3D prestack migration of profiles

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ABSTRACT

The 3D one-way wave operator downward continues a 2D seismic dataset in the 3D Cartesian coordinates. The 3D prestack migration gives a correct image of the earth's reflectivity interior since the time events are migrated to the correct location in the (x,y,z) space.

3D prestack migration in Cartesian coordinates is the depth downward continuation of both a 2D set of receiver profiles and a 2D set of source profiles in the frequency domain and the (S-G) coordinates.

This processing is mainly designed to correct the effects of severe depth and lateral velocity variations, wide angle offsets and steeply dipping beds. Most of the energy focuses at non-zero offset lines of the 2D shot profile dataset because the reflectors have significant dips.

Since we alternately downward continue the 2D sets of shot and geophone profiles, this energy focuses on a zero-offset line of another 2D set of shot profiles and so there is no loss of energy on the edges.

INTRODUCTION

Oil exploration has to image increasing complex structures with severe lateral velocity variations and steeply dipping reflectors such as tilted blocks below salt domes. 3D conventional processing, which consists of the NMO correction, stacking and 3D depth migration, has the same problem as its 2D analog, both in the velocity analysis and the correct imaging of the earth's reflectivity interior for complex structures.

Cartesian coordinate prestack migration handles severe lateral velocity variations and wide angle offsets better than all its competitors since it uses a local operator.

3D downward continuation of the pressure wavefield $\psi(x,y,z,w)$ recorded at the earth's surface applies for each z-step NMO (normal move-out) corrections in the x and y directions, and time to depth conversion through the delay pressure function.

Therefore, it migrates the dip events to their correct position in the 3D space and focuses the diffractions at their apex. The downward continuation operator we will now derive uses Muir's 45 degree approximation of the square-root and Ma's splitting technique. We will also demonstrate that the NMO corrections in the x and y directions may be split.

The 3D One-Way Wave Equation

We assume a constant density model and apply the 3D full wave equation to a pressure deviation wavefield $\psi(x,y,z,w)$ (we work in the Fourier domain) in a medium with acoustic velocity $v(x,y,z)$.

$$\frac{\partial^2\psi}{\partial z^2} + \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = - \frac{w^2}{v(x,y,z)^2}\,\psi \tag{1}$$

We then factorize the 3D full wave equation in order to derive the two 3D one-way wave equations which are used to continue either up or down-going waves. The 3D one-way wave equation which downward continues an up-going wave is:

$$\frac{\partial\psi}{\partial z} - \frac{iw}{v}\left[1 + \frac{v^2}{w^2}\frac{\partial^2}{\partial x^2} + \frac{v^2}{w^2}\frac{\partial^2}{\partial y^2}\right]^{1/2}\psi = 0 \tag{2}$$

The difference between this and the 2D one-way wave equation is the presence of the term $\frac{v^2}{w^2}\frac{\partial^2}{\partial y^2}$, which makes this square-root operator a bit more complicated to compute.

Formally, we can solve equation (2) , i.e. find the pressure wave at depth $z+\Delta z$ by the following relation :

$$\psi(x,y,z+\Delta z,w) = \exp\left(\frac{iw\Delta z}{v}\left[1 + \frac{v^2}{w^2}\frac{\partial^2}{\partial x^2} + \frac{v^2}{w^2}\frac{\partial^2}{\partial y^2}\right]^{1/2}\right)\psi(x,y,z,w) \tag{3}$$

We factorize the above expression in order to get the delay pressure operator:

$$\psi(x,y,z+\Delta z,w) = exp\left[\frac{iw\Delta z}{v}\right]exp\left(\frac{iw\Delta z}{v}\left(-1 +\left[1 + \frac{v^2}{w^2}\frac{\partial^2}{\partial x^2} + \frac{v^2}{w^2}\frac{\partial^2}{\partial y^2}\right]^{1/2}\right)\right)\psi \tag{4}$$

where the first term of the right side of equation(4) is the delay pressure operator.

45 Degree Expansion of The Square-Root

The 2D migration operators operate on vector which can represent either a set of receivers or a set of shots. The 3D migration operators downward continue a matrix $D(z,w)$ whose rows represent the data in the x direction and columns the data in the y direction. Therefore, the operator $\frac{\partial^2}{\partial x^2}$ operates on the rows of D and its analog in the y direction on the columns.

