Velocity Analysis Using Prestack Migration

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Introduction

Among other processes, the conventional processing sequence involves three major steps : NMO, stacking, and migration, in that order. The velocities used in the migration process are those used in stacking which in turn are obtained by the conventional velocity stacks.

This procedure has few advantages. The two main ones are the signal to noise ratio improvement and data compression to a manageable and displyable size. Since this data resembles (or is supposed to resemble) a zero offset section, then the exploding reflector model can be used and migration involves downward continuation of the upgoing field only.

In regions of very small dips, this procedure is quite accurate and resorting to other methods (like prestack migration) is usually unjustifiable, considering the cost involved. It is a well known fact however that CMP stacking loses its accuracy in a variety of situations. For example, ambiguity exists between multiples and primaries which share the same vertical travel time. Also, for dipping beds a higher velocity is needed $(v/\cos\alpha)$, which introduces an ambiguity between velocity and dip as shown in Figure 1. A more serious problem is that in a laterally varying medium, the whole concept of CMP stacking fails.

It would be useful to have a scheme in which velocity estimation is independent of CMP stacking. Interweaving this velocity estimation with migration is our ideal goal. Since our final result is common geophone gathers, the dependence of reflectivity on the angle of incidence can be readily obtained since in this gather we can see the reflectivity $R(x,z)$ for different sources which illuminate that point at different angles. This is a helpful peace fo information in stratigraphic interpretation.

Prestack Migration

Prestack migration has been treated fairly extensively in the literature (Clærbout and Yilmaz 1979, Snyder 1979, Thorson 1980.) Most of these attempts have been done in the midpoint-offset (y-h) coordinate. Radial and slant-stacked midpoint coordinates have also

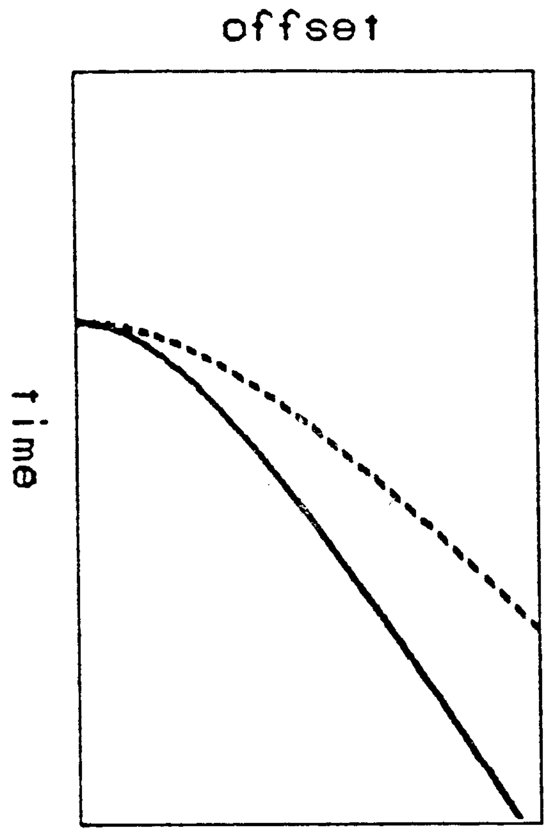

FIG. 1. Ambiguity between dip and velocity.

been used (Ottolini 1983). The data is usually transferred to the Fourier space, thus excluding lateral velocity variation. The source-geophone (s-g) space is not usually favored in prestack migration methods. The reasons usually stated are sensitivity to velocity variations and truncation effects. As far as the first reason is concerned, We have no reason to believe that sensitivity to velocity is more serious in the (s-g) space than in the (y-h) space. As far as the second reason is concerned, if it is the only problem, then it is not impossible to require more geophones per cable and wider apertures than is done nowadays.

