Kinematics[†] and Dynamics[‡] of Dip Move-Out

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Abstract

The dispersion relation for the full migration of a constant-offset section will be derived using kinematic arguments. Residual migration arguments will be used to decompose the constant-offset migration operator into pre-stack partial migration followed by post-stack zero-offset migration. A kinematicly exact dip move-out (DMO) will then be developed from the pre-stack partial migration. I then continue from that kinematic result to improve handling of amplitudes and to develop an offset extrapolation relation.

Kinematic DMO

The migration of a spike, $\delta(x)\delta(y)\delta(t-t_h)$, on a common-offset section, for constant velocity, produces the ellipsoid

$$\left[\frac{x}{vt_h/2}\right]^2 + \left[\frac{y}{vt_n/2}\right]^2 + \left[\frac{z}{vt_n/2}\right]^2 = 1 \tag{1-1}$$

Here t_h is the arrival time of the spike on the common-offset section, t_n is the normal move-out (NMO) time,

$$\left[vt_h/2\right]^2 = \left[vt_n/2\right]^2 + h^2 \tag{1-2}$$

and the offset vector is assumed to be in the x direction; no generality is lost by this choice of coordinates. Eliminating t_h from equation (1-1) produces

† Seismologists adopt the word kinematic from physicists, when they handle correctly only the travel time.

‡ Dynamics handles also the amplitudes correctly.

$$\frac{x^2}{1 + \left(2h / vt_n\right)^2} + y^2 + z^2 = \left(vt_n / 2\right)^2 \tag{1-3}$$

The (time-dependent) change of variable

$$\chi^2 = \frac{x^2}{1 + \left(2h / vt_n\right)^2} \tag{1-4}$$

compresses the ellipsoid (1-3) to the sphere

$$\chi^2 + y^2 + z^2 = \left(vt_n / 2\right)^2 \tag{1-5}$$

which is the zero-offset migration of a spike at the NMO time t_n. Zero-offset migration has the well known dispersion relation §

$$\left(\frac{v}{2}\right)^2 \left(k_\chi^2 + k_y^2 + k_z^2\right) = \omega_n^2 \tag{1-6}$$

Inserting the Fourier-transform of change of variables (1-4),

$$k_\chi^2 = k_x^2\left[1 + \left(2h / vt_n\right)^2\right] \tag{1-7}$$

into equation (1-6) gives the important result

$$\left(\frac{v}{2}\right)^2 \left(k_x^2 + k_y^2 + k_z^2\right) = \omega_n^2 - \left(\frac{h}{t_n}\right)^2 k_x^2 \tag{1-8}$$

This is a dispersion relation for full migration of a common offset section; it maps the normal moved-out common-offset section $p_n(t_n,x,y)$ directly to the migrated section $p_m(z,x,y)$. This can be done in two steps:

(1) Dip move-out

Substituting ω_0^2 in the right hand side of equation (1-8), when

$$\left(\frac{\omega_n}{h / t_n}\right)^2 = k_x^2 + \left(\frac{\omega_0}{h / t_n}\right)^2 \tag{1-9}$$

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§ Variables subscripts notation in time and Fourier domains is

$$P_\alpha(\omega_\alpha,k_x,k_y) = \int dx e^{ik_x x} \int dy e^{ik_y y} \int dt_\alpha e^{-i\omega_\alpha t_\alpha} p_\alpha(t_\alpha,x,y)$$

where α is a general subscript. I will later use equations like (1-5) and (1-6) as time domain and Fourier domain expressions of the same process (in this case three-dimensional zero-offset migration). The connection between these two equations is that equation (1-5) is the shape of the Green function (or impulse response) of the differential equation that Fourier-transforms to the dispersion relation (1-6).

Equation (1-9) is time dependent: ω_n and t_n appear together, it is velocity independent (but constant velocity was assumed !), otherwise it is similar to two-dimensional zero-offset migration,

$$\left(\frac{\omega}{v}\right)^2 = k_x^2 + k_z^2 \qquad (1\text{-}10)$$

Table 1 summarizes that similarity.