Equation (4) leads to the following 3D one way wave operator (OP):

$$(OP) = exp[\frac{iw\Delta z}{v}] \exp\left(\frac{iw\Delta z}{v} \left[-1 + [1 + \frac{v^2}{w^2}\frac{\partial^2}{\partial x^2} + \frac{v^2}{w^2}\frac{\partial^2}{\partial y^2}]^{1/2} \right] \right) \tag{5}$$

This operator downward continues an up-going pressure deviation wavefield recorded on a 2D grid by a step of Δz. In the above equation, the term with the square-root represents the *NMO correction vector*, i.e. NMO in both x and y directions.

We define the operator S to be the argument of the square-root operator, i.e. the sum of the second partial derivative operators scaled by the frequency and the the velocity matrix as it appears in equation (5). Muir's 45 degree approximation of the square-root yields:

$$I - (I + S)^{1/2} = \frac{S}{2 + \frac{S}{2}} \tag{6}$$

We plug the above expression in the exponential of equation (5) and we expand the exponential by its second order partial rational expansion which is exactly the Crank-Nicholson algorithm. This yields:

$$\left[I - \frac{iw\Delta z}{2V} \frac{S}{2 + \frac{S}{2}} \right] D(z+\Delta z, w) = \left[I + \frac{iw\Delta z}{2V} \frac{S}{2 + \frac{S}{2}} \right] exp[\frac{iw\Delta z}{V}] D(z,w) \tag{7}$$

Where I represents the identity matrix and V the velocity matrix. Since the matrices V and S don't commute, in order to simplify the above expression we first multiply both sides by V, then by the matrix $2 + \frac{S}{2}$, and obtain :

$$\left[S\ (\ V - iw\Delta z\ ) + 4V \right] D(z+\Delta z, w) = \left[S(\ V + iw\Delta z\ ) + 4V \right] exp[\frac{iw\Delta z}{V}]\ D(z,w) \quad (8)$$

The reader may notice that the 2 operators containing the matrix S are complex conjugates. Since we want to simplify these operators, we first define S_x and S_y to be $\frac{V^2}{w^2}\frac{\partial^2}{\partial x^2}$ and $\frac{V^2}{w^2}\frac{\partial^2}{\partial y^2}$ respectively. Then, we use *Ma's philosophy* to split the operators respectively to the right and left of equation(8). This yields:

$$\left[I + S_x\ \frac{(\ V - iw\Delta z\ )}{4V} \right] \left[I + S_y\ \frac{(\ V - iw\Delta z\ )}{4V} \right] D(z+\Delta z) = \quad (9)$$

$$\left[I + S_x\ \frac{(\ V + iw\Delta z\ )}{4V} \right] \left[I + S_y\ \frac{(\ V + iw\Delta z\ )}{4V} \right] exp[\frac{iw\Delta z}{V}]\ D(z+\Delta z)$$

Thus, we may define the $X-NMO$ and $Y-NMO$ operators to be respectively:

$$(X-NMO) = \frac{I + S_x\ \frac{(\ V + iw\Delta z\ )}{4V}}{I + S_x\ \frac{(\ V - iw\Delta z\ )}{4V}} \quad (10a)$$

$$(Y-NMO) = \frac{I + S_y\ \frac{(\ V + iw\Delta z\ )}{4V}}{I + S_y\ \frac{(\ V - iw\Delta z\ )}{4V}} \quad (10b)$$

Finally, we have split the 3D downward continuation operator into three operators. The *delay pressure operator* (analog to the delay pressure function described by Claerbout in FGDP), the $X-NMO$ operator which corrects the normal move-out in the x direction and the $Y-NMO$ operator which corrects the NMO in the y direction.

At each step of the downward continuation, one applies sequentially the *delay pressure operator* on the entire 2D dataset grid, the $X-NMO$ operator on each set of shot profiles(or receiver profiles), then the $Y{-}NMO$ operator on the transposed 2D dataset as described by Brown in SEP (15).

The downward continuation operator we have derived is always stable since it is split into 3 operators which are all unconditionally stable. The reader may notice that the $X{-}NMO$ and $Y{-}NMO$ operators are exactly the analog of the 45 degree operators in the 2D case.

CONCLUSION

The previous demonstration may be generalised to higher order operators. The 3D downward continuation at each z-step first corrects the NMO on the *inline sections*, then corrects the NMO on the resulting *cross line* sections of the previous processing.

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REFERENCES

Brown, David, 1978, Splitting and separation of differential equations with applications to three dimensional migration and lateral velocity variation: SEP-15, p.214-229.

