Imaging the Earth's Interior

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Stanford, California 94305
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Produced as report SEP-40.

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Available after January 1, 1985
from a commercial publisher.

Blackwell Scientific Publications, Inc.
P. O. Box
Palo Alto, CA

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This book is dedicated to Cecil and Ida Green who enjoyed the excitement of being among the pioneers, had the foresight to garner a fortune, and now have the fun of giving it away to universities throughout the world.

Contents

Title		*i*
Dedication		*iii*
Contents		*iv*
Preface		*vi*
Introduction		*x*

Introduction to Imaging

1.1	Exploding Reflectors	1
1.2	Wave Extrapolation as a 2-D Filter	15
1.3	Four Wide-Angle Migration Methods	25
1.4	The Physical Basis	40
1.5	The Paraxial Wave Equation	54
1.6	Mastery of 2-D Fourier Techniques	63
1.7	Sample Programs	67

Why Time and Space ? 76

2.1	Wave-Extrapolation Equations	81
2.2	Finite Differencing	90
2.3	Monochromatic Wave Programs	103
2.4	Splitting and Full Separation	113
2.5	Recursive Dip Filters	122
2.6	Retarded Coordinates	127
2.7	Finite Differencing in (t, x, z)-Space	132
2.8	Introduction to Stability	139

Offset, Another Dimension 144

3.1 Absorption and a Little Focusing 152
3.2 Introduction to Dip 160
3.3 Survey Sinking with the DSR Equation 173
3.4 The Meaning of the DSR Equation 183
3.5 Stacking and Velocity Analysis 192
3.6 Migration with Velocity Estimation 208
3.7 Lateral Velocity Variation in Bigger Doses 220

The Craft of Wavefield Extrapolation 230

4.1 Physical and Cosmetic Aspects of Wave Extrapolation 233
4.2 Anisotropy Dispersion and Wave-Migration Accuracy 246
4.3 Frequency Dispersion and Wave-Migration Accuracy 257
4.4 Absorbing Sides 267
4.5 Tuning up Fourier Migrations 272
4.6 Impedance 279
4.7 Accuracy — the Contractor's View 298
4.8 The Bulletproofing of Muir and Godfrey 305

Some Frontiers 309

5.1 Radial Traces 311
5.2 Slant Stack 315
5.3 Snell Waves and Skewed Coordinates 326
5.4 Interval Velocity by Linear Moveout 336
5.5 Multiple Reflection — Current Practice 346
5.6 Multiple Reflection — Prospects 363
5.7 Profile Imaging 376
5.8 Predictions for the Next Decade 382

Appendix: Sponsors of the Stanford Exploration Project 385

References 386

Index 392

Preface

Some History

Reflection seismologists make images of the earth's interior. Through the 1960s this was done in an ad hoc fashion. Between 1968 and 1972, I conceived and field tested a new method of image making based directly on the wave equation of physics. Previously the wave equation had been used to predict observations starting from simplified, hypothesized models. It was not used in routine data analysis. My imaging method using finite differences soon came into widespread use in the petroleum exploration industry. Many other people quickly became involved and made important improvements. The earlier ad hoc methods were reinterpreted and they too improved in the light of wave theory.

An industrial affiliates group, known as the Stanford Exploration Project (SEP), was founded at Stanford University to pursue the new developments. Of the forty-eight sponsoring organizations, many have substantial research departments of their own. Thus began the decade of the 1970s, in which much progress was made. In the 1980s progress continues to be made at a rapid pace.

The Place for This Book

This textbook was born of the need to teach the best of the many new ideas to those entering the industry. Because so many people enter geophysics from outside the field, I have kept specialized geophysical terminology to a minimum and defined everything. So this book should be useful, not only to those interested in petroleum exploration, but also to professionals in all disciplines in which waves are analyzed.

My previous book, *Fundamentals of Geophysical Data Processing* (FGDP), was published in 1976. It covered more basic aspects of reflection seismic data processing such as Z-transforms, Fourier transforms, discrete linear system theory, matrices, statistics, and theory of the stratified earth.

FGDP also introduced wave-equation imaging, but extensive supplements became necessary. The supplements evolved into this book. The two books are about ninety percent different, the ten percent overlap being necessary to keep this book fully self-contained.

