The Pre-Stack Migration Operators.

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ABSTRACT

We describe different schemes for the pre-stack migration of profiles in (S-G) space and for an inhomogeneous medium. The central part of the downward continuation analysis is the description of each method including derivation, dispersion relation, stability and applicable numerical algorithms. In this paper, we present two downward continuation operators. Several analytical and numerical examples are given to illustrate them. The Raphson-Newton and Muir's expansion of the square-root operator are compared for relative error phase velocity and error group velocity . Since the pre-stack migration of profiles in the Cartesian coordinate system has been analysed in detail by Jacobs (SEP-34) , we will restrict ourselves to the essentials .

Introduction

Snyder (SEP-16) has shown that lateral velocity variations and steeply dipping beds invalidate the standard industry assumption that a zero-offset section is identical to a CDP stack. Finite difference operators have been designed to handle both lateral velocity gradients and complex structures. As described by Jacobs (SEP-34), the Cartesian coordinate extrapolation downward continues both the upgoing wave recorded at the geophones of a profile, and a downgoing wave originating at the shot point using either the principle of reciprocity, or by modeling a source.

Energy migrates towards the zero-offset trace so that the migration of the unstacked data results in the imaging of reflectors on the zero-offset traces. The stack appears as one takes the sum of the frequency domain quotients of the upgoing wave over the downgoing wave . The imaging deteriorates as the phase velocity error increases since the accuracy of the imaging depends upon the correct determination of where the up- and downgoing waves are in phase.

Another point concerns the group velocity error which can be viewed as a measure of the energy dispersion. This dispersion is highly undesirable for two main reasons: First, the imaging may be altered since dispersed energy can interfere with constructive energy. Second, energy dispersion may give rise to errors in the wave equation residual velocity analysis since this processing is done concurrently with the downward continuation of the pressure deviation wavefield.

We study first the downward continuation of the up and down-going waves and review the 4 approximations usually used to derive the extrapolation operators. Later we will study the derivation of the operators. The first operator has been previously designed by Claerbout , Muir and Jacobs. Stability properties are demonstrated using the eigenvalues of the tridiagonal matrix T. The second operator has been designed for its higher accuracy than the first one and the simple algorithm it leads to.

The One-Way Wave Equation In The (x,z,w) Domain

We assume that the density model is a constant. Clayton (SEP-27) has shown that the density variations modify the amplitudes of the wavefield but not the phase, so that the imaging is not affected by this approximation.

The two-way wave equation for a pressure deviation wavefield $\psi(x,z,w)$ in a medium with acoustic velocity $v(x,z)$ is:

$$\frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^2 \psi}{\partial x^2} = - \frac{w^2}{v(x,z)^2} \psi \tag{1}$$

This equation can be factorized in order to get the two one-way wave equations which are used to continue either up or down-going waves.

$$\left(\frac{\partial}{\partial z} - \frac{iw}{v}\left[1 + \frac{v^2}{w^2}\frac{\partial^2}{\partial x^2}\right]^{1/2}\right)\left(\frac{\partial}{\partial z} + \frac{iw}{v}\left[1 + \frac{v^2}{w^2}\frac{\partial^2}{\partial x^2}\right]^{1/2}\right)\psi = 0 \tag{2}$$

The first term of equation (2) is the one-way wave equation which allows one to continue up-going wave downwards in depth.

$$\frac{\partial \psi}{\partial z} - \frac{iw}{v}\left[1 + \frac{v^2}{w^2}\frac{\partial^2}{\partial x^2}\right]^{1/2}\psi = 0 \tag{3}$$

Formally, we can solve equation (3), i.e. find the pressure wave at depth $z+\Delta z$ by the following relation:

$$\psi(x,z+\Delta z,w) = \exp\left(\int_z^{z+\Delta z} \frac{iw}{v} \left[1 + \frac{v^2}{w^2} \frac{\partial^2}{\partial x^2} \right]^{1/2} \psi \; dz \right) \tag{4}$$

In order to simplify the downward continuation of the up-going waves, 4 approximations are usually done. The first of them concerns the argument of the exponential and we simplify this argument with the following:

$$\int_z^{z+\Delta z} \frac{iw}{v} \left[1 + \frac{v^2}{w^2} \frac{\partial^2}{\partial x^2} \right]^{1/2} \psi \; dz \approx \frac{iw}{v} \left[1 + \frac{v^2}{w^2} \frac{\partial^2}{\partial x^2} \right]^{1/2} \psi \; \Delta z \tag{5}$$

This first assumption requires that the variations of $\psi(x,z,w)$ in the interval $[z,z+\Delta z]$ be negligible, i.e. the wave length along the depth axis should be much greater than Δz. This is the requirement not to have aliasing in the z axis direction. It leads to the following condition:

$$\Delta z \ll 2\pi \frac{v}{w \cos(\theta)} \tag{6}$$

Here, θ is the angle a wave vector makes with the z axis.

The second assumption is the approximation of the square-root operator $\left[1 + \frac{v^2}{w^2} \frac{\partial^2}{\partial x^2} \right]^{1/2}$ by a finite continued fraction using either Muir's or the Raphson-Newton expansion:

$$\left[1 + \frac{v^2}{w^2} \frac{\partial^2}{\partial x^2} \right]^{1/2} = R\left(\frac{v^2}{w^2} \frac{\partial^2}{\partial x^2} \right) \tag{7}$$

