An analysis of p-Stolt stretch

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Introduction

In SEP-35 Claerbout suggests a generalization of Stolt stretch that is designed to tune the trace stretching of input to Stolt migration to other than near zero dips. In this method, based on a vertically-stratified medium, a Snell parameter p is chosen about which to optimize. Here I rederive this result in a way that shows clearly that this new p-Stolt stretch correctly positions such constant p events laterally but not vertically. A small numerical example verifies the claim. I then present a straightforward modification that corrects the vertical positioning error. In the process I discover a fundamental weakness of both the original and modified forms of p-Stolt stretch. The difficulty, illustrated in Figure 1, is that the boundary between events with distinct dips, e.g. a disconformity or a fault plane reflection, is misplaced. Finally, I suggest possible methods to ameliorate this imaging problem.

Derivation

When a ray travels in a stratified medium its path is governed by Snell's law

$$p = \frac{\sin\theta}{v} = constant \qquad (1)$$

where v is half the medium velocity. The arrival time slope dt/dx of the associated plane wave at the surface or any datum intermediate is p. The arrival time and offset at the surface of such a ray emanating from a subsurface exploding reflector are

$$t = \int_0^{\tau_0} \frac{d\tau}{\sqrt{1 - p^2 v^2(\tau)}} \qquad (2a)$$

$$x = \int_0^{\tau_0} \frac{p v^2(\tau) d\tau}{\sqrt{1 - p^2 v^2(\tau)}} \qquad (2b)$$

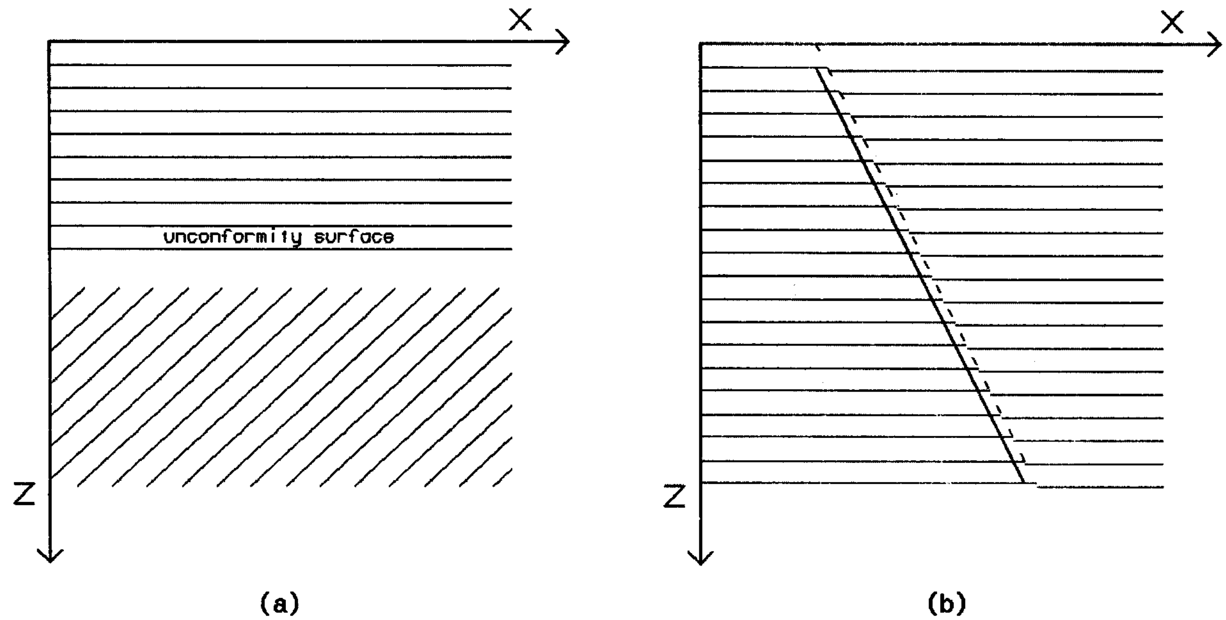

FIG. 1. Some structures which p-Stolt stretch migration won't correctly image. These are characterized by being defined by two or more distinct dips. Figure (a) shows how p-Stolt stretch would handle an angular unconformity. The dipping beds, while correctly positioned laterally, are terminated deeper than the unconformity surface against which they should truncate. In (b) a dipping reflector appears laterally mispositioned because p-Stolt migration places it below the fault plane which gave rise to it.

where τ_0 is the traveltime of a vertical ray.