This book provides a coherent overview of the whole field of data processing as it is used in petroleum exploration, and it is the basic textbook in exploration geophysics at Stanford University. I make no claim, however, that the book is encyclopedic in scope. Some important processes such as *deconvolution* and *statics* are lightly sketched, as are the experimental applications of *tomography,* while other techniques such as *ray tracing* and many kinds of *modeling* are omitted altogether. Regrettably, the migration literature alone has grown so large that significant contributions such as the theoretical side of Kirchhoff migration (see Berkhout[1980]) are omitted.

Organization

Seismic imaging is a subject that draws much from mathematics and physics. These subjects build one idea upon another in a logical progression. I chose to organize this book likewise. This organization favors the new student who wants to understand the material thoroughly. A detailed index and more than a hundred cross references are included to link the practical topics that a logical organization has left somewhat scattered about.

More encyclopedic, topic-oriented textbooks on reflection seismology are those of Waters [1981] and Sengbush [1983]. Books focused on petroleum prospecting that treat reflection seismology descriptively are Sheriff [1980] and Anstey [1980]. Recommended books on earthquake seismology with a mathematical level comparable to this book are Aki and Richards [1980] and Kennett [1983].

I have also tried to reach readers who want to learn concepts while skimming the mathematics. Individual sections (which are lectures) carry practical and descriptive matters as far as possible before the mathematical analysis. The chapters themselves are also organized in this way, so for example, when you get to the middle of Chapter 1, you can skip forward to Chapter 2.

As it happens, waves are marvelously geometrical objects, and much can be learned with little mathematical analysis. But you should begin the book having previous familiarity with calculus, complex exponentials, and Fourier transformation.

Philosophy

There is always a gap between theory and practice. Many books give you no clue as to the size and location of the gap — even books in exploration geophysics. This gap is nothing to be embarrassed about. It represents the current life of the subject — the current life of any science. It is a moving target, and its size is a matter of opinion. So it is risky for me to tell you what works, what doesn't, what is important, and what isn't. Opinions go beyond facts. Your knowledge won't be complete if you don't know some opinions as well as the facts. You will be getting opinions as well as facts when I explain the discrepancies between theory and industrial practice, and when I explain what should work, but doesn't seem to.

Thanks

This book is less my personal creation than was FGDP. I am indebted to many people. My colleagues Francis Muir and Fabio Rocca collaborated extensively in research. The following Stanford students contributed figures, exercises or ideas: David Brown, Junyee Chen, Robert Clayton, Steve Doherty, Raul Estevez, Paul Fowler, Bob Godfrey, Alfonso Gonzalez, Dave Hale, Bill Harlan, Bert Jacobs, Einar Kjartansson, Walter Lynn, Larry Morley, Dave Okaya, Richard Ottolini, Don Riley, Shuki Ronen, Dan Rothman, Chuck Sword, Jeff Thorson, John Toldi, Oz Yilmaz, and Li Zhiming. Industrial teaching helped me clarify the material in this book. Many useful ideas on presentation came from those who assisted me in industrial teaching, namely, Rob Clayton, Walt Lynn, Einar Kjartansson, Rick Ottolini, Fabio Rocca, Dave Hale, and Dan Rothman. Thanks are also due those who helped arrange the classes: Phil Hoyt, Lee Lu, A. Lamer, Aftab Alam, Gary Latham, and Mike Graul.

JoAnn Heydron Pluemer was first my secretary, and at a later date, the editor. Terri Ramey provided drafting and graphics assistance. Pat Bartz maintained order in ways too numerous to mention. I did most of the typing myself, as computerized typesetting has lightened that load and allows a more heavily revised and debugged final product. Peter Mora, Stew Levin, Dan Rothman, and Bill Harbert assisted in final proof reading.

Major thanks are due to Stanford University and the sponsoring organizations. Without them, much less would have been achieved. A list of sponsor names is given in an appendix. Many sponsors also donated data for our research. This is acknowledged in the figure caption. Many figures were based on data from a world-wide assortment prepared for academic use by Yilmaz and Cumro [1983]. Where figures were prepared by students, I have also acknowledged them in the caption. I prepared most of the figures myself

often using outstanding graphics software utilities prepared by Rick Ottolini (*Tiplot, Movie*), Shuki Ronen (*Thplot*), Rob Clayton (*pen*), Joe Dellinger (*Ipen*), and Dave Hale (*Graph*).