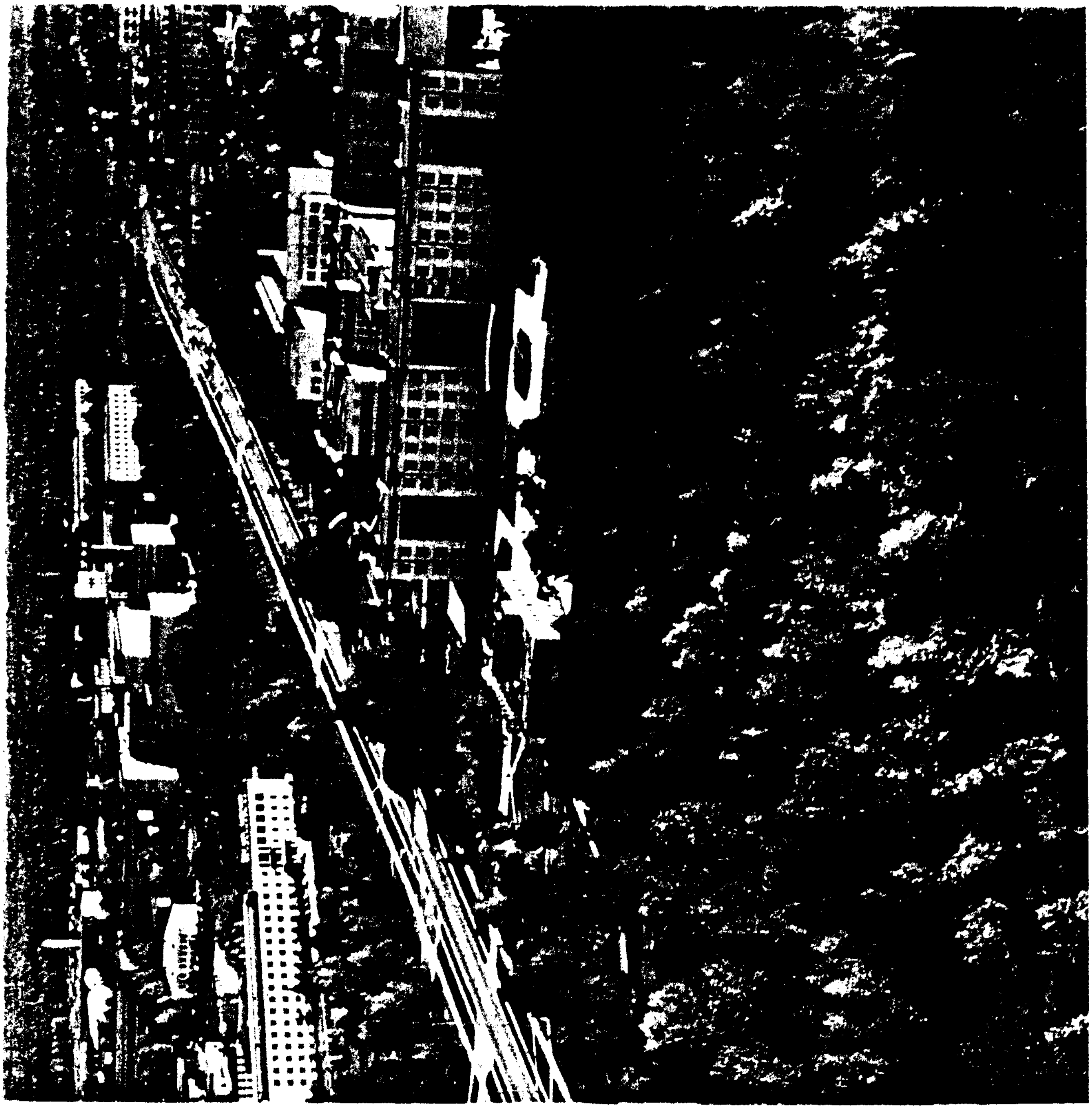
Claerbout, Jon F., 1983, Imaging the earth's interior.

Claerbout, Jon F., 1976, Three dimensional wave equation: SEP-8, p.15-16.

Jacobs, Allan, 1982, The pre-stack migration of profiles: Ph.D. thesis, Stanford University.

Muir, F. and Jacobs, B., 1981, Continued fraction for the square-root function: SEP-26, p.183-195.

Muir, F. and Jacobs, B., 1981, High order migration operators for laterally homogeneous Media: SEP-26, p.163-181.



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