Where R represents a finite continued fraction. This approximation of the square root operator as a finite rational fraction expansion is the key to derive the extrapolation operators. The third approximation concerns the evaluation of the second partial continuous derivative $\frac{\partial^2}{\partial x^2}$ by Claerbout's β trick:

$$\delta_{xx} = -\frac{1}{\Delta x^2} \frac{T}{1 - \beta T} \tag{8}$$

Following Jacobs, the parameter β may be considered as a function of w since you want to fit $\frac{\partial^2}{\partial x^2}$ to the largest k_x for which propagation will occur at the frequency w. The fourth approximation is the expansion of the exponential in the equation (4). Let's consider

consider the second order finite rational fraction expansion of the exponential:

$$exp(x) = \frac{1}{1 - \dfrac{x}{1 + x/2}} \tag{9}$$

If we replace x by the operator $\Delta z \frac{\partial}{\partial z}$, then we find the following Crank-Nicholson scheme:

$$\psi(x,z+\Delta z,w) = \frac{1}{1 - \dfrac{\Delta z \frac{\partial}{\partial z}}{1 + \frac{1}{2}\Delta z \frac{\partial}{\partial z}}} \psi(x,z,w) \tag{10}$$

Usually, the first order of the Taylor series expansion is simply considered and the other terms of the exponential series are neglected. The Crank-Nicholson scheme leads to a stable operator (OP_1), accurate to third order terms in Δz, the discretization parameter for the depth axis. In this paper we consider the third order of the Taylor expansion of the exponential in the equation (4) to derive a new operator (OP_2).

Derivation, stability and accuracy of the operators

We define the matrix T as the positive definite tridiagonal matrix arising from the second order discretization of ∂_{xx}. Some stability results may be derived by considering the eigenvalues of this matrix. The eigenvalues λ of this matrix are solution of the following equation:

$$det(\lambda I_n - T) = 0 \tag{11}$$

where n is the number of receivers to downward extrapolate. Let us define $D(n)$ as the determinant of the matrix $\lambda I_n - T$, and the parameter u by the following relation:

$$u = \lambda - 2 \tag{12}$$

If we develop the determinant $D(n)$ by rows, it leads to the difference relation:

$$D(n) = u\ D(n-1) - D(n-2) \tag{13}$$

If we define the angle α as:

$$u = 2\cos(\alpha) \tag{13a}$$

Then, it can be shown that:

$$D(n) = \frac{\sin(n+1)\alpha}{\sin\alpha} \tag{14}$$

Therefore the eigenvalues λ_k of the matrix T are given by the following relation:

$$\lambda_k = 2\left[\ 1 + \cos\left(\frac{k\ \pi}{n+1}\right)\right] \qquad 1 \le k \le n \tag{15}$$

Thus, we can write the T matrix as:

$$T = O\ \Lambda(\lambda_k)\ O^t \tag{16}$$

where O is an orthonormal matrix and $\Lambda(\lambda_k)$ a diagonal matrix whose coefficients are the eigenvalues of the matrix T. Let us define Q(z,w) as the data set vector to be extrapolated:

$$Q(z,w) = \begin{bmatrix} \psi(x_1,z,w) \\ \psi(x_2,z,w) \\ \cdot \\ \cdot \\ \cdot \\ \psi(x_n,z,w) \end{bmatrix} \tag{17}$$

$Q(z,w)$ may be considered either as a common shot point gather vector or a common receiver gather where n is the number of receivers. The simplest operator (OP) which can be derived by a first order Taylor expansion of the exponential in equation (4) is:

$$(OP) = I_n + \Delta z \frac{\partial}{\partial z} \tag{18}$$

Using equation (16), we may write the square root operator defined in equation (7) in a similar decomposition:

$$[\, I_n - \frac{V^2}{w^2 \Delta x^2} \frac{T}{1-\beta T}]^{1/2} = O \, \Lambda[\, (1 - \frac{v^2}{w^2 \Delta x^2} \frac{\lambda_k}{1 - \beta \lambda_k})^{1/2} \,]\, O^t \tag{19}$$

For convenience, this relation has been derived in the case of no lateral velocity variations since the matrix O and the diagonal velocity matrix V commute in this case, and this allows the square-root operator to be computed easely. The goal is to write the operator (OP) in a similar form to the matrix T in equation (16). Then we will be able to discuss its stability in the Von Neuman sense. Using the equations (3) and (19), we may write the operator (OP) in the following form:

$$(OP) = O \, \Lambda[\, 1 + \frac{iw \, \Delta z}{v} (1 - \frac{v^2}{w^2 \, \Delta x^2} \frac{\lambda_k}{1 - \lambda_k})^{1/2} \,]\, O^t \tag{20}$$

The above relation shows that the operator (OP) has n eigenvalues γ_k given by:

$$\gamma_k = 1 + \frac{iw \, \Delta z}{v} (1 - \frac{v^2}{w^2 \, \Delta x^2} \frac{\lambda_k}{1 - \lambda_k})^{1/2} \qquad 1 \le k \le n \tag{21}$$

The Von Neuman stability criterion of the operator (OP) requires that $|\gamma_k|$ be less than 1. Therefore, it yields:

$$\Delta x < \frac{v}{w} \frac{\pi}{n+1} \tag{22}$$

Derivation of the operator (OP_1)

The condition (22) leads to very small value of Δx (a few decimeters for usual seismic velocity and frequency). Another point is the weak accuracy of this operator. This is why a Crank-Nicholson scheme is used, providing both stability and accuracy. Following the equation (10), this scheme may be viewed as the second order finite rational fraction expansion of the exponential, leading to the operator (OP_1):

$$(OP_1) = \frac{I_n}{I_n - \dfrac{\Delta z \dfrac{\partial}{\partial z}}{I_n + \dfrac{I_n}{2}\Delta z \dfrac{\partial}{\partial z}}} \tag{23}$$

The eigenvalues μ_k of this operator may be directly calculated as for the operator (OP) if the velocity has no lateral variations. Their values are:

$$\mu_k = \frac{1 + \dfrac{iw\,\Delta z}{2}\left(1 - \dfrac{v^2}{w^2\,\Delta x^2}\,\dfrac{\lambda_k}{1-\lambda_k}\right)^{1/2}}{1 - \dfrac{iw\,\Delta z}{2}\left(1 - \dfrac{v^2}{w^2\,\Delta x^2}\,\dfrac{\lambda_k}{1-\lambda_k}\right)^{1/2}} \qquad 1 \le k \le n \tag{24}$$