There are basically two approaches to prestack migration in the the (s-g) space. The first is to downward continue the geophones to some level, use the reciprocity to exchange the geophones and sources, downward continue the sources, and use the imaging principle $t = 0$. This is the theme of the double-square-root equation which, in (s,g) space is

$$\frac{\partial U}{\partial z} = \{[\frac{1}{v^2} - [\frac{\partial t}{\partial g}]^2]^{\frac{1}{2}} + [\frac{1}{v^2} - [\frac{\partial t}{\partial s}]^2]^{\frac{1}{2}}\} \frac{\partial U}{\partial t} \tag{1}$$

A serious shortcoming of this method is the insufficient sampling of the source axis. The second approach, which will be used here, is to migrate *profiles* one by one (Jacobs 1982.) (A *profile* or *field record* is a common *source* gather). This method has some advantages.

1. It does not depend on the the reciprocity principle, which might not work due to a variety of reasons.
2. Since each profile is migrated independently then sampling along the source axis can be sparse, thus making it economically more practical.
3. The profiles independence on each other localizes the error.
4. The size of data that is used at a particular time is small.
5. Each source is dealt with separately, so there is no interaction between the sources. This feature may also be useful in checking for errors in deconvolution.

Velocity Analysis

The conventional processing sequence uses iteration to find the correct velocity function. We will use iterations, but our method will be based on the wave equation, and we will abandon the assumption of hyperbolic arrivals.

Since we would like to stay in the space domain, Fourier methods of extrapolation, e.g. phase shift and Stolt methods have to be excluded. Whether we stay in time or go to the frequency domain is not crucial. The method that we used in the downward continuation is the bullet-proof 45-degree equation (Jacobs et al 1979). At wide apertures a more accurate equation, like the 65-degree equation (a fourth order equation), may be necessary. Our scheme of interweaving migration and velocity analysis can be summarized in these steps :

1. Start with profiles $p_1, p_2, ..., p_n$ obtained from sources $s_1, s_2, ..., s_m$. Thus our data is

$$p_j = p_j(g_i, t, z = 0) \qquad i = 1, n \qquad j = 1, m$$

2. Fourier transform all profiles to the temporal frequency domain to get

$$\hat{p}_j(g_i, w, z = 0) \qquad i = 1, n \qquad j = 1, m$$

3. Using a single frequency, downward continue all profiles to a given *predetermined* travel time τ using a velocity function $v(x, z)$ chosen by a reasonable guess.
4. Downward continue all sources to the same travel time.
5. Do steps 3 and 4 for all frequencies.
6. Use the imaging principle that the reflectivity is $c = \sum_\omega U/D$, where U is the upgoing energy and D is the down going energy. Note that since D can be zero for some frequencies (if there are lateral velocity variations), other forms of c should be used, e.g $\sum_\omega U/(D+\epsilon)$ where ϵ is a small value added for stability (see Snyder (1978)).
7. Sort the data to obtain common geophone gathers. Examine these gathers to determine whether the velocity that was used in the migration to this τ level is correct. If the correct velocity was used, then the images from all sources will be aligned and flat, otherwise they will be misaligned, and a smile or a frown will be observed. The criterion used to check for such alignment might be visual inspection or coherency measurement.
8. Once the velocity has been determined for this travel time, repeat steps 3-8 for the next travel time τ.
9. Now, stack the data over g to produce an image $R(x, z)$. This stacking is expected to increase the signal to noise ratio and also attenuate coherent noise like multiples.

A modification to this method was made so that steps 3-6 are replaced by raytracing. In doing so, we use the first square root in equation (1) to downward continue the geophones and instead of using the second square root to downward continue the sources, we use the imaging principle $t = \tau_{tr}$ where τ_{tr} is the travel time between the source and the point (x, z), obtained by raytracing. Using this alternative eliminates the instability associated with the vanishing of D for some frequencies, but on the other hand is not very useful for strongly laterally varying media. We shall call such method a hybrid method, because it is a combination of wave equation downward continuation and kinematic time shifting. It should be noted that even though we used time shifting, this method is quite different from NMO correction, since we make no assumption regarding hyperbolic arrivals. The only assumption we make is our knowledge of the ray path, which depends on the velocity function we use.