	Migration	DMO
velocity	v	h / t_n
variables	k_x	k_x
	ω	ω_n
	vk_z	ω_0
input	$P_u(\omega,k_x)$	$P_n(\omega_n,k_x)$
output	$P_m(k_z,k_x)$	$P_0(\omega_0,k_x)$

TABLE 1. Variables and parameters of migration and of DMO.

The DMO is a two-dimensional operation (for a fixed y); the input is normal moved-out, common-offset section $p_n(x,t_n)$; the output in the zero-offset $p_0(x,t_0)$ plane. The offset is along the x direction; The DMO moves in t and x directions; nothing is moved in the y direction. In the case of no dip, $k_x = 0$ and the DMO just copies the input to the output; $\omega_0 = \omega_n$.

(2) Zero-offset migration

Migrating the extrapolated zero-offset data, $p_0(t_0,x,y)$, to the migrated section $p_m(z,x,y)$, with the dispersion relation

$$(v/2)^2\left(k_x^2 + k_y^2 + k_z^2\right) = \omega_0^2 \qquad (1\text{-}11)$$

The similarity between DMO (equation (1-9)) and migration (equation (1-10)), can be used to derive a Stolt like DMO method.

Recall Stolt's method for migration:

$$P_m(k_z,k_x) = \frac{\partial(\omega,k_x)}{\partial(k_z,k_x)} P_u\left[\omega(k_x,k_z),k_x\right] \qquad (1\text{-}12)$$

where $p_m(x,z)$ is the migrated section and $p_u(x,t)$ is the unmigrated section. $\omega(k_x,k_z)$ is given by equation (1-10). Table 1 is used to translate the variables used in equation (1-12), yielding the following Stolt-DMO

$$P_0(\omega_0,k_x) = \frac{\partial(\omega_n,k_x)}{\partial(\omega_0,k_x)} P_n\left[\omega_n(k_x,\omega_0),k_x\right] \tag{1-13}$$

$\omega_n(k_x,\omega_0)$ is given by equation (1-9). This is not the final result, however, because the dispersion relation (1-9) is time-dependent and relation (1-10) is time independent; therefor the DMO does not correspond to a physical time independent wave equation like migration. Rewrite equation (1-9) as

$$\omega_n(k_x,\omega_0) = \omega_0\left[1 + \left(\frac{hk_x}{\omega_0 t_n}\right)^2\right]^{1/2} = \omega_0 A \tag{1-14}$$

where $A(h,k_x,\omega_0,k_z)$ is implicitly defined. The Jacobian is also time dependent:

$$\frac{\partial(\omega_n,k_x)}{\partial(\omega_0,k_x)} = \left|\frac{\omega_0}{\omega_n}\right| = A^{-1} \tag{1-15}$$

Equation (1-13) holds for every t_n separately. The input is decomposed to t_n layers, each of which Fourier-transforms like

$$P_n(\omega_n,k_x) = e^{-i\omega_n t_n} p_n(t_n,k_x) \tag{1-16}$$

Substituting equations (1-15) and (1-16) into (1-13) and integrating, we obtain

$$P_0(\omega_0,k_x) = \int dt_n\, A^{-1} e^{-i\omega_0 A t_n} p_n(t_n,k_x,h) \tag{1-17}$$

which the result of chapter 1 of Hale's thesis (1983).

Dynamic DMO

Equation (1-1) says nothing about the reflection coefficient amplitude along the ellipsoid. The dispersion relation (1-8), however, will produce a certain amplitude, which corresponds to a uniform amplitude on the sphere (1-5). Stew Levin called to my attention that this amplitude is not necessarily correct; the question was raised: what distribution of reflection coefficients along the ellipsoid will reflect an isotropic spherical wave, diverging from the source, to an isotropic spherical wave converging on the receiver?