Since μ_k always has a modulus lower than 1, the operator is unconditionally stable in the Von Neuman sense. An estimate of the accuracy may be provided by a Taylor series expansion of $Q(z)$ which leads to:

$$|\ Q(z,w) - (OP_1)\ Q(z - \Delta z)\ | \ \le \ \frac{1}{12}\Delta z^3 \ ||\ \frac{\partial^3 Q}{\partial z^3}\ ||_\infty \tag{25}$$

where $||.||_\infty$ is the infinite norm on the interval $[\ z - \Delta z, z]$. In the case of a plane wave travelling with an angle θ to the depth axis z, this leads to:

$$|\ Q(z,w) - (OP_1)\ Q(z - \Delta z, w)\ | \ \le \ \frac{1}{12}\,\frac{w\ \Delta z^3}{v^3}\ |sin(\theta)|^3\ Q(z - \Delta z, w)^3 \tag{26}$$

This estimate is somehow pessimistic since it always exceeds the error. However, it provides an approximation of the numerical dispersion. Furthermore, the previous inequality shows how the accuracy depends upon the value $\frac{\Delta z \; w}{v}$, the expression already encountered in the non-aliasing condition. For a monochromatic plane wave travelling in a medium of velocity $2400 m/s$, with an angle of 45 degrees and a frequency of 30 hertz, it leads to a relative error of 0.4% for a discretization parameter Δz of 10 meters and one downward extrapolation. If we extrapolate, say up to 1000 meters, the relative error is about 40%. This estimate of the numerical dispersion may seem quite pessimistic, but we must realize that the splitting technique used for the downward extrapolation reduce the accuracy of our scheme to second order in Δz.

Derivation of the operator (OP_2)

The second operator (OP_2) we describe now, leads to an exact splitting algorithm and the strategy is accurate to fourth order in Δz. This operator is the third order Taylor expansion of the exponential.

$$(OP_2) = I_n + \Delta z \frac{\partial}{\partial z} + \frac{\Delta z^2}{2} \frac{\partial^2}{\partial z^2} + \frac{\Delta z^3}{6} \frac{\partial^3}{\partial z^3} \tag{27}$$

This operator may be factorized and it leads to:

$$(OP_2) = 6 \; I_n \; (\Delta z \frac{\partial}{\partial z} - \alpha_3) (\Delta z \frac{\partial}{\partial z} - \alpha_2) (\Delta z \frac{\partial}{\partial z} - \alpha_1) \tag{28}$$

In the appendix, we demonstrate that the stability of this operator requires:

$$\Delta z \le (1 - 4\beta)^{1/2} \Delta x \tag{29}$$

Since the parameter β is taken in the interval [1/12,1/6], the above criteria is easy to handle and does not require small Δz. Also, the accuracy is improved by almost an order of magnitude. As for the operator (OP_1), an estimate of the accuracy may be provided by a Taylor series expansion and it leads to:

$$| \; Q(z,w) - (OP_2)Q(z-\Delta z,w) \; | \; \le \; \frac{1}{24} \Delta z^4 \; || \; \frac{\partial^4 Q}{\partial z^4} \; ||_\infty \tag{30}$$

This equation may be viewed as a control of the accuracy. For the same monochromatic plane wave used to estimate the accuracy of the operator (OP_1), the new operator provides a relative error less than 0.06%.

Jacobs has described the downward continuation algorithm with the operator derived from the Crank-Nicholson scheme. His technique is to approximate the square-root operator by its partial fraction expansion and to plug it into the operator (OP_1). Then, he needs the Ma splitting to derive a useful procedure. Ma strategy is to solve each z-step as a sequence of 45-degree split equations. Thus, the algorithm is accurate to second order in Δz. The operator (OP_2) does not need to split the square-root expansion in its component pieces so that the accuracy to fourth order in Δz is not destroyed.

As shown by Jacobs, Muir's partial fraction expansion of the square-root may be written as a sum of 45- degree operators:

$$R\left(\frac{v^2}{w^2}\frac{\partial^2}{\partial x_2}\right) = \sum_{j=1}^{k} r_j^{(k)} \tag{31}$$

where $r_j^{(k)}$ is a 45-degree operator and k the order of the Muir's expansion used. We then plug the previous expression in equation (28) and it yields:

$$(OP_2) = \frac{I_n}{6}(op_3)(op_2)(op_1) \tag{32.a}$$

where the operators (op_i) are defined by:

$$(op_i) = \frac{jw\Delta z}{v}\sum_{j=1}^{k} r_j^{(k)} - \alpha_i \tag{32.b}$$

Thus, we may extrapolate $Q(z,w)$ to the depth $z+\Delta z$ by the sequence of both shifting and focusing equations:

$$Q_1 = (op_1)\ Q(z,w) \tag{33.a}$$

$$Q_2 = (op_2)\ Q_1 \tag{33.b}$$

$$Q(z+\Delta z) = \frac{I_n}{6}(op_3)\ Q_2 \tag{33.c}$$

where $Q(z+\Delta z,w)$ is the output wavefield at depth $z+\Delta z$ and the subscripted Qs are vectors used in implementing the procedure. An algebraic matrix manipulation shows that we may write each 45-degree operator $r_j^{(k)}$ in the reduced form:

$$r_i^{(k)} = \alpha_i^{(k)} + \frac{\mu_i^{(k)}}{\gamma_i^{(k)} + T_i^{(k)}} \tag{34}$$

All the parameters encountered in the right side of the previous equation are diagonal matrices except the tridiagonal matrix $T_i^{(k)}$. Thus, the main work is to solve tridiagonal linear systems. Accuracy and stability of the above operators ((op_i) , (OP_1) and (OP_2)) greatly depend upon the approximation of the square-root operator that one uses.