Special thanks must also go to Western Electric Company, who donated the typesetting software that I used to prepare my lecture notes at Stanford. This software enabled me to go through many generations of improvements, eradicating scores of errors, rarely introducing new ones, all the while seeing (and having the students see) the material in almost final form. I am confident that this book has many fewer errors than did my previous book even after its second printing. When these lectures are in final form, I will simply mail a camera-ready copy and you my readers should have the book within eight weeks. So long, I'll see you at the SEG meeting.

Jon F. Claerbout

Introduction

Seek truth from facts. - Deng Xiaoping

Prospecting for oil begins with seismic soundings. The echoes are processed by computer into images that reveal much geological history. Worldwide, echo sounding and image making constitute about a four-billion-dollar-per-year activity.

Meaning of the Measurements

The presence of oil and gas has little *direct* effect on seismic reflections. The volume of rock is much larger than the volume of hydrocarbon. The reflections are well correlated, however, with interfaces between various rock types. In porous rocks, the hydrocarbons are free to flow. Fluids tend to rise. The shapes of rock interfaces tell us where hydrocarbons may accumulate. The discovery of oil and gas in the middle of the North Sea is a remarkable success story for the reflection seismic method. As the first exploratory wells, located by reflection seismology, were being drilled, it was impossible to predict whether they would hit oil. But if oil was to be found anywhere under the North Sea, there was great confidence that these initial drill sites were much more favorable than random locations. And as it turned out, of course, oil was soon found.

After a well has been drilled and logged, the reflection images become even more valuable, because then it is known what rock type corresponds to each echo. Seismology is usually able to provide a remarkably accurate mapping of rock types at some distance from the well. It is particularly valuable to know in which direction the rocks tilt upward, and where the strata are broken by faults. Seismology provides this information at a much lower cost than more drilling. When petroleum prospecting moves offshore the cost of

seismology goes down by an order of magnitude, while the cost of drilling goes up by an order of magnitude.

Interested readers can purchase three volumes of reflection seismic images of the earth from the American Association of Petroleum Geologists (Bally [1983]).

Reproducibility of the Data

Reflection seismic data is voluminous. It is not like pencil marks on a sheet of paper. It is row upon row of high density magnetic tapes. Much seismic data is readily comprehensible. But much remains that is not, especially on the first try. Although much data is incomprehensible and seems noisy and random, it is remarkable that the data is experimentally repeatable. And we find that by working with this data we learn more and more, and are encouraged to continue. Because there is so much still hidden in the data which is routinely collected, this book concentrates on the mainstream data geometry: single survey lines with conventional near-surface sources and receivers. Experimental techniques are indicated but not examined.

Computers as Imaging Devices

Philosophers ask the question, "What is knowledge?" As technologists, our answer is that there is a real world and there is also an image of it in our minds. Knowledge means that the two are similar. To help form images we use imaging devices, such as microscopes, telescopes, cameras, television, etc. In this book computers are imaging devices for seismic echo soundings.

As an imaging device, a computer is in many ways ideal. A telescope is limited by the quality of its components. The image created by a computer is limited more by our understanding of mathematics, physics, and statistics, than by limitations inherent in the computer. For imaging radar or ultrasonics, computer capacity would be a real problem. It happens that the information content (bandwidth) of seismic echo soundings is about matched by today's computers.

Why is it Fun?

Many young people seem to enjoy tackling tough theoretical problems, but when the time comes for application they are often disappointed to find that the theory is in some ways irrelevant or inadequate to the problem at hand. At first this causes a diminished interest in practical problems. But eventually many come to see the real problems as more interesting than the original mathematical models. Why is this?

Maybe life is like a computer game. I have noticed that the games students like best are not those with a predetermined, intricate logical structure. They like the games which allow them to gradually uncover the rules as they play. It is really fun when a period of frustration with a game is ended by some application of a personal idea. But to be fun, a game must *have* rules, and you must *be able* to uncover them with a reasonable amount of effort. Luckily, reflection seismology, together with modern computers, provides us with a similar environment. Sometimes a game can be too frustrating, and you need a hint to get you over some obstacle into a new and deeper level. Reading this book is not like playing the game. It is more like being given a collection of hints, a bag of tricks, to help you to the deeper level.