When $v \equiv v_0$ (constant velocity), we can check our formulas:

$$\frac{\tau}{t} = \sqrt{1 - p^2 v^2(\tau)} = \sqrt{1 - \sin^2\theta} = \cos\theta \tag{3a}$$

$$\frac{x}{v\tau} = \frac{pv}{\sqrt{1 - p^2 v^2(\tau)}} = \frac{\sin\theta}{\cos\theta} = \tan\theta \quad . \tag{3b}$$

Now the idea of a Stolt stretch is to posit a stretching function $f(\tau)$ to make the actual point diffractor moveout (summation) trajectories match constant velocity hyperbolas as well as possible. Claerbout's idea is to match tangents (dt/dx) for a fixed, non-zero Snell parameter. Thus we want the condition that a constant velocity hyperbola of the form

$$f^2(t_p) = f^2(\tau_0) + \frac{x^2}{v_0^2} \quad , \tag{4a}$$

where t_p is the arrival time of a ray with parameter p from vertical traveltime depth τ_0 at offset x, matches the slope of the stretched diffraction curve at the correct offset and arrival time; i.e.,

$$\frac{d[f(t_p)]}{dx} = pf'(t_p) = \frac{x}{v_0^2 f(t_p)} \quad . \qquad (4b)$$

Equations (2) give us

$$\frac{dx}{dt} = pv^2(\tau) \qquad (5)$$

and so we may rewrite (4b) in the form

$$pf(t_p)f'(t_p) = \frac{p\int_0^{\tau_0} v^2(\tau)\,dt}{v_0^2} \qquad (4b')$$

verifying Jon's generalized p-RMS velocity formulation.

The problem with the above is that equation (4a) is not satisfied, as the following layer over a halfspace model will show.

An example

Let the medium consist of a slab with velocity v_0 and traveltime thickness τ_0 overlying a homogeneous halfspace with velocity v_1. From equations (2) we may calculate

$$x(\tau,p) = \begin{cases} \dfrac{p\tau v_0^2}{\sqrt{1-p^2v_0^2}} & \tau\le\tau_0 \\[2ex] \dfrac{p\tau_0 v_0^2}{\sqrt{1-p^2v_0^2}} + \dfrac{(\tau-\tau_0)p\,v_1^2}{\sqrt{1-p^2v_1^2}} & \tau>\tau_0 \end{cases} \qquad (6)$$

and

$$t(\tau,p) = \begin{cases} \dfrac{\tau}{\sqrt{1-p^2v_0^2}} & \tau\le\tau_0 \\[2ex] \dfrac{\tau_0}{\sqrt{1-p^2v_0^2}} + \dfrac{\tau-\tau_0}{\sqrt{1-p^2v_1^2}} & \tau>\tau_0 \end{cases} \quad . \qquad (7)$$

The interesting case is when $\tau>\tau_0$ which is equivalent to

$$t \ge t_0 = \frac{\tau_0}{\sqrt{1-p^2v_0^2}} \quad . \qquad (8)$$

Eliminating τ from equations (6) and (7) lets us express x as a function of t (and p)

$$x = \begin{cases} p v_0^2 t & t \le t_0 \\ p v_1^2 t + p(v_0^2 - v_1^2) t_0 & t > t_0 \end{cases} \qquad . \tag{9}$$

Now, for a frame velocity v_f, the stretching function is given by

$$[f^2(t)]' = \frac{2x}{v_f^2 p} = \begin{cases} \dfrac{2v_0^2 t}{v_f^2} & t \le t_0 \\ \dfrac{2v_0^2 t_0}{v_f^2} + \dfrac{2v_1^2 (t - t_0)}{v_f^2} & t > t_0 \end{cases} \tag{10}$$

which can be directly integrated to give

$$f^2(t) = \begin{cases} \dfrac{v_0^2 t^2}{v_f^2} & t \le t_0 \\ \dfrac{v_0^2 t^2}{v_f^2} + \dfrac{(v_1^2 - v_0^2)(t - t_0)^2}{v_f^2} & t > t_0 \end{cases} \qquad . \tag{11}$$

Place a reflection point at depth τ between τ_0 and t_0. The arrival times from it follow from equations (2). Let us take the following numbers: $p = 0.0003$, $v_0 = 1500$, $v_1 = 2500$, $v_f = 2000$ and $\tau = 1.1$. We may then calculate

$$\begin{aligned} t_0 &= 1.11978502 \\ t_p &= 1.66304368 \\ x_p &= 1303.97743 \\ f(t_p) &= 1.42678459 \\ f(\tau) &= 0.82500000 \\ \left[f^2(\tau) + \frac{x_p^2}{v_f^2} \right]^{1/2} &= 1.05152950 \end{aligned} \tag{12}$$

showing that the summation hyperbola from the stretched apex position, $f(\tau)$, passes about 350 msec *above* the point $f(t_p)$ that we'd like it to pass through.

So we see that we cannot guarantee that simple trace stretching will make the diffraction patterns become tangent to constant velocity hyperbolas at a non-zero p and the apex $p=0$ simultaneously. Indeed, what equation (4b) really does is ensure the correct *lateral* positioning of constant p reflection energy after migration. Nothing is guaranteed about *vertical* positioning (and therefore dip).

A fix

One solution is to not place the trajectory sum at the apex location but to shift the sum vertically to correctly position at $f(\tau)$. This is equivalent to saying that after migration in the stretched coordinates, we unstretch according to a different formula than the input p-Stolt stretch. So if we go ahead and migrate after p-Stolt stretch, the output would be at

$$\eta = \left[f^2(t) - \frac{x^2}{v_f^2} \right]^{1/2} \tag{13}$$

and we want it to show up at τ_0, as determined by equation (2a). This gives us the relation

$$\begin{aligned} \eta^2(\tau_0) &= f^2(t) - \frac{x^2}{v_f^2} \\ &= 2\int_0^t \frac{x(t')}{p\, v_f^2} dt' - \frac{x^2(t)}{v_f^2} \end{aligned} \tag{14}$$

or, differentiating,

$$\frac{d\eta^2}{d\tau_0} = \frac{2\,x(t)}{p\, v_f^2} \left[\frac{dt}{d\tau_0} - p\, \frac{dx}{d\tau_0} \right] \quad . \tag{15}$$

Plugging in equations (2a) and (2b), this simplifies to

$$\begin{aligned} \frac{d\eta^2}{d\tau_0} &= \frac{2\,x(t)}{p\, v_f^2} \sqrt{1 - p^2 v^2(\tau_0)} \\ &= 2\int_0^{\tau_0} \frac{v^2(\tau)}{v_f^2} \left[\frac{1 - p^2 v^2(\tau_0)}{1 - p^2 v^2(\tau)} \right]^{1/2} d\tau \end{aligned} \quad . \tag{16}$$

Thus, integrating (16), we may convert the migrated p-Stolt stretched traces back to ordinary vertical traveltime.

But to what avail?

So far we have derived a Stolt stretch tuned to some non-zero Snell parameter. We have shown how this Snell parameter may then be accurately positioned both laterally and vertically. Unfortunately, this can only be accomplished by sacrificing accuracy at other dips. In particular, flat events would be, incorrectly, shifted up or down. Thus faults and angular unconformities would be no better defined than if we used ordinary migration - the dipping event would migrate correctly but the beds that should truncate against it would be shifted vertically away from the plane of contact.

One possible improvement suggested by Fabio Rocca is to apply a dip filter, D, to select low-dip events and apply ordinary Stolt migration to $D*data$ and p-Stolt migration to $(1-D)*data$, adding the two panels when done. Of course, care should be taken not to choose a p that might correspond to evanescent energy at the higher interval velocities. If one generalized this to a suite of narrow bands of p, the results should be similar to the dip-domain migration of Robinson and Robbins (1978).

Another possibility is to use a more generalized dispersion relation to produce a summation trajectory with the same apex curvature but with additional free parameters. This would let me adjust the summation path to be more nearly tangent to the true moveout curve at the correct offset; i.e. to better carry out the objectives that led to equations (4a,b). This generalizes the game of adjusting Stolt's "W" factor to best fit the data.

Conclusions

We have found that for the purposes of imaging subsurface structures involving angular unconformities, p-Stolt stretch should work no better than ordinary Stolt stretch. A couple of possible fixes have been suggested but not, at present, tried out.

REFERENCES

Claerbout, J.F., 1983, #4.5 Stretching Tricks: SEP-35, p. 191-194.

Robinson, J.C., and Robbins, R.R., 1978, Dip-domain migration of two-dimensional seismic profiles: Geophysics, v. 43, no. 1, p. 77-93.