Expansion of the square-root operator and dispersion relation

The approximation of the square-root operator by either Muir's or the Raphson-Newton expansion leads to wave extrapolators which are not frequency dispersive but angle dispersive. Since Raphson-Newton converges quadratically to the square-root, it provides higher-order approximations. Raphson-Newton recursion is defined by:

$$R_{k+1}^N = \frac{R_k^N}{2} + \frac{F}{2\,R_k^N} \tag{35a}$$

$$F = 1 + S \tag{35b}$$

$$S = \frac{v^2}{w^2}\frac{\partial^2}{\partial x^2} \tag{35c}$$

Following Dubrulle, it may be shown that R_1^N coincides to the 15-degree approximation of Muir's expansion and R_2^N with the third approximation. Raphson-Newton approximations of the square-root are never used for two reasons. First, no stability proof may be given easily and second, for each Raphson-Newton iteration, the equivalent (in computation cost) Muir approximation is more accurate.

Claerbout shows (Imaging the Earth's Interior) that the dispersion equation arising from the extrapolation operators is not frequency dispersive but angle dispersive. The true dispersion equation is:

$$k_z = \frac{w}{v}[\,1 - \cos^2(\theta)\,]^{1/2} \tag{36}$$

If we assume that Claerbout's approximation of $\frac{\partial^2}{\partial x^2}$ (i.e. the β trick) is correct (things are worse), then a Fourier transform over x shows that the parameter S used in Muir approximation is $\sin^2(\theta)$. Thus, we may write the above equation as:

$$k_z = \frac{w}{v}\, f(\sin^2(\theta)) \tag{37}$$

where the function f represents either Muir or Raphson-Newton approximation of the square-root. Therefore, both the phase velocity and the group velocity presents an angle error. It is easy to derive the relative error of the wavenumber k_z. We denote it by $g(\theta)$.

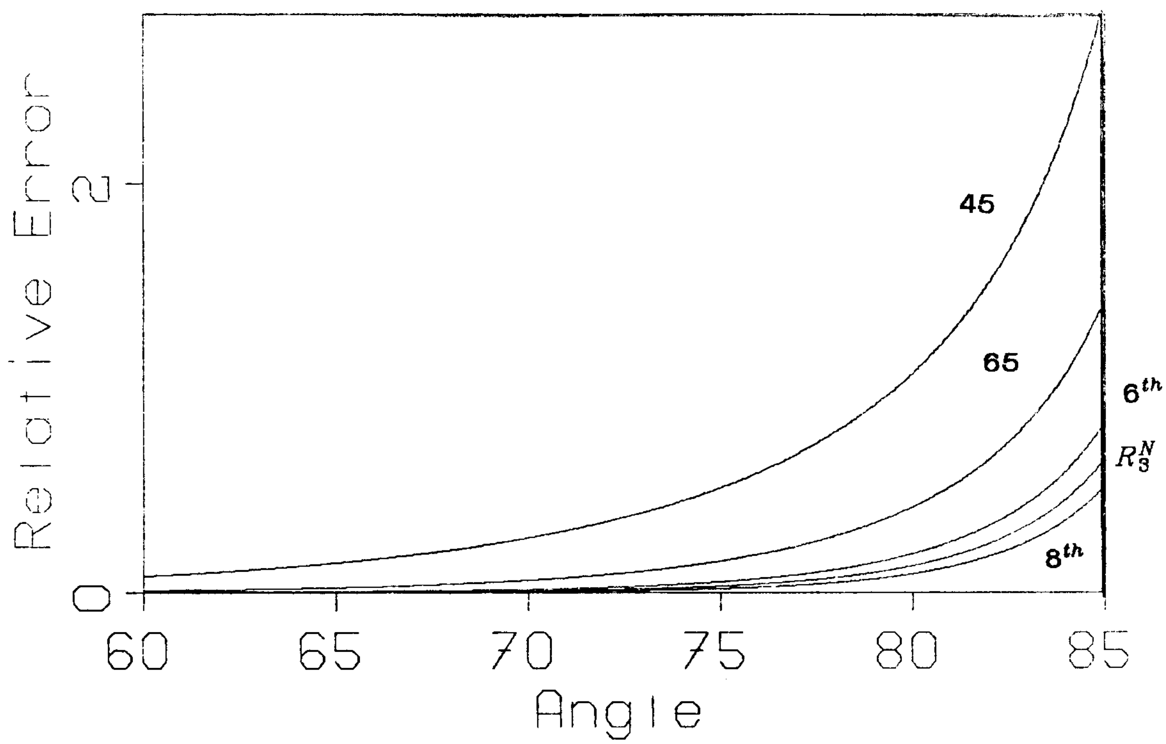

FIG. 1. Relative error of the wavenumber k_z. The curves have been drawn for positive degree angles since g is an even function of the angle θ. The 65 degree approximation is much more accurate than the 45, and Muir's 6^{th} approximation to the square-root is significantly better than the 65.

$$g(\theta) = \frac{f(\theta) - \cos(\theta)}{\cos(\theta)} \tag{38}$$

These relative errors are represented by the curves of fig. 1, for the 45 degree, 65 degree, 6^{th} and 8^{th} Muir approximations and for the third Raphson-Newton approximation. It shows us first that it is of weak interest to use higher approximation orders than the 6^{th} Muir approximation. Second, 8^{th} Muir approximation is better than third Raphson-Newton ones for the same computation cost since both of them require the inversion of 4 partial fractions.

Derivation of the group velocity vector

Following Claerbout (FGDP p. 16), we derive the group velocity vector V_g from an implicit equation $\Omega(w, k_x, k_z) = 0$ by the following relation:

$$V_g = - \frac{\nabla_k \Omega}{\frac{\partial \Omega}{\partial w}} \tag{39}$$

A simple implicit function Ω may be defined as:

$$\Omega(w, k_x, k_z) = k_z - \frac{w}{v} f\left(\frac{v}{w} k_x\right) \tag{40}$$

where the function f is the approximation of the square-root (equation (37)). The definition of the group velocity, equation (38) and the definition of Ω, equation (40) yields:

$$V_g = \frac{v}{\frac{vk_x}{w} f'\left(\frac{vk_x}{w}\right) - f\left(\frac{vk_x}{w}\right)} \left[f'\left(\frac{vk_x}{w}\right) e_x - e_z \right] \tag{41}$$

where f' denotes the first derivative of the function f and the vectors e_z, e_x an orthonormal basis of the vertical plane (X-Z). Let's now replace the expression $\frac{vk_x}{w}$ by $sin(\theta)$ in equation (40) in order to derive the parametric equations of the group velocity vector.

