Velocity stack DMO: an addendum

Paul Fowler

In a recent paper (Fowler, 1984) I described a method for processing seismic data which incorporated dip moveout (DMO) and migration into the velocity analysis process. The formulation of DMO described therein operated on F-K transformed velocity stacks and executed an end run around the need to do (velocity dependent) NMO before (supposedly velocity independent) DMO. This formulation of dip moveout was based on a simple, heuristic ray model which suggested correcting the apparent, or stacking, velocity of a dipping bed by the cosine of the dip. However, although the algorithm was easy to understand and to implement, its theoretical validity was not demonstrated formally. In this paper I want to show briefly the equivalence, in the case of constant velocity, of this formulation of DMO to the Fourier domain method introduced by Hale (1983).

Hale derived an exact F-K domain DMO algorithm which would, in theory, yield results identical to full prestack migration for constant velocity media. He also demonstrated a practical, asymptotically correct, implementation of this formulation of DMO. His algorithm acted on transformed constant offset sections. The relation between Hale's DMO algorithm and the velocity space DMO algorithm may not be obvious, but the two formulations are, for the case of constant velocity, formally equivalent. The key insights to demonstrating this equivalence are present in the work of Jakubowicz (1984), who recast the Hale algorithm in terms of dip decomposed data. In what follows, I shall first write the velocity space algorithm in terms of its action upon a given dip component of the data. I will then summarize Jakubowicz's reformulation of Hale's algorithm; the formal difference between the methods will be seen to reduce to a change of order of integration.

The velocity space algorithm can be summarized by the following sequence of steps: normal moveout (NMO) and stack over a range of velocities, dip decomposition of the stacks by two dimensional Fourier transformation, and correction of the velocities by the cosine of the appropriate dip by shifting data between stacks. Let us express this more formally for the case of a constant medium velocity v_c. The stacked, DMO corrected image will be found in the v_c stack after the velocity corrections have been applied. Call this image $q_{DMO}(t, y, v_c)$, where t is time, y is midpoint, and v_c is the medium velocity. The computations are performed in Fourier transformed space; before inverse transforming back to time and midpoint, the data will be given by $q_{DMO}(\omega_0, k_y, v_c)$, where the subscript on the frequency ω_0 is used to indicate a zero offset frequency, for consistency with Hale's notation. Define a dip variable D by

$$D \equiv \frac{k_y}{\omega_0} = \frac{2\sin\theta}{v_c}$$

where θ is the earth dip of an event. Also define a narrow band dip filter operator $\mathbf{F}_D$ which will select a specified dip component of the data:

$$\mathbf{F}_D \left[q(\omega_0, k_y, v_c) \right] \equiv \begin{cases} q(\omega_0, k_y, v_c) & \text{if } k_y = \omega_0 D \\ 0 & \text{if } k_y \neq \omega_0 D \end{cases}$$

The entire image can be recomposed by the superposition of all such dip components:

$$q_{DMO}(\omega_0, k_y, v_c) = \int dD \; \mathbf{F}_D \left[q_{DMO}(\omega_0, k_y, v_c) \right]$$

In the velocity space DMO algorithm, each dip component of the DMO corrected image is extracted from an appropriate constant velocity stack, with the stacking velocity related to the earth velocity by

$$v_{stack} = v_c \left(1 - \frac{D^2 v_c^2}{4} \right)^{-1/2}$$

or in terms of the earth dip θ,

$$v_{stack} = v_c / \cos\theta$$

Call the data before this DMO correction q_{stack}. Then

$$q_{DMO}(\omega_0, k_y, v_c) = \int dD \; \mathbf{F}_D \left[q_{stack}(\omega_0, k_y, v_{stack} = v_c / \cos\theta) \right]$$

Transforming ω_0 back to zero offset time t_0,

$$q_{DMO}(\omega_0, k_y, v_c) = \int dD \; \mathbf{F}_D \left[\int dt_0 \, e^{i\omega_0 t_0} \, q_{stack}(t_0, k_y, v_{stack} = v_c / \cos\theta) \right]$$

The stacked data $q_{stack}(t_0,k_y,v_{stack})$ is just an integral over offset h of NMO corrected data, with a moveout velocity of v_{stack}, so in terms of the original data $q(t,k_y)$

$$q_{DMO}(\omega_0,k_y,v_c) = \int dD \ \mathbf{F}_D \left[\int dt_0 \, e^{i\omega_0 t_0} \int dh \ \ q\left(t = (t_0^2 + 4h^2/v_{stack}^2)^{1/2}, k_y, h\right) \right]$$

$$= \int dD \ \mathbf{F}_D \left[\int dt_0 \, e^{i\omega_0 t_0} \int dh \ \ q\left(t = (t_0^2 + 4h^2\cos^2\theta/v_c^2)^{1/2}, k_y, h\right) \right] \quad (1)$$