Results

Figure 2 shows a synthetic profile obtained from three reflectors at $50m$ intervals in a constant velocity medium with $v = 400m/sec$. Since the medium is layered, all profiles are identical. Our first goal is to downward continue the profiles to a certain depth. Figure 3 shows the result of downward continuing a profile to 40 meters. Note that the energy has

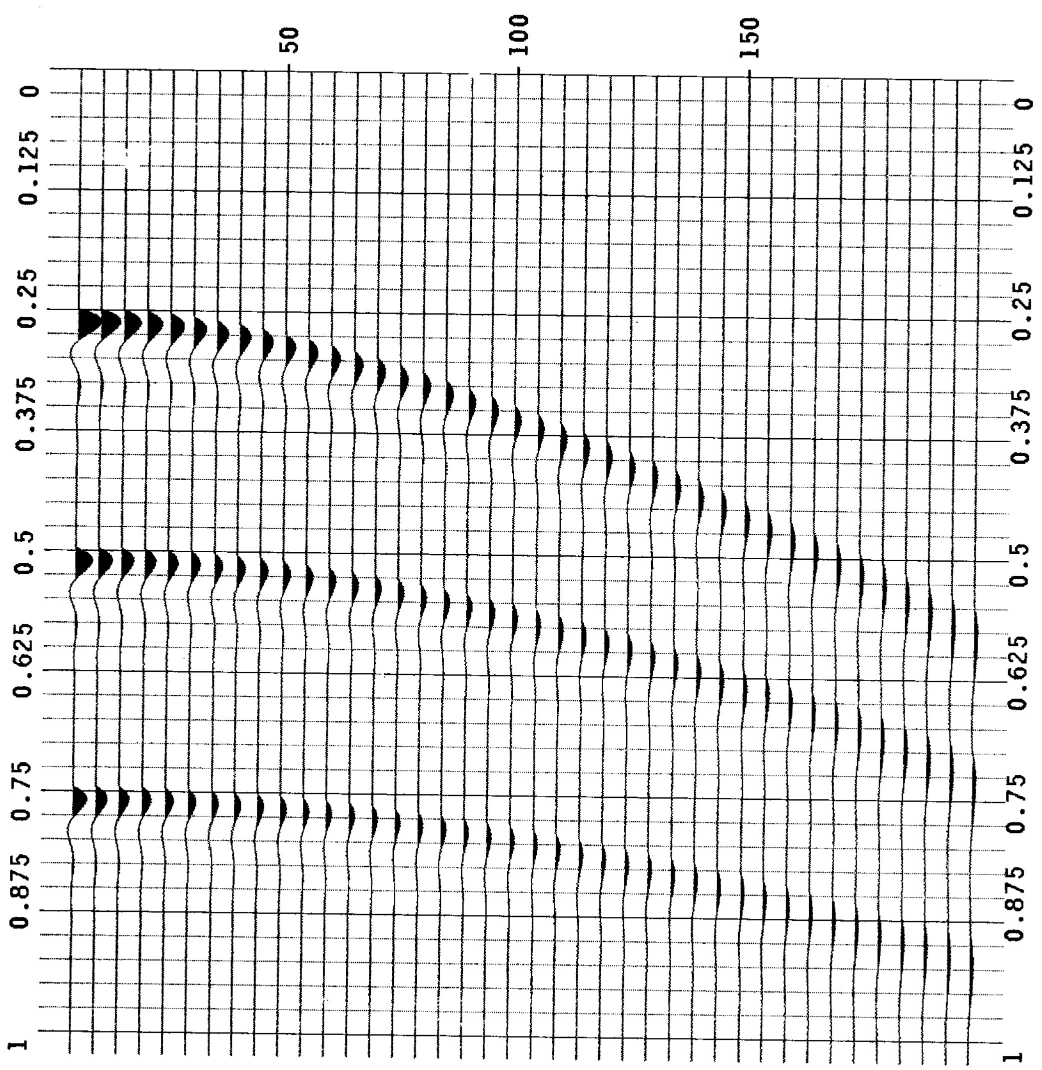

FIG. 2. A synthetic profile for a medium with three reflectors with a velocity of 400 m/sec.

started to migrate towards the zero offset. This is expected since as we downward continue we also push the field sideways.