The evanescent waves may be characterized by an imaginary complex angle θ. Thus, instead of parametrized the group velocity components V_x and V_z by the parameter $sin(\theta)$, we must use a parameter u whose interval of variations is $[-\infty, \infty]$. When $|u|$ is greater than 1, we are in the evanescent wave domain and this waves don't propagate but are responsible of the familiar tails behind the impulse response of the migration operators.

Figure 2.a shows the group velocity parametric curve for the so-called 45 degree approximation of the square-root inscribed within the semi-circle which is the exact parametric curve of the group velocity vector. As described by Claerbout (Imaging the Earth's Interior, chapter 4.2), the evanescent waves are above the semi-circles which indicate waves with $|u| = 1$, i.e. waves propagating in the x direction. Things are worse than for the phase velocity approximation or for the wavenumber approximation k_z (Figure 1.) since the wavefront drastically diverges from the correct group velocity parametric curve for small angles. The impulse response of the operator (OP_1) for the 45 degree approximation (Figure 2.b) in a medium with constant velocity presents exactly the same wavefront and we may notice that the dispersion waves occur as predicted by the theoretical group velocity parametric curve.

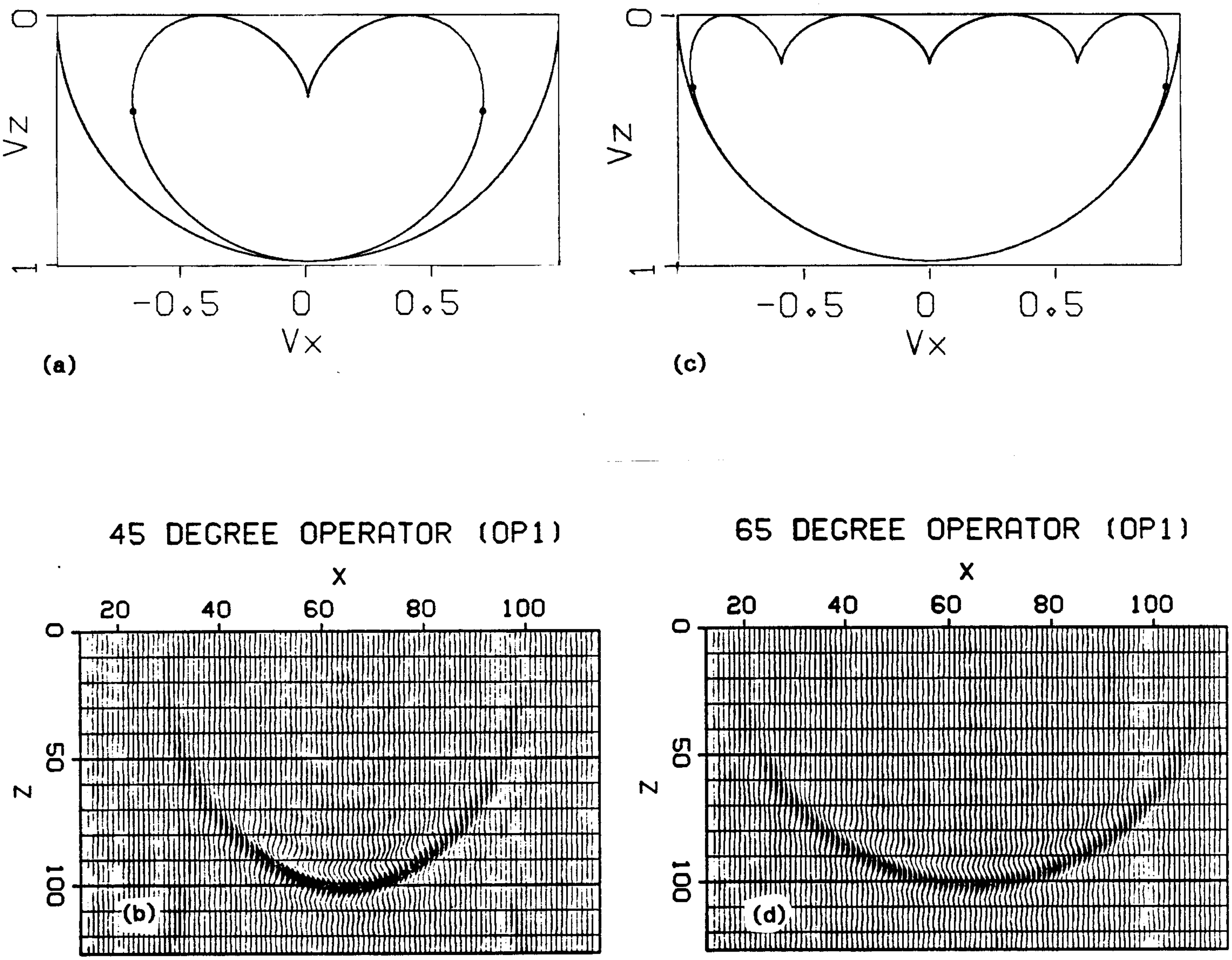

FIG. 2. (a) Theoretical wavefront for the Muir 45 degree approximation. The impulse responses of the 45 and 65 degree Muir expansion, respectively Figure (b) and (d) have been generated by the downward continuation of a spike (frequency domain $[15_{Hz}, 40_{Hz}]$) with $\beta = \frac{1}{6}$, then filtered both in the x and z directions by a *cosine Hanning window*. Burg's extrapolation technique has been implemented for the side boundary conditions. Figure (c) is the theoretical wavefront for the Muir 65 degree approximation.