Hale's DMO algorithm in the notation of this paper is given by

$$q_{DMO}(\omega_0,k_y) = \int dh \int dt_n \, A^{-1} \, e^{iA\,\omega_0 t_n} \ q(t_n, k_y, h) \quad (2)$$

where t_n is the NMO corrected time

$$t_n^2 = t^2 - \frac{4h^2}{v_c^2}$$

and

$$A = \left(1 + \frac{h^2 k_y^2}{\omega_0^2 t_n^2}\right)^{1/2}$$

The outer integral over offset h is just the usual stack, so Hale's algorithm reduces to performing NMO, evaluating the indicated integral over t_n, and stacking over offset. The t_n integral represents the DMO operator; leaving out this step gives just the ordinary processing sequence of NMO and stack.

Jakubowicz rewrote Hale's algorithm in terms of its operation upon a given dip component of the input data; I will now summarize his argument. Let D and $\mathbf{F}_D$ be defined as above, and write equation (2) as

$$q_{DMO}(\omega_0,k_y) = \int dD \ \mathbf{F}_D \left[\int dh \int dt_n \, A^{-1} e^{iA\,\omega_0 t_n} \ q(t_n, k_y, h) \right] \quad (3)$$

where now

$$A = \left(1 + \frac{h^2 D^2}{t_n^2}\right)^{1/2}$$

The change of variables from t_n to t_0 with

$$t_0^2 = t_n^2 + h^2 D^2$$

and Jacobian

$$\frac{dt_n}{dt_0} = \left(1 + \frac{h^2 D^2}{t_n^2}\right)^{1/2} = A$$

converts the integral in equation (3) into a Fourier transform over t_0 :

$$q_{DMO}(\omega_0,k_y) = \int dD \ \mathbf{F}_D \left[\int dh \int dt_0 e^{i\omega_0 t_0} \ q_0(t_0{=}(t_n^{\ 2}+h^2D^2)^{1/2},k_y,h) \right] \tag{4}$$

where

$$q_0(t_0{=}(t_n^{\ 2}+h^2D^2)^{1/2},k_y,h) = q_{NMO}(t_n,k_y,h)$$

Here the data with normal moveout applied at the medium velocity v_c is indicated by q_{NMO} and the data with the dip corrected moveout velocity is indicated by q_0. To express the q_0 in terms of the original recorded data q, note that

$$t^2 = t_n^{\ 2} + \frac{4h^2}{v^2}$$

$$= \left(t_0^2 - \frac{4h^2\sin^2\theta}{v^2} \right) + \frac{4h^2}{v^2}$$

$$= t_0^2 + \frac{4h^2}{v^2}\,(1-\sin^2\theta)$$

$$= t_0^2 + \frac{4h^2}{v^2}\,\cos\theta$$

Thus for a fixed dip component θ

$$q_0(t_0,k_y,h) = q\left(t=(\ t_0^2 + 4h^2\cos^2\theta/v^2\ )^{1/2},k_y,h,\right)$$

Equation (4) thus can be rewritten as

$$q_{DMO}(\omega_0,k_y,v_c) = \int dD \ \mathbf{F}_D \left[\int dh \ \int dt_0 \ e^{i\omega_0 t_0} q\left(t=(t_0^2+4h^2\cos^2\theta/v_c^{\ 2})^{1/2},k_y,h\right) \right] \tag{5}$$

Written this way, the only formal difference between the Hale-Jakubowicz algorithm (equation 5) and the velocity space formulation (equation 1) is the interchange of order of the integral over offset h (stacking) and the Fourier transform over t_0. Clearly, the two steps commute, so the methods are formally equivalent.

Jakubowicz showed that the Hale algorithm can be implemented by dip dependent NMO combined with dip filtering. The principal difference between this formulation and the velocity space algorithm is whether the dip correction is done before or after stacking. In practice, the Hale-Jakubowicz algorithm is generally a more efficient method of implementing DMO if the velocity is known; the velocity space algorithm has the advantage of postponing velocity analysis until DMO and migration are done.

REFERENCES

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Hale, I.D., 1983, Dip-moveout by Fourier transform, Ph.D. dissertation, Stanford University, also SEP-36

Jakubowicz, H., 1984, A simple exact method of pre-stack partial migration, talk presented at the 46th Meeting of the EAEG in London

"Snell's Bat"

(what flew out of Cheop's pyramid!)