Offset-dependent near-surface velocity corrections

John Toldi

Introduction

Seismic velocities in the near surface often have extremely strong lateral variation. Such a zone of heterogeneous material greatly corrupts the images of all the underlying reflectors. Although one can't actually remove this heterogeneous material, one can remove its effects from the data through a process of downward continuation.

Downward continuation of the seismic experiment through the heterogeneous near surface zone forms the basis of all near surface velocity correction and estimation methods. In using this process, one simulates the seismic data that would have been recorded, had the seismic experiment been conducted from a datum beneath the heterogeneous zone. Some methods follow this downward continuation with an upward continuation through a constant velocity medium. The corrections then depend only on the difference between the near-surface velocity and the replacement velocity. This results in smaller corrections than for the simple downward continuation.

Assumptions about the near-surface velocities and the geometry of the reflectors can simplify the downward and upward continuation. The standard assumption is that energy in the near-surface travels along vertical rays. Downward and upward continuation move energy along these vertical rays and therefore cause simple static time shifts to the data. The magnitude of the shift for each trace is the sum of the static shifts attributable to the corresponding shot and geophone. Note that no energy moves from one trace to another; making the corrections is a single-trace process.

The vertical raypath assumption is often a reasonable one. Snell's law tells us that as long as the near-surface velocities are much slower than those below, the rays in the near-surface will be nearly vertical. They will also be nearly vertical when the reflectors are flat and the offset angles are not too large.

There are also cases where the raypaths are clearly not vertical in the near surface. Only for a near-surface velocity of zero would the near-surface rays be strictly vertical. In the arctic regions, permafrost conditions provide an anomalously high velocity, laterally variable near-surface. Rays encountering this near surface will actually refract away from vertical.

In this paper I examine how the vertical raypath assumption can be relaxed. I begin by looking at near-surface corrections from the point of view of wave theory. The multichannel nature of the corrections is emphasized: some energy naturally moves from one trace to another when we downward or upward continue through the near-surface. Only in an zero-thickness, zero-velocity near surface would the energy stay on one trace.

I next assume the point of view of ray theory. In this view, the corrections are seen to move each point of the input data to a different time and horizontal position. Again, only when incidence is vertical is the change in lateral position zero, and the time shift constant for an entire trace. One can, however, adjust the time corrections in a way that minimizes the effect of the lateral motion. This final approximation leads to a method in which making near-surface corrections is a time-varying, single-trace process.

In the first two parts of this paper it is assumed that the near-surface velocities are known. Velocities can sometimes be determined through refraction studies, or, in the case of buried sources, from uphole times. Even then, the velocity information is often inexact or incomplete. Usually, then, one must try to estimate the near-surface velocities from the reflection times themselves. The final part of the paper discusses means of incorporating offset-dependent corrections into near-surface velocity estimation.

Experiment sinking - Wave theory

The basic process of correcting for near-surface velocity variations involves downward or upward continuing the shots or geophones through the near-surface zone. I will begin by looking at one step of the overall process: the downward continuation of the geophones. The shots are fixed at the surface, then the wavefield is reconstructed as if the geophones had been buried beneath the laterally variable near-surface.

This process of downward continuation is based on the diffraction model of figure 1. As the waves come up to the surface, each point at the datum acts as a secondary source to send the energy radiating upward through the near-surface. The wavefronts in figure 1 are irregularly shaped to emphasize the lateral variability of the near-surface velocities.