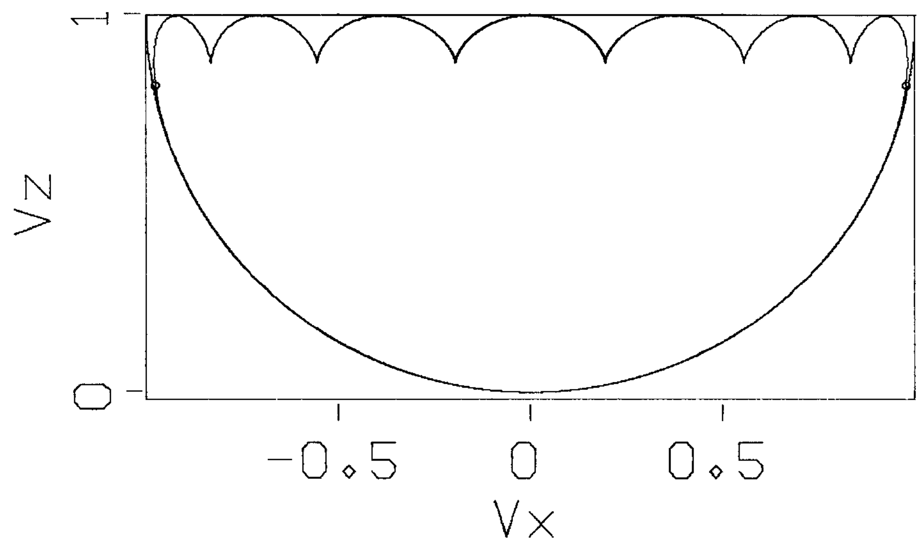

FIG. 3. Group velocity parametric curve for third order Raphson-Newton approximation.

The 65 degree approximation of the square-root leads to a noticeable insight of the convergence towards the semi-circle as shown by Figure 2c and the impulse response of the operator (OP_1) for the 65 degree approximation (Figure 2d) still presents the same wavefront and the anisotropic dispersion for large angles as predicted by the theory.

Figure 3. displays the group velocity parametric curve for the third order Raphson-Newton approximation. This operator handles well dips up to 80 degrees. Its numerical implementation leads to the inversion of four three tridiagonal matrices at each downward continuation step. Muir's 6^{th} approximation has the same accuracy than the previous operator (Figure 1) for angles up to 80 degrees. Since it requires the inversion of three tridiagonal matrices at each step, it is the highest operator of interest for our seismic data processing.

Impulse response in severe velocity variation medium

The grid dimensions used in generating these impulse responses have 128 traces both in the x and in the z directions, with a sample interval of 18 meters in the x and 8 meters in the z direction. 45 and 65 degree (OP_1) impulse responses are displayed in Fig. 4, 5 and 6 for three different velocity models.

Fig. 4 (b) and (c) show 45 and 65 degree impulse responses for a simple fault model (a). The wavefronts are no longer semi-circles because there are both depth and lateral