These tricks are mostly new, many being less than ten years old. They have been selected because they really work, not always, but often enough. I have repressed the urge to include many promising tricks which have not been sufficiently tested.

Practical problems are not only deeper than theoretical problems, but ultimately they yield more interesting theory. For example, in freshman physics laboratory I learned to deduce Newton's laws of motion from simple experiments. I should have found experimentally that force equals mass times acceleration. Of course I didn't find exactly that. The experiment didn't seem to work out too well because of friction. *Friction,* now there's a really interesting subject for you. Physicists, chemists, metallurgists, earth scientists, they all know Newton's laws but wish they understood friction!

The theoretical book you are now holding wouldn't have been written except that two earlier theoretical approaches, (1) the theory of mathematical physics in stratified media, and (2) time series analysis, couldn't touch some of the most interesting aspects of our data. Some people thought we just had dirty data! Reflection seismic data are repeatable. Most of our problems really arise from the theory, not the data.

Computers and Movies

This book includes a number of computer programs. These programs are used for illustration and as exercises, but they should also be useful for reference. While they cannot be guaranteed, they worked for me when I generated many of the figures in this book, and they should work for you. You will notice that they are in a language similar to Fortran. The language is described at the beginning of Section 1.7. Since everyone has different facilities for graphical output, to use these programs yourself, you will have to understand them well enough to direct their outputs into your plotting equipment.

A movie is really a stack of pictures. In a computer it is just a three-dimensional matrix of floating point numbers which must somehow be converted to brightness *pixels* (*pic*ture *el*ements). At the time of writing, few people are equipped to directly convert such a three-dimensional matrix into a movie. In our laboratory (Ottolini et al. [1984]) this is done on a high-quality video computer terminal (AED 512). Movie capability is a valuable asset. It enhances our understanding of our data and of the processing. Students are inspired by seeing their programming work result immediately in a movie, which is easily videotaped. Compared to other graphical devices this one is easy to maintain. It is used by both the research students and the students in the master's degree program, who use it for homework exercises.

The cost of such equipment, including the direct memory access (DMA) computer interface, is less than $10,000. For a really good experience with movies, you should also have physical control of a computer with a memory greater than a few megabytes. If you don't already have this, the price (1985) increases by about a factor of ten.

Will There Be Jobs ?

The main use of reflection seismic imaging is petroleum prospecting. Unlike nuclear energy, hydrocarbons are a non-renewable resource and there is evidence that petroleum production must decline during the lifetimes of the young generation. Does this mean that young people should avoid these studies? I think not. Taking the long-range view, with the population of the earth continuing to increase, it is not easy to imagine people losing interest in the earth's crust. A view of more intermediate range is that as the resource declines in abundance, there will be greater efforts to seek it. A short-range view is that workers are needed today, and there are no coal- or nuclear-powered airplanes. In any case, the skills developed in this book, computer implementations of concepts from physics, will always be of general utility.

Guide to This Book

Chapters 1 and 3 describe the basic concepts of imaging in reflection seismology. Chapters 2 and 4 cover computer techniques for the analysis of observed wavefields. Chapter 5 describes advanced imaging concepts. At Stanford University, Chapters 1-3 are taught to master's students in a course that runs for one quarter. These students also take a class from FGDP either before or after the class from this book.

You may want to understand concepts without learning about techniques. You could try reading just Chapters 1 and 3, but Chapter 2 will increase your comprehension because of its concrete nature and the examples

it includes. Chapter 4 is for craftsmen who want to know what is involved in a high-quality implementation, or for unusually skilled interpreters who wish to understand artifacts and the accuracy limitations of various techniques. Chapter 5 describes seismic imaging concepts that are novel and seem correct in principle, but for various reasons (many unknown to me) have not yet come into widespread practical use. Interpreters who can stand the mathematics may appreciate Chapter 5 because it claims to explain how and why things often work out the way they do in practice. But the main attraction of Chapter 5 will be for those who wish to develop new echo imaging techniques.