A short note on implementing hyperbolic velocity filters

Paul Fowler

Abstract

Tatham et al. (1982) and Noponen and Keeney (1983) have described a method of applying a normal moveout velocity filter during slant stacking (tau-p transform). This paper discusses briefly a few features of formulating this hyperbolic velocity filter in a vectorized manner suitable for implementation in an array processor.

Introduction

Within the limits of the usual approximation, continuous reflectors appear in common midpoint gathers as hyperbolas. The parameters generally used to to describe these hyperbolas are the zero-offset intercept time and the normal-moveout (NMO) velocity which characterizes the curvature. At any specified point on such a hyperbola, the velocity may be alternatively defined not by the NMO value, but by the local moveout, that is, by the slope of the tangent to the hyperbola at that point. Common midpoint stacking at a given NMO velocity, or the constructive integration of the data along hyperbolic trajectories, is a velocity selective filter. Slant stacking (tau-p transforming) can be thought of as summation along the tangents to the hyperbolas instead of along the hyperbolas themselves. Tatham et al. (1982) and Noponen and Keeney (1983) pointed out that the local moveout characterization of velocity in a midpoint gather can be exploited to incorporate a hyperbolic velocity selectivity into slant stacking of the gather similar to that of NMO stacking.

Consider an hyperbola in a (h,t) gather specified by the parameters of zero offset time t_0 and normal moveout velocity v:

$$t^2 = t_0^2 + \frac{h^2}{v^2} \qquad (1)$$

We can differentiate (1) to get the tangent to such a hyperbola:

$$\frac{dt}{dh} = \frac{h}{tv^2} \tag{2}$$

Identifying dt/dh as the ray parameter, p, yields

$$v^2 = \frac{h}{tp} \tag{3}$$

This equation tells us how to assign a normal moveout velocity to a point (h,t) if the ray parameter p is specified; it thus provides the key to incorporating moveout velocity information into slant stacks. Essentially, equation (3) shows whether each each combination of a point (h,t) and a ray parameter p can possibly correspond to a tangent to a hyperbola whose moveout velocity meets desired criteria. If they can, that point is included in the slant stack sum for that value of p; if not, it is excluded.

Vectorized implementation of hyperbolic velocity filters

The algorithm described above is inefficient: it requires a computation and decision step for each (h,t,p) point. The most efficient implementation of slant stacking in general is achieved in the Fourier transform domain (Ottolini and Claerbout, 1984; Harlan, 1983); however, the velocity filtering step described, being time and space variable, requires realization on untransformed data. Slant stacking in the time-space domain can be accomplished much faster if the algorithm is vectorized to allow use of an array processor.

The slant stack algorithm in the time domain can be readily vectorized. Consider a midpoint gather as a two dimensional data array, $data(h,t)$. This may equally well be described as a single vector, or one-dimensional array, $\overline{data}(h)$; each of the elements of the one dimensional array is then a vector parametrized by time, namely the corresponding column of $d(h,t)$. The output of a slant stack algorithm can similarly be seen as another data vector, $\overline{stack}(p)$, whose elements also are vectors parametrized by (zero-offset) time. The slant stack process is then implemented simply by successively applying a time shift of $t=ph$ to each element of $\overline{data}(h)$ and summing it into the appropriate element of $\overline{stack}(p)$. The size of the ph shifts will not in general correspond to the interval of time discretization of the data, so an interpolation step is implicit in each shift as well, although we do not explicitly describe the interpolation in the algorithm descriptions which follow. In summary the time domain slant stack algorithm becomes:

```
for each p {
    for each h {
        shift data(h) by ph
        add shifted data(h) into stack(p)
    }
}
```

To incorporate the velocity filtering step into the vectorized slant stack, we rewrite equation (3) as

$$t = \frac{h}{pv^2} \tag{4}$$

Suppose we wish to include in our slant stack only those points corresponding to NMO velocities between some bounds v_{min} and v_{max}. Let us assume that these velocity bounds are constants independent of h and t. To apply these bounds, we need only limit our stack to those points in each vector $\overline{data}(h)$ for which

$$\frac{h}{pv_{max}^2} = t_{min} \le t \le t_{max} = \frac{h}{pv_{min}^2} \tag{5}$$

We thus need to create for each p, a second array $\overline{limits}(h)$, whose elements are vectors that contain zeroes outside the appropriate time limits and ones inside. Then for a given h, taking the inner product (pointwise multiplication) of the vector $\overline{data}(h)$ with the vector $\overline{limits}(h)$ truncates the data being included in the stack as desired. The algorithm for constant velocity bounds thus becomes:

```
for each p {
    for each h {
        truncate data(h) between t_min and t_max
        shift data(h) by ph
        add shifted data(h) into stack(p)
    }
}
```