This figure shows that the downward continuation clearly will move energy not only in time within the common shot gather, but also from one trace to another. Only for vertically

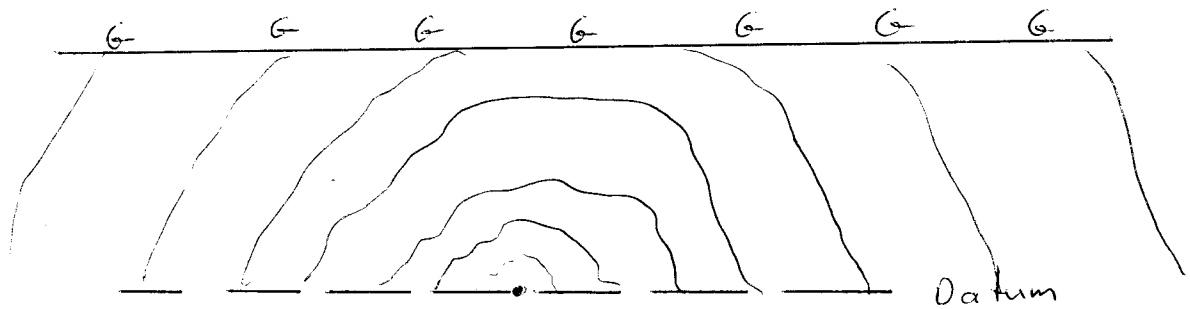

FIG. 1. A diffraction model for near surface propagation. Each point on the datum acted as a secondary source, sending waves upward through the near-surface. Every geophone within the common shot gather received some energy from this secondary source.

incident plane waves would there be no lateral motion. This diffraction model is theoretically correct, but takes no particular account of the fact that one is trying to undo only the last bit of propagation. The near-surface layer is thin enough that most of the geophones receive horizontally propagating waves from the secondary source. Because the contribution from the energy travelling nearly horizontally in the near-surface is relatively unimportant, the secondary source shown in figure 1 will affect only a small range of points on the surface.

Although the downward continuation of the geophones will move energy from one trace to another, the lateral motion will not be great. That is, for the near-surface corrections, the downward continuation is a very local operation. This local quality could be used to advantage by a finite difference method. A local differencing scheme could be used for the lateral derivatives, then only a few steps in depth taken. Energy would thus be moved across only a couple of traces.

A ray-theory method of making near-surface corrections

Another way to exploit the local nature of the downward continuation is to use a ray method. According to ray theory, the downward continuation moves the energy back along the rays that carried it up to the surface. Thus, the immediate problem becomes one of determining those raypaths. As will be discussed further, determining the raypaths requires assuming a model of the velocity distribution and geometry of reflectors below the datum. A main advantage of the wave-equation method just discussed is that it requires no such assumptions: the only required input are the data and an estimate of the near-surface model.

I once again begin the discussion by looking at the downward continuation of the geophones. Shown in figure 2 is a ray arriving at the geophone at an angle γ_g, having refracted from γ_g', at depth Δz. Each data point involving this geophone, $P(s,g,t,z=0)$, will move to a new point, $P(s,g',t',z=dz)$. Note that the shot coordinate remains the same: one holds the shots fixed in this process, exactly as in the wave-theory development.

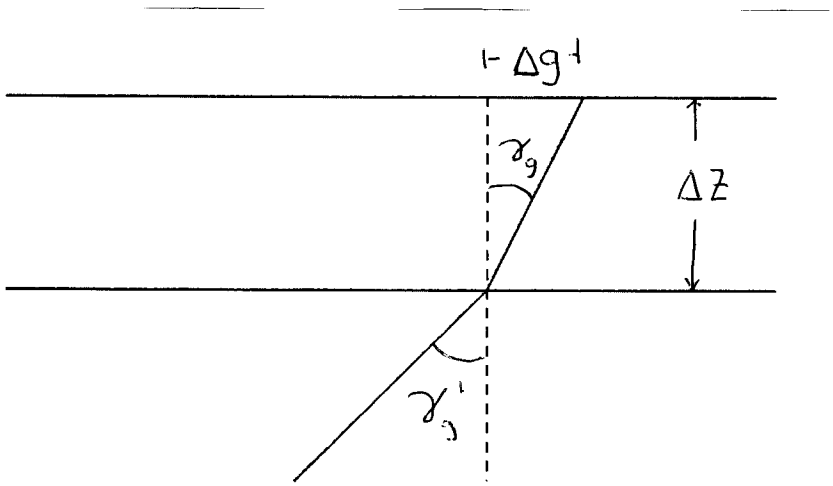

FIG. 2. Ray arriving at the geophone at an angle γ_g, having refracted from γ_g', at depth Δz.