An approximate answer for high frequency waves may be obtained by assuming flux conservation within ray tubes The geometry is shown in Figure 1. The calculation can be done in the x,z plane and then rotated around the x axis to give the three-dimensional

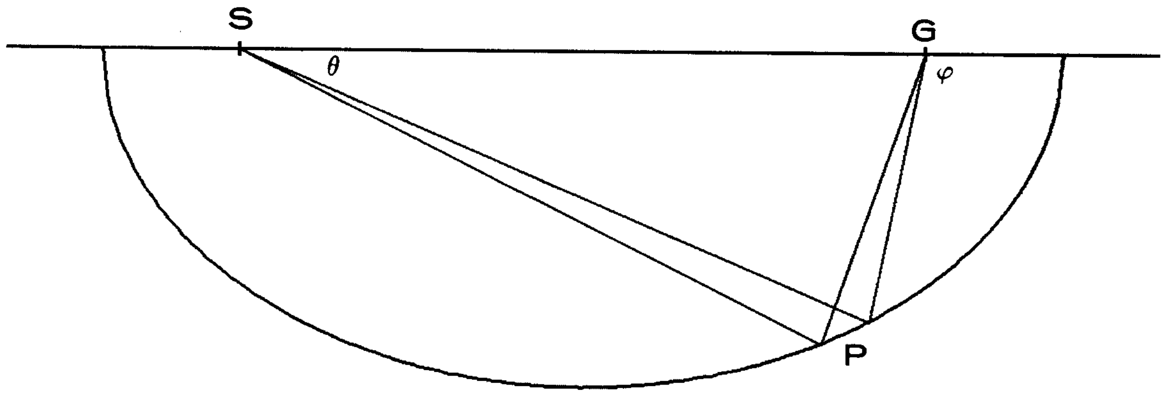

FIG. 1. The ray tubes.

picture. The energy in the diverging ray tube is $d\theta$. If $R(\theta)$ is the the square of the absolute value of the reflection coefficient at point P, then the energy in the converging ray tube is $R(\theta)d\theta$. Isotropic converging wave requires that $R(\theta)d\theta = d\varphi$, hence

$$R(\theta) = \left| \frac{d\varphi}{d\theta} \right| \tag{2-1}$$

To evaluate this expression, refer to Figure 1. The offset is

$$SG = 2h \tag{2-2a}$$

The total travel time is

$$SP + PG = vt_h \tag{2-2b}$$

The sine theorem for ΔSPG gives

$$\frac{SG}{\sin(\varphi-\theta)} = \frac{GP}{\sin\theta} = \frac{SP}{\sin\varphi} \tag{2-2c,d}$$

Equations (2-2) simplify to

$$2h(\sin\theta + \sin\varphi) = vt_h \sin(\varphi - \theta) \tag{2-3}$$

Differentiating equation (2-3) gives the result

$$R(\theta) = \frac{vt_h \cos(\varphi - \theta) + 2h\cos\theta}{vt_h \cos(\varphi - \theta) - 2h\cos\varphi} \tag{2-4}$$

shown in Figure 2.

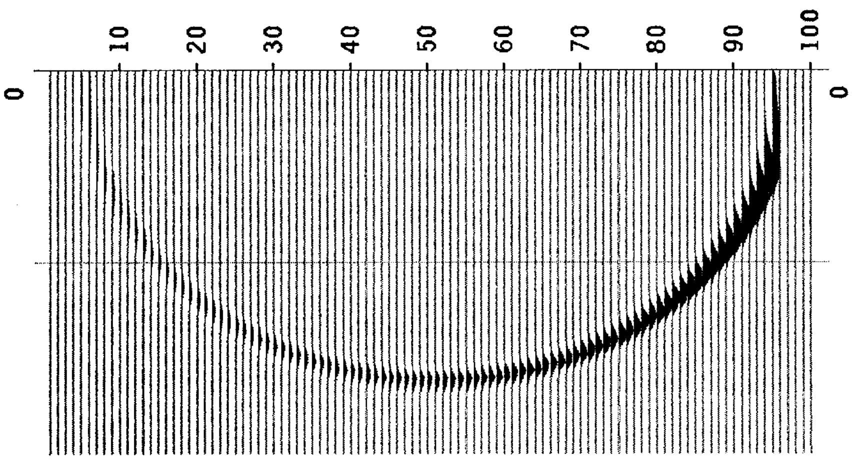

FIG. 2. The asymmetric ellipse

Can this be right ? Superposing such asymmetric ellipses to Kirchhoff-migrate before stack, would bias the amplitudes of reflectors; reflectors dipping away from the receiver toward the shot would image stronger than reflectors dipping the other way; migration of plane dipping reflectors would have wrong amplitudes.

The validity of the ray approximation near caustics like the foci of an ellipse is questionable but this analysis considers the field in a finite distance from the singular points of the ray approximation. This problem is related to Eisner's acoustic reciprocity paradox (1983); Reciprocity predicts that exactly the same data would be recorded if the locations of shot and geophone were reversed; i.e., the data is an even function of h. The problem here is not with the field at the points S and G, but around these points, and this field may not be a subject to reciprocity.

Stew Levin (this report) may have the solution; in short, superposition of the coefficient $R(\theta)$ is not valid. the exploding reflectors concept allows superposition of reflectors because it assumes that the reflectors are the wavefield at time $t=0$; the coefficient R is the reflection coefficient (or even the absolute value of it) but not a wavefield. Equation (1-17) followed by zero-offset migration will produce a symmetric impulse response. The reflection coefficient has yet to be imaged by summing (stacking) over all the offsets. This implies that the extrapolated zero-offset sections p_0 of equation (1-17) depend on the offset from which they were extrapolated, $p_n(t_n,k_x,h)$, even though they all correspond to same earth model.

Offset extrapolation

The previous section ended in concluding that the extrapolated zero offset section p_0

$$P_0(\omega_0,k_x,h) = \int dt_n \, A^{-1} e^{-i\omega_0 A t_n} p_n(t_n,k_x,h) \tag{1-17}$$

is not offset independent; the variable h cannot be dropped in the left hand side. This is unfortunate because reflection seismology data contain a finite range of offsets. We would like to believe that all the common-offset sections contain the same information except random and aliasing noise but it seems that is not true at least when we used equation (1-17) to extrapolate the zero-offset from a general offset. In practice we make this assumption when we have a narrow range of offsets. I will show directly that this assumption is only approximately correct.

If $P_0(\omega_0,k_x,h)$ of equation (1-17) is independent of h then for all ω_0 and k_x we have

$$0 = \frac{\partial}{\partial h} P_0(\omega_0,k_x,h) = \frac{\partial}{\partial h} \int dt_n \, A^{-1} e^{i\omega_0 A t_n} P_n(t_n,k_x,h) \tag{3-1}$$

A is defined by equation (1-14). For all t_n we have

$$0 = \frac{\partial}{\partial h}\left[A^{-1} e^{i\omega_0 A t_n} P_n(t_n,k_x,h)\right] \tag{3-2}$$

Differentiating this equation, using

$$A\frac{\partial A}{\partial h} = \frac{hk_x^2}{\omega_0^2 t_n^2} \ ,$$

gives

$$\frac{\partial P_n}{\partial h} = \left\{ A^{-1}\frac{\partial A}{\partial h} - i\frac{k_x^2 h}{\omega_0 t_n} \right\} P_n \tag{3-3}$$

that can be solved to yield the familiar result

$$P_n(t_n,k_x,h) = A \exp\left[-i\frac{k_x^2 h^2}{2\omega_0 t_n}\right] P_0(\omega_0,k_x) \tag{3-4}$$

which is known to be an approximation; the phase of equation (3-4) was shown (Ronen, 1983) to give a parabolic impulse response, instead of the elliptical exact one. The amplitude term enhances dipping events.

Conclusions

The pre-stack partial migration is a two dimensional, in-line, operator. It is velocity independent and time dependent. Attempts to go beyond kinematic analysis led me to a paradoxical amplitude, and to an approximate offset extrapolation. No approximation was made except in assuming that the extrapolated zero-offset sections were independent of the offset from which they were extrapolated. There still may be an exact offset extrapolation but it is not given by the kinematicly exact relation of equation (1-17).

REFERENCES

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