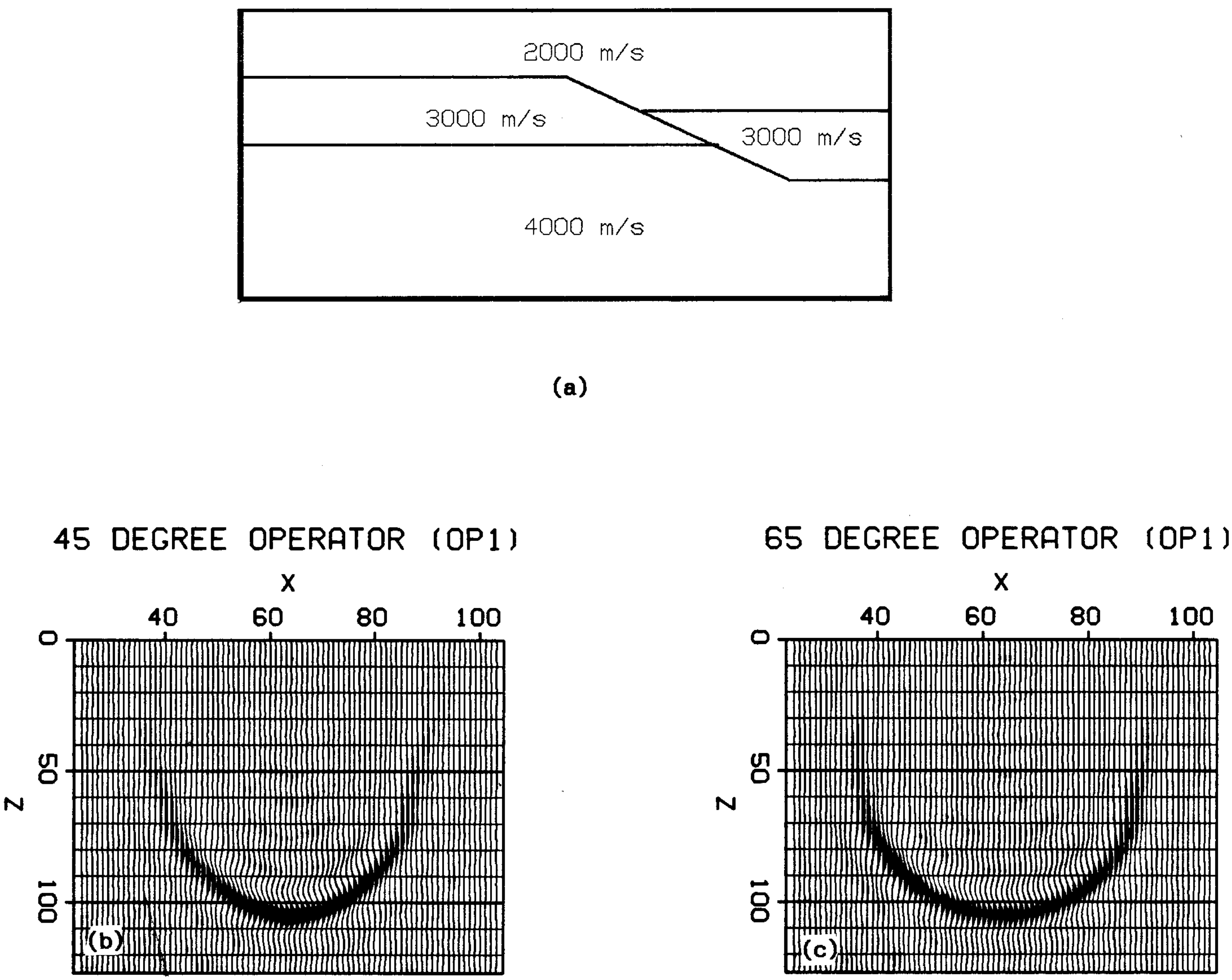

FIG. 4. (a) The velocity model. (b) 45 and (c) 65 degree (OP_1) impulse responses. These were generated by the same input as used for figure 2.

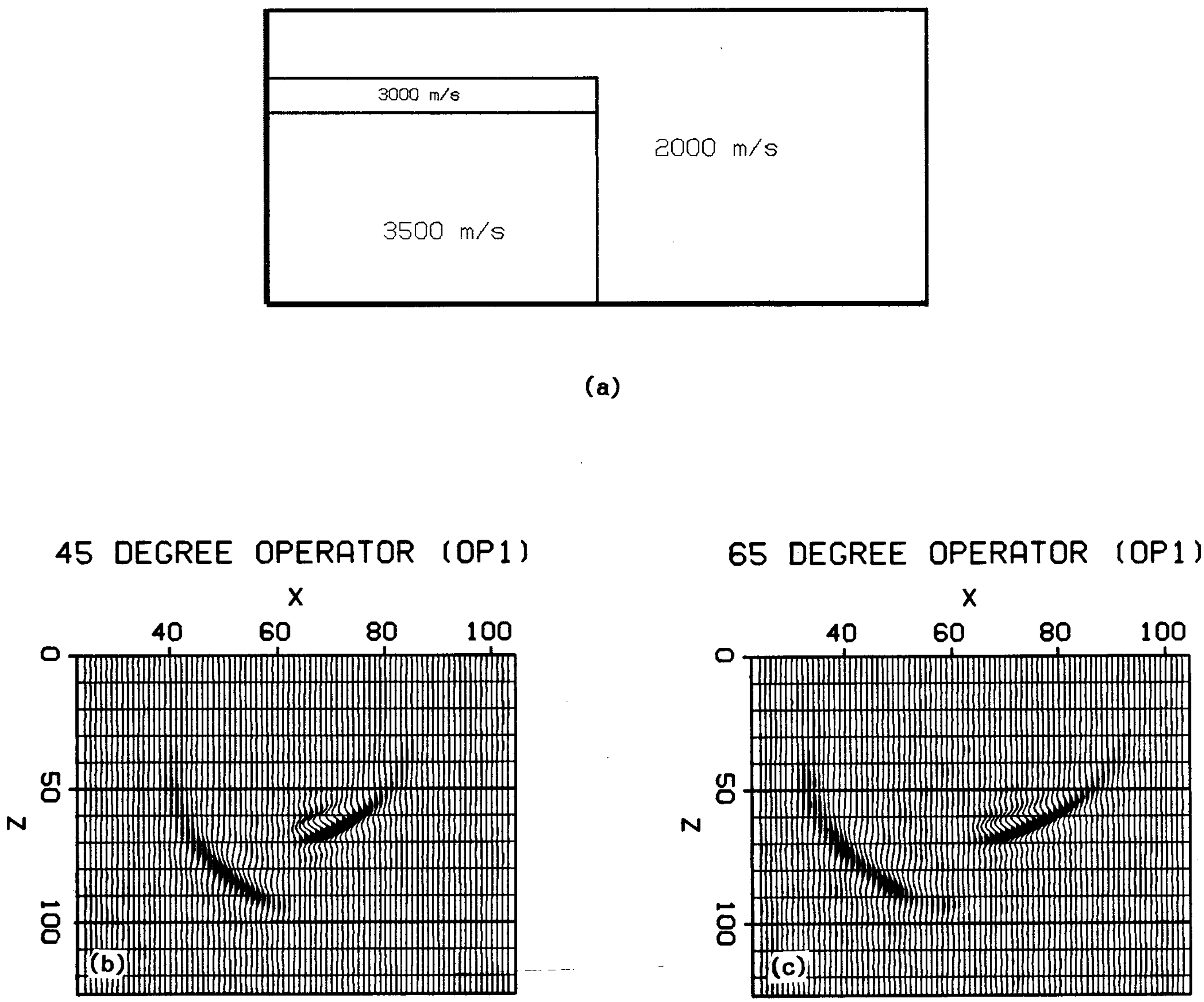

FIG. 5. (a) The velocity model. The impulse responses (b) and (c) have been split into two wavefronts since waves propagate at different velocities in different parts of the medium.

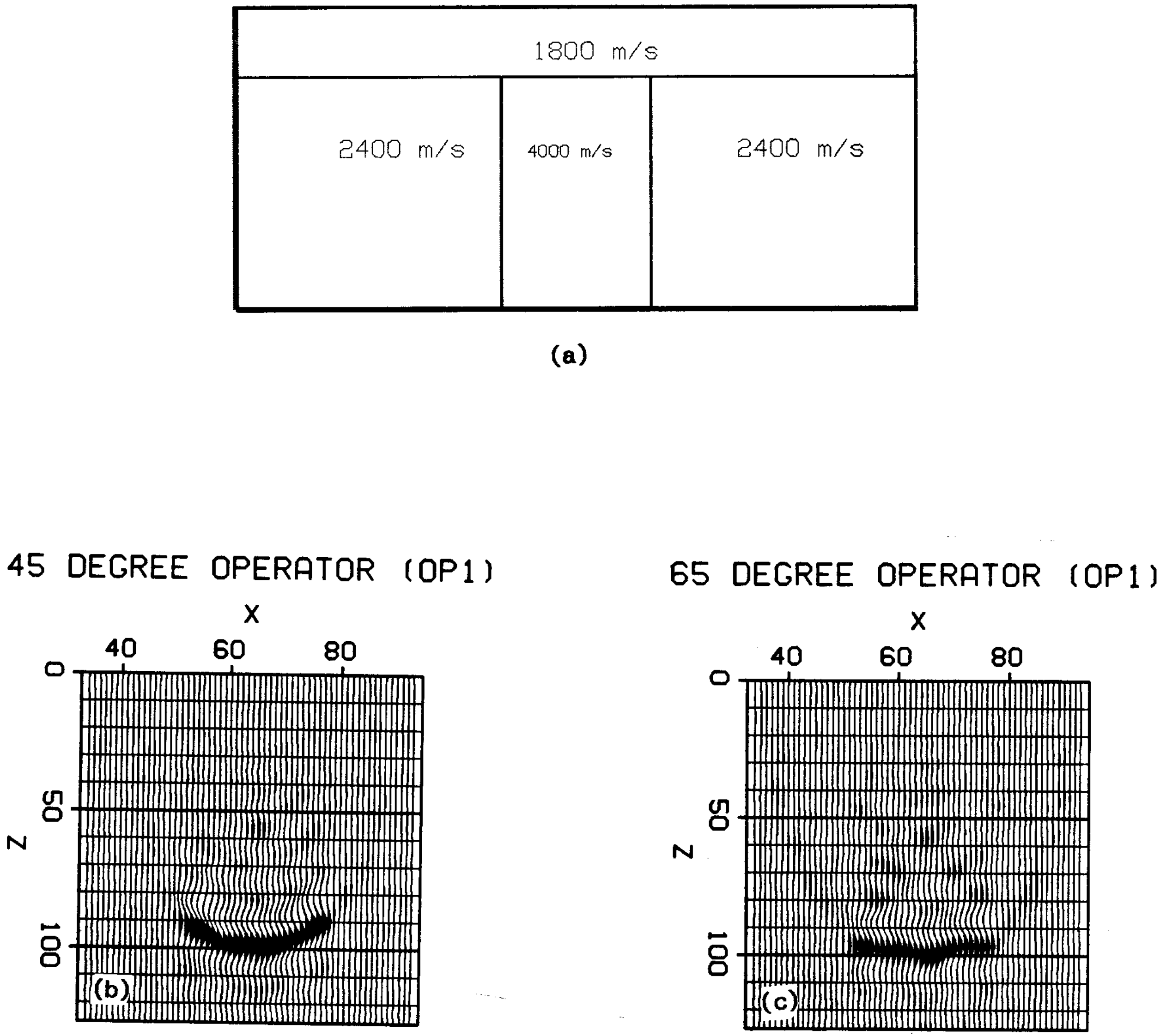

FIG. 6. (a) The velocity model. The impulse responses (b) and (c) no longuer have curvature. All the energy seems to propagate in the narrow salt dome model whose velocity is 4000m/s.

velocity variations.

Fig. 5 (b) and (c) display 45 and 65 degree impulse responses in a medium with both a severe lateral and depth velocity variation media, shown in (a). The impulse responses for both the 45 and 65 have split into two wavefronts along the vertical fault and these wavefronts have different curvatures since they have propagated at different velocities. The behaviors of the 45 and 65 degree impulse responses are significantly different.

Fig. 6 (b) and (c) show 45 and 65 degree impulse responses in a media representing a salt dome model, shown in (a). The wavefronts no longer curved and the energy is trapped in the narrow salt dome.

These three examples show that higher operators than 45 degree (OP_1) must be used for downward extrapolation in a medium with complex tectonics. Muir's (65) degree approximation has a poor behavior for angles beyond 60 degrees. One must used the 80 degree approximation in media with severe lateral velocity variations.

CONCLUSION

Ma's spitting technique and Muir's approximation of the square-root give an operator (OP_1) which is always stable and whose accuracy is to second order in Δz. Third order Taylor series expansion and Muir's approximation of the square-root give an operator (OP_2) whose stability has been demonstrated for a homogeneous medium. It is accurate to fourth order in Δz. Muir's 45, 65 and 6^{th} approximations of the square-root are the three square-root operators of interest for seismic data processing. Muir's 6^{th} approximation handle dips up to 80 degrees well.

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APPENDIX

We derive the stability criterion of the operator (OP_2) in a homogeneous medium. If we define the parameter x to be:

$$x \;=\; \frac{w\Delta z}{v}\left[\, 1 - \frac{v^2}{w^2\Delta x^2}\,\frac{\lambda_k}{1-\beta\lambda_k}\right]^{1/2} \qquad \text{(A-1)}$$

Then, the eigenvalues of (OP_2) are:

$$\gamma_k \;=\; 1 + jx - \frac{x^2}{2} - \frac{jx^3}{6} \qquad \text{(A-2)}$$

If the argument of the square-root in equation ($A-2$) is negative, then:

$$\gamma_k \;=\; \frac{1}{6}[\, -x^3 + 3x^2 - 6x + 6\,] \qquad \text{(A-3)}$$

The Von Neuman stability criterion $|\gamma_k| \le 1$ leads to:

$$\frac{w\Delta z}{v} \le \frac{1}{2} \qquad \text{(A-4)}$$

The above requirement has been already encountered in the non-aliasing condition, equation (6).

If the argument of the square-root in equation (A-1) is negative, the modulus of γ_k is defined by:

$$|\gamma_k|^2 \;=\; (\,1 - \frac{x^2}{2})^2 + (\,x - \frac{x^3}{6})^2 \qquad \text{(A-5)}$$

Here, the Von Neuman stability criterion leads to:

$$\Delta z \;\le\; [\,1 - 4\beta\,]^{1/2}\,\Delta x \qquad \text{(A-6)}$$

where β is Claerbout "β trick" parameter.