From figure 2,

$$\Delta g = g - g' = \Delta z \tan\gamma_g \tag{1}$$

$$\Delta t = t - t' = \frac{-\Delta z}{v_g}\left(\cos\gamma_g\right)^{-1} \tag{2}$$

$$\Delta s = s - s' = 0 \tag{3}$$

In deriving these equations, I have assumed that a ray in the near-surface is affected by only one velocity, v_g. Δz must be small enough to allow Δg to be less than one geophone interval: otherwise the ray would travel under many geophones, encountering various velocities along the way. In many cases, this limitation on Δz would be too restrictive. For example, for making topographic corrections (i.e., corrections designed to eliminate the effects of topography by reducing it to a flat-datum), the elevations dictate what Δz to use. Furthermore, because rays actually bend towards horizontal when they encounter the high velocity

near surface of permafrost, one should expect fairly large horizontal displacements in such a case.

One can eliminate the restriction on Δz by using a near-surface velocity derived by averaging the velocity over a suitable range of geophone positions. With this average velocity replacing v_g, equations 1,2, and 3 require no restrictions on Δz. Using an average velocity does carry the implicit assumption that rays follow straight lines in the near surface.

Flat reflectors

To use equations 1,2 and 3, one must know the near-surface propagation angle γ_g. The general correction process described by these equations thus requires a large amount of ray tracing. By making a flat dip approximation, one can derive a much simpler correction process.

Begin by rewriting equations 1 and 2 as

$$\Delta g = \Delta z \tan\gamma_g = \Delta z \sin\gamma_g \left(1 - \sin^2\gamma_g\right)^{-1/2} \tag{4}$$

$$\Delta t = \frac{-\Delta z}{v_g}\left(\cos\gamma_g\right)^{-1} = \frac{-\Delta z}{v_g}\left(1 - \sin^2\gamma_g\right)^{-1/2} \tag{5}$$

The near-surface propagation angle γ_g can be expressed in terms of the incident angle, γ_g' through Snell's law:

$$\frac{\sin\gamma_g}{v_g} = \frac{\sin\gamma_g'}{v} \tag{6}$$

Substituting Snell's law into equations 4 and 5 gives

$$\Delta g = \Delta z \frac{v_g}{v}\sin\gamma_g' \left(1 - \left[\frac{v_g}{v}\sin\gamma_g'\right]^2\right)^{-1/2} \tag{7}$$

$$\Delta t = \frac{-\Delta z}{v_g}\left(1 - \left[\frac{v_g}{v}\sin\gamma_g'\right]\right)^{-1/2} \tag{8}$$

The substitutions have allowed Δg and Δt to be expressed in terms of the angle of incidence γ_g'. A flat dip assumption will now allow γ_g' to be expressed as a simple function of offset and time.

Shown in figure 3 is the simplest possible model: flat reflectors with velocity = v constant below the datum. In this case

$$\sin\gamma_g' = \sin\gamma_s' = \sin\gamma' = \frac{x_1}{vt_1}$$

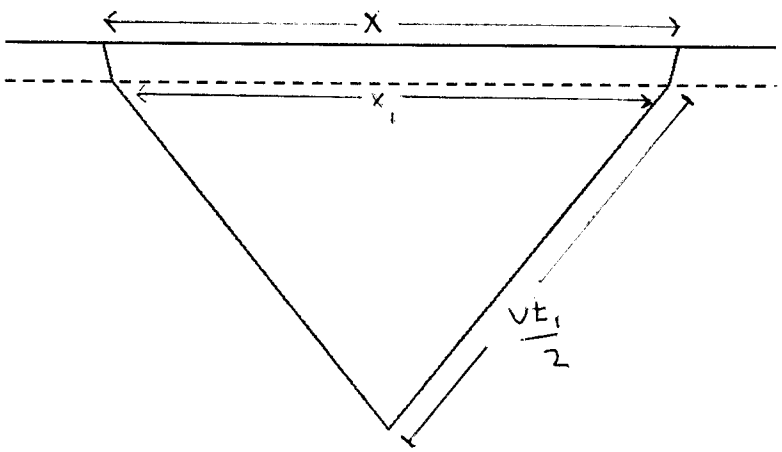

FIG. 3. Raypath for flat reflector, with velocity = v, constant below the datum.

where x_1 and t_1 are the offset and time measured from the datum.