Incorporating depth-variable velocity bounds

For practical applications such as multiple suppression we often might want to use velocity bounds that are not constant. The most common uses would be of velocities which vary with depth, or with zero-offset travel time. This case can be incorporated into the vectorized algorithm described above if we make the reasonable assumption that the velocity bounds are monotonic, non-decreasing functions of zero-offset travel time. Equation (1), which describes the hyperbola with NMO velocity v which passes through a given point (h,t), can be rewritten using the substitution from equation (3) as

$$t^2 = t_0^2 + hpt$$

The zero intercept, or zero-offset travel time, for a hyperbola passing through (h,t) with tangent p is then given by

$$t_0 = \sqrt{t(t-hp)} \tag{6}$$

This tells us how to relate $v(t)$ to $v(t_0)$ for a given h and p. Let us use a prime notation on velocities to designate the assignment indicated by equation (6) of a corresponding zero-offset velocity to a point (h,p,t) with non-zero offset: $v'(t) = v\left[t_0=\sqrt{t(t-hp)}\right]$. Suppose that for some time t_1 and a specified h and p,

$$\sqrt{\frac{h}{pt_1}} = v'(t_1) = v'_{\max}(t_1) = v_{\max}\left[t_0=\sqrt{t_1(t_1-hp)}\right]$$

Then for any $t_2 \geq t_1$,

$$v'(t_2) = \sqrt{\frac{h}{pt_2}} \leq \sqrt{\frac{h}{pt_1}} = v_{\max}\left[t_0=\sqrt{t_1(t_1-hp)}\right]$$

But $t_1 \leq t_2$ implies that

$$\sqrt{(t_1(t_1-hp)} \leq \sqrt{(t_2(t_2-hp)}$$

We can now utilize our assumption that $v_{\max}$ is a monotonic function of t_0 to conclude that

$$v_{\max}\left[t_0=\sqrt{t_1(t_1-hp)}\right] \leq v_{\max}\left[t_0=\sqrt{t_2(t_2-hp)}\right]$$

Hence we have shown that

$$v'(t_2) \leq v_{\max}\left[t_0=\sqrt{t_2(t_2-hp)}\right] = v'_{\max}(t_2)$$

In summary, we have shown that, for a given p and h, if we can find a time t_1 for which the hyperbola through the point (h,t_1) with slope p has the maximum allowable zero-offset NMO velocity, then at no subsequent time t at that offset h can a hyperbola of lower velocity pass through (h,t) with slope p. In other words, for each p and h, truncation of the data vector before a suitable lower cutoff time value restricts the maximum velocities included in the slant stack, just as in the case of velocity bounds which were independent of time. A similar statement and proof will hold for minimum velocities and maximum times, so the algorithm outlined above for constant velocity bounds will also work for velocity bounds that vary as monotonic, non-decreasing functions of zero offset time. The actual values of the time cutoffs to use may be more difficult to find for this case than for constant velocity bounds, but once they are established the same algorithm may be used for implementing the filter.

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Thompson honored for geophysics work

George A. Thompson

The Geological Society of America has chosen Prof. George A. Thompson, chairman of the Department of Geophysics, as recipient of the George P. Woollard Award "in recognition of distinguished contributions to geology through the application of the principles and techniques of geophysics."

Thompson is the first holder of this award, which is named for one of the pioneers in the plate tectonics revolution. In the nomination, Prof. Burt Slemmons of the University of Nevada said that Thompson comes closer than anyone else "to being a renaissance earth scientist."

Slemmons cited Thompson's "outstanding research results obtained from applying geophysical techniques to the solution of regional geologic problems of the lithosphere" and his impact on student achievements. The nomination mentioned Thompson's national visibility, his outstanding academic record, and his versatility in teaching a broad range of subjects, from sedimentary petrology to advanced courses in geophysics.

Thompson's early geophysical studies on the tectonics of the Basin and Range Province established the framework for later research by Thompson and his students that established two periods of extension, each characterized by a different stress regime. "In addition," Slemmons pointed out, "he and his students have made important contributions to thermal crustal evolution, plateau uplift, and the interpretation of COCORP seismic reflection data."

Thompson has been an NSF postdoctoral fellow at the Lamont Observatory, Columbia University, and a Guggenheim fellow in New Zealand. As a graduate student with degrees from Penn State and MIT, Thompson taught Stanford's first geophysics course in 1947.

He joined the faculty as an assistant professor upon receiving his Ph.D. in 1949, became chairman of geophysics in 1967, and chaired both geophysics and geology from 1979 to 1982. In 1980 Thompson received an endowed chair as the Otto N. Miller professor of earth sciences. Recently, Dean Allan Cox delegated to Thompson responsibility for tracking faculty appointments and promotions in the School of Earth Sciences.

Thompson accepted the Woollard Award at the annual meeting of the Geophysics Division of the Geological Society of America in Indianapolis on Nov. 2.