the result of turning on dip-filtering in the migration program I was using.

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* Analogically speaking one might better describe Einar's subtraction method as a mousetrap, i.e. a bar that smashes down on everything along a thin line.

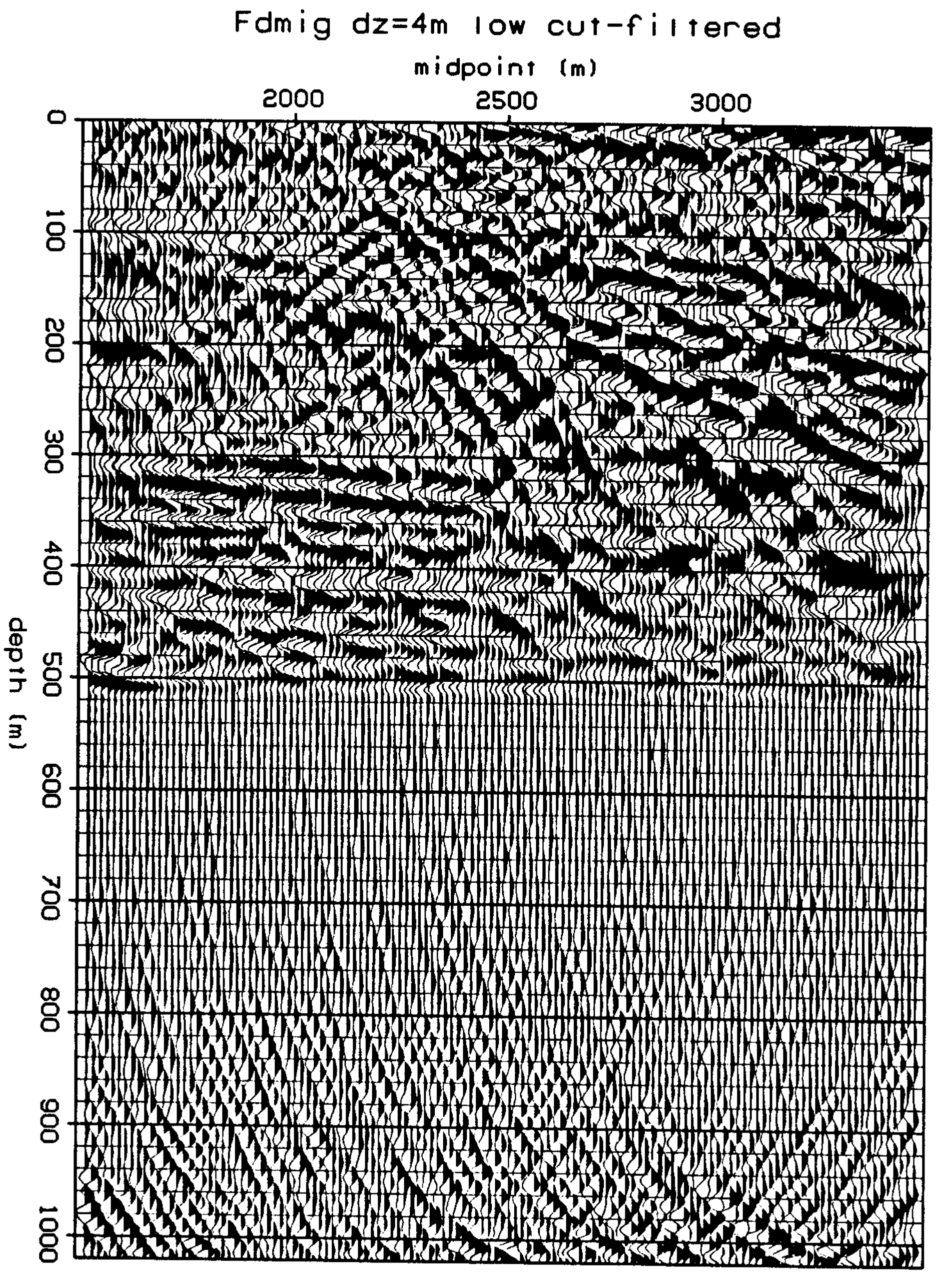

FIG. 5. First of three proposed alternatives to t=0 wraparound subtraction. Here the migrated image in Figure 2a has been low-cut filtered at 5 cycles per kilometer (equivalent to a 5 Hz lowcut on the unmigrated data in Figure 1.) As anticipated, the near vertical wraparound overlaying the migrated section has been attacked. The wraparound in the zero padding is little changed. Clearly a reasonable amount of zero padding is needed for this approach to work.

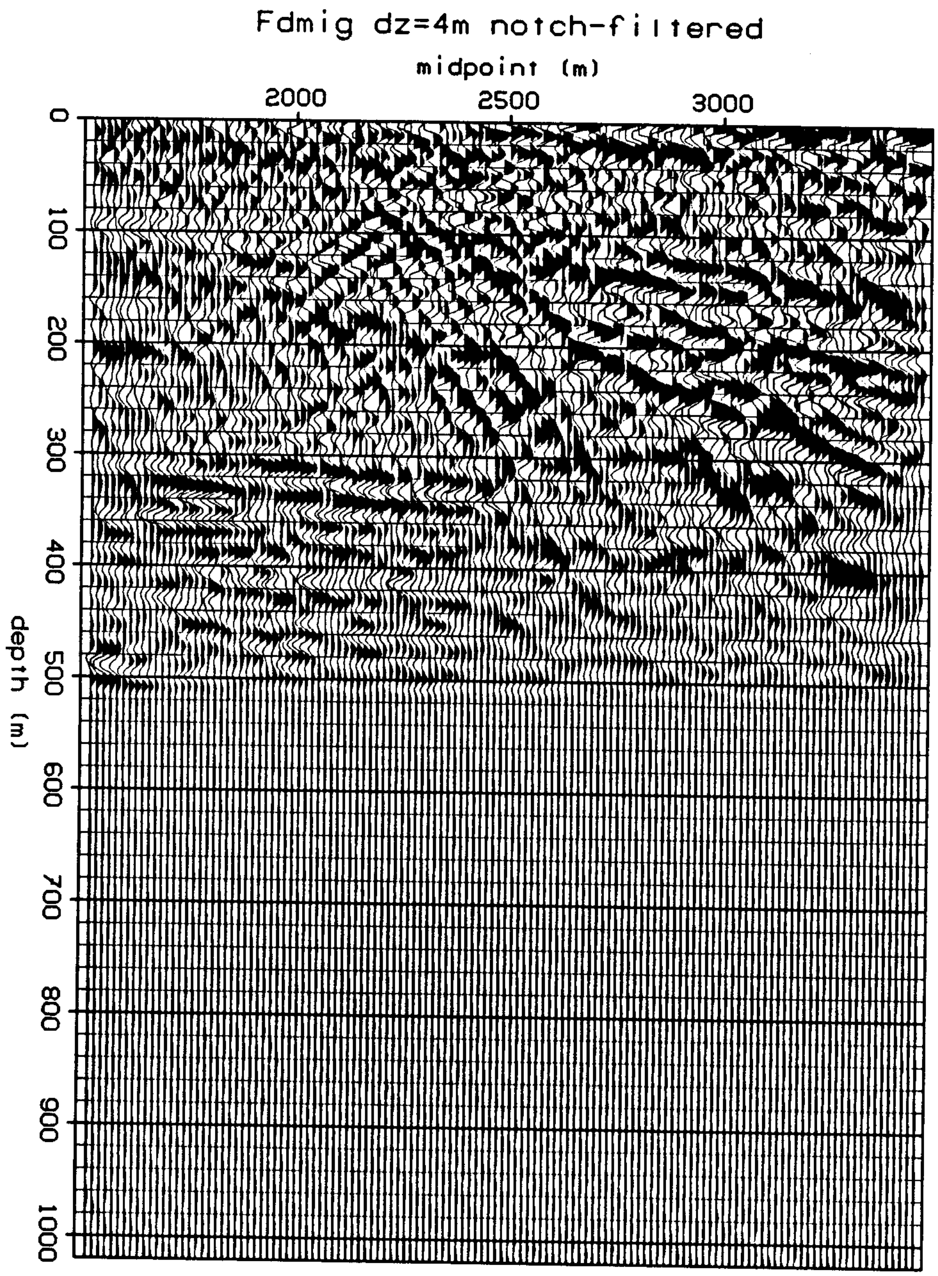

FIG. 6. Second of three proposed alternatives to t=0 wraparound subtraction. Instead of subtracting just t=0, the width of the zone of subtraction was increased by means of a temporal notch filter across the center of the zero padding. Wraparound has been suppressed somewhat moreso than with t=0 subtraction. The vertical streaks overlying the data are still there however.

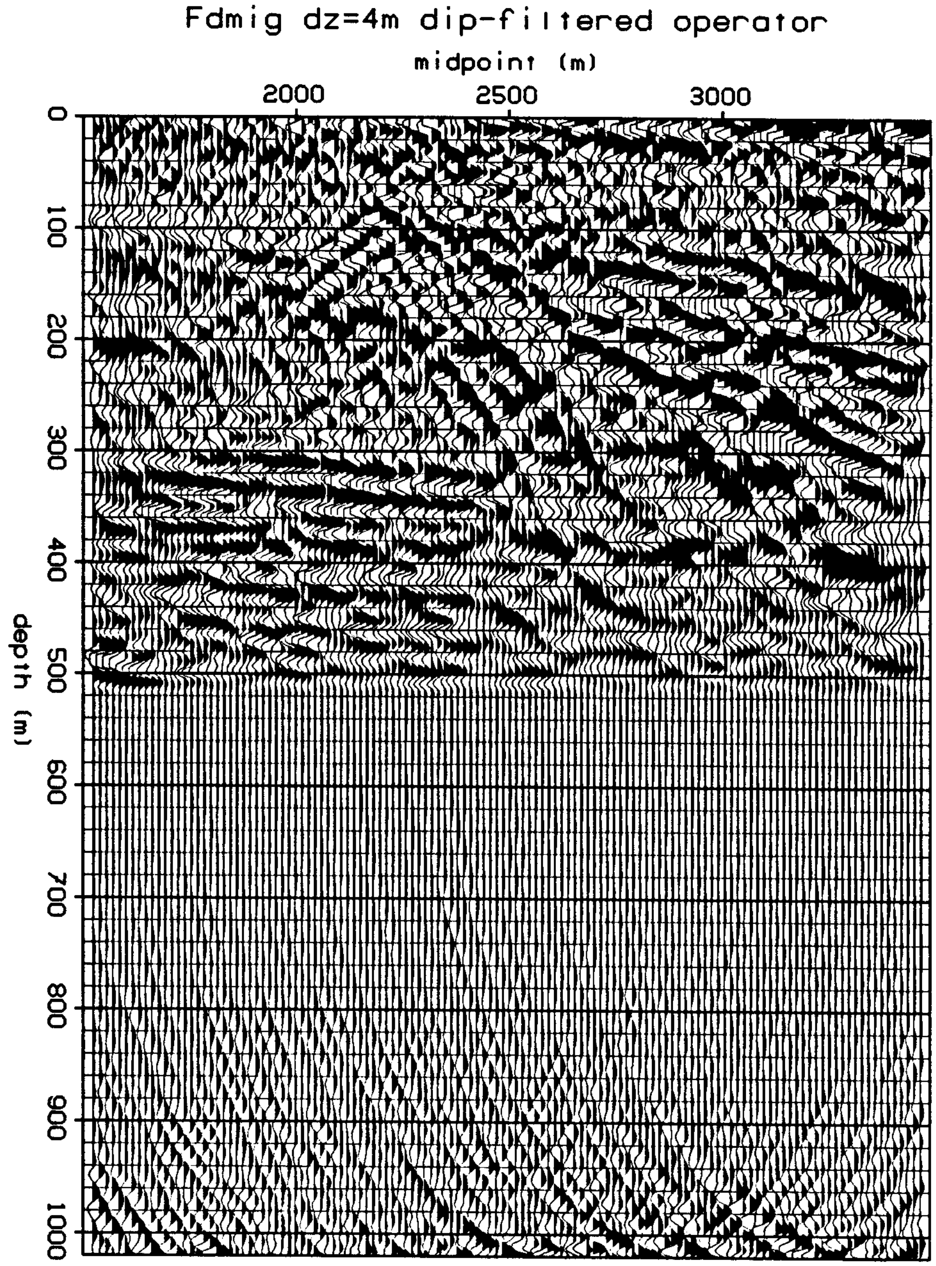

FIG. 7. Last of three proposed alternatives to t=0 wraparound subtraction. Here one of the dip filtering parameters in the 45 degree migration operator has been turned on to attenuate steeply dipping events as the are migrated. Here the effects are reversed from the previous figure: the wraparound in the zero padding has been little affected but the near vertical streaks on the migrated data are noticeably reduced.

SUMMARY

Wraparound artifacts can be a problem in ω–x migration. Previously proposed solutions to these problems are not always effective and can even be damaging if used incautiously. This is because they are well designed to handle gentle dip wraparound whereas steeper dips are the real problem. I have given examples of three* alternative low cost methods of enlarging the dip suppression range for ω–x migration for those sections where wraparound will be a problem.

REFERENCES

Jacobs, A., and Muir, F., 1981, High order migration operators for laterally homogeneous media: SEP-26 p. 163-181.

Kjartansson, E., 1978, Modeling and migration with the monochromatic wave equation -- variable velocity and attenuation: SEP-15 p. 1-19.

Levin, S., 1983, Avoiding artifacts in phase shift migration: SEP-73 p. 27-35.

* When I showed these examples at a recent seminar, Frances Muir pointed out that my three processing alternatives implicitly assumed an immutable amount of wraparound I had to deal with. He proposed a fourth alternative: smoothly extrapolate the unmigrated data into the region before time zero so as to reduce artifacts arising merely from truncation of the data at time zero. I haven't had the opportunity to try out this suggestion.

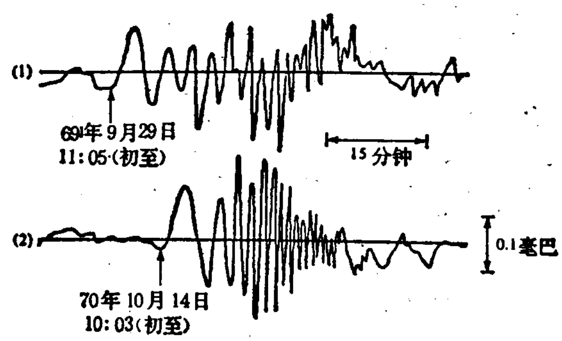

图 1-11

1-5 相关和谱

一个时间函数的谱，是该函数的付里叶变换的模的平方。对于实函数，它的付里叶变换有一个偶实部 RE 和一个奇虚部 IO，取其模的平方，就有

$$(RE+iIO)(RE-iIO)=(RE)^2+(IO)^2$$

偶函数的平方显然仍是偶函数，而奇函数的平方也是偶函数。因此，一个实时间函数的谱总是偶函数，它在正频率的函数值与在负频率的函数值相对称，也就是说，对它讨论负频没有多大意义。

虽然，应用中多数时间函数是实函数，但是，丢开复值时间函数来讨论相关和谱，在数学上是不完全的，何况复值时间函数在研究有关转动的许多物理问题中确实是非常有用的。例如，研究两个向量分量的风速计，它的一个分量指北，记为 n_t，另一个分量指西，记为 w_t。若构成一个复值时间序列 $v_t=n_t+iw_t$，则复数的模和相角都有明确的物理解释。变换的 $(RE+iIO)$ 部分相应于 n_t，而 $(RO+iIE)$ 部分相

From the Chinese translation of Jon Claerbout's *Fundamentals of Geophysical Data Processing*. The caption to Figure 1.11 reads: "Figure 1.11". Times and dates are given.