Because Δz is small, a further simplification is possible. A Δt of 20 msec is considered large, whereas the smallest times of interest are several hundred msec. Similarly, the maximum Δx, which will only occur at large offsets, will be a couple of trace spacings. Thus,

$$\frac{x_1}{vt_1} \approx \frac{x}{vt}\left[1 - \frac{\Delta t}{t}\right]\left[1 + \frac{\Delta x}{x}\right] \approx \frac{x}{vt}\left[1 - \frac{\Delta t}{t} + \frac{\Delta x}{x}\right]$$

The corrections $\frac{\Delta t}{t}$ and $\frac{\Delta x}{x}$ have the same sign (both negative in figure 3), and are about the same size. Thus,

$$\frac{x_1}{vt_1} \approx \frac{x}{vt}$$

and finally,

$$\sin\gamma_g' \approx \frac{x}{vt} \tag{9}$$

Substituting for $\sin\gamma_g'$ in equations 7 and 8 gives the following equations for the downward continuation of the geophones:

$$\Delta g = \Delta z \frac{v_g}{v}\tan\gamma_g = \Delta z \frac{v_g}{v}\frac{x}{vt}\left[1 - \left[\frac{v_g}{v}\frac{x}{vt}\right]^2\right]^{-1/2} \tag{10}$$

$$\Delta t = \frac{-\Delta z}{v_g}\left[\cos\gamma_g\right]^{-1} = \frac{-\Delta z}{v_g}\left[1 - \left[\frac{v_g}{v}\frac{x}{vt}\right]^2\right]^{-1/2} \tag{11}$$

and

$$\Delta s = s - s' = 0 \tag{12}$$

Although equations 10 and 11 are considerably more complicated looking than the simple expressions of equations 1 and 2, they are much easier to implement. Rather than determining the propagation angle in the near surface through ray tracing, they express the angle as a function of time and offset.

The corrections of equations 11 and 12 carry out the downward continuation of the geophones by stretching the dataset in the time and geophone directions. Because the shots are handled in an analogous manner, the entire near-surface correction process involves a series of such stretches along the shot, geophone, and time axes. The output data points will not generally coincide with the location of input points. Thus one must follow the stretch with an interpolation between traces and time points to get the data back onto a regular grid.

Making the corrections thus requires a multi-trace process: to find the corrected data at an output point, one must interpolate between a set of input points, taken from nearby traces and times. Thus far, the ray theory corrections show no advantage over those derived through wave theory. The ray theory has provided interpolation coefficients based on an assumed subsurface model and the near-surface velocities. In contrast, the wave theory supplied a set of interpolation coefficients based strictly on the near-surface velocities. The real benefits of the ray theory approach become apparent only after further approximations concerning the near-surface raypaths are made.

Near-surface models

So far the discussion has centered on approximations for the subsurface geometry and velocity model. Now I will add some assumptions about the near-surface raypaths. Once again I begin by looking at the geophone correction; the shot correction is analogous.

Consider the process in which one downward continues the geophones through the heterogeneous near surface, then upward continues them through the replacement medium. The simplest approximation assumes that the rays always travel along the direction given by $\sin\gamma'$; that is, there is no ray bending in the near-surface. This approximation is used commonly in tomography: calculate the time correction by following the unperturbed path, encountering the anomalous material along the way. Because of the assumption of no ray bending, the Δg from the downgoing path is eliminated when the ray comes back up. Thus the entire correction is

$$\Delta t = \frac{\Delta z}{v \cos\gamma'}\left(\frac{1}{v} - \frac{1}{v_g}\right) \tag{13}$$

$$\Delta g = 0 \tag{14}$$

This result provides the main motivation for following the downward continuation with an upward continuation: because Δg is zero, the corrections are single trace processes. Note also that Δt in equation 13 depends only on the difference between the near surface velocity v_g and the replacement velocity v. Thus the upward continuation has reduced the correction to a residual effect: if $v_g = v$, no correction is required.