Figure 4 shows the final result of the imaging process, the migrated profile. Since the reflectors are flat in this example, the images of all geophones occur at the same depth. As mentioned above, we see that the energy has moved towards the zero offset. In fact, little energy is seen after half the cable length. This is because when energy is extrapolated to the reflector it is put back to where it came from and this happens at half the offset. Moreover, again because the reflectors are flat, this profile can also be thought of as a common *geophone* gather. Looking at it this way we see that the images from all sources

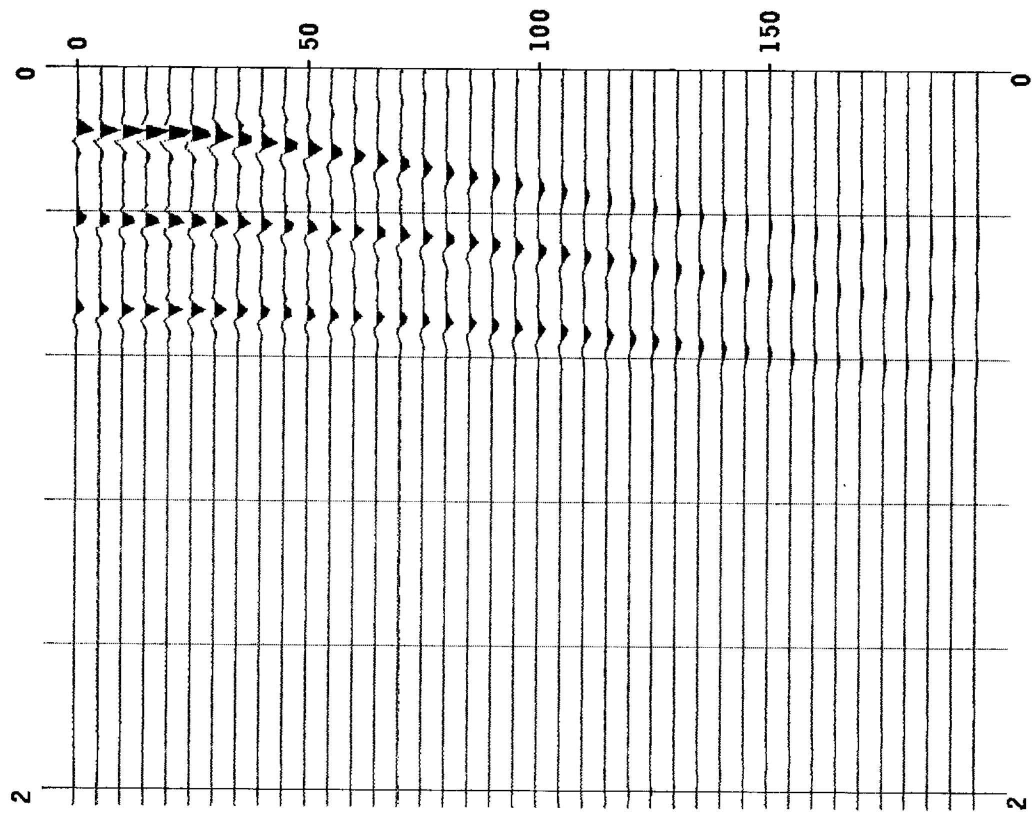

FIG. 3. The profile of Figure 2 after downward continuation to $40m$.

are aligned and flat. It should be stressed that the flatness of the common geophone gather is independent of structure. They are flat merely because they represent reflectivities for the same physical point and this point need not be a reflector.

Figure 5 shows a common geophone gather when the wrong velocity was used, in this case $300m/sec$. We see that the result in this case is a smile, since the velocity is too low. Had we used a too high velocity then we would expect to see a frown. This fact can be used to subsequently correct the velocity function.

Conclusion

We have seen that it is possible to make the velocity analysis interactive with migration. The example shown above is a simple illustration that does not warrant use of this method. We hope to apply this technique to a more complicated example in the near future.

The cost involved in such a process is still high. The factors that determine the cost