Although the simplifications offered by equations 13 and 14 are very attractive, having to make the assumption of no ray bending in the near surface is not very satisfying. After all, statics is not a bad model, and it predicts that all rays bend to the vertical in the near surface. For the high-velocity near surface of permafrost, equations 13 and 14 do provide more reasonable corrections than static shifts. On the other hand, for the more common case of a low-velocity near surface, they overestimate the importance of the offset variation.

A better approximation assumes that the rays travel through the near-surface in a direction predicted by some assumed background velocity v_0. Note that this contains the previous approximation as the special case of $v_0 = v$. On the downward path the rays encounter the anomalous near surface velocity v_g, on the way back up, the replacement velocity v. Once again, the Δg accumulated on the downward path will be subtracted on the upward path, so only a time correction is left:

$$\Delta t = \frac{\Delta z}{\cos\gamma_g}\left(\frac{1}{v_g} - \frac{1}{v}\right) \tag{15}$$

$$\Delta g = 0 \tag{16}$$

The assumption that γ_g is determined by the background velocity v_0, combines with Snell's law to give:

$$\frac{\sin\gamma_g}{v_0} = \frac{\sin\gamma_g'}{v} \tag{17}$$

Substituting for $\cos\gamma_g$ in equation 15 then leads to

$$\Delta t = \Delta z \left(1 - \left(\frac{v_0}{v}\sin\gamma'\right)^2\right)^{-1/2}\left(\frac{1}{v_g} - \frac{1}{v}\right) \tag{18}$$

The analogous treatment of the shot corrections leads to:

$$\Delta t = \Delta z \left[1-\left[\frac{v_0}{v}\sin\gamma'\right]^2\right]^{-1/2}\left[\frac{1}{v_s}-\frac{1}{v}\right] \tag{19}$$

The correction method described by equations 18 and 19 has several desirable features. First, the corrections are single trace processes, so the correction for each trace is the simple sum of the Δt in equations 18 and 19. Thus, for any trace with shot coordinate s and geophone coordinate g,

$$\Delta t = \Delta z \left[1-\left[\frac{v_0}{v}\sin\gamma'\right]^2\right]^{-1/2}\left[\frac{1}{v_s}+\frac{1}{v_g}-\frac{2}{v}\right] \tag{20}$$

Second, because the velocity appearing inside the square root in equations 18,19, and 20 is v_0, the background velocity determines the near-surface raypath. Setting $v_0 = 0$ in equation 20 leads to the static shifting case:

$$\Delta t = \Delta z \left[\frac{1}{v_g}-\frac{1}{v}\right]$$

Thus, a single parameter leads one from the time dependent to the static shift corrections. Finally, the method has the property that the size of the corrections depends on the difference between the near-surface velocities and the replacement velocity. This is certainly a desirable property: if these velocities are the same one would like to leave the data untouched.

Source of velocity information

So far, this paper has discussed methods of correcting for near-surface velocity variations. It is natural to ask where this near-surface velocity information might come from. Some velocity information comes from use of the refractions, or in the case of a buried source, from uphole times. For these cases, one could use the methods described in this paper directly.

In many cases, however, this independent information is either unavailable or incomplete. Then one must try to estimate the velocities directly from the seismic reflection times. Typically, one begins by using so-called field statics, which correct for the effects of topography and known near-surface velocity variations. The remaining near-surface velocity effects are estimated and applied as residual statics.

Estimation

The typical statics estimation program cross-correlates traces, then picks the peak of the cross-correlation function as the measured time shift. This time shift is distributed among its possible sources, which are typically assumed to be shot and geophone statics, residual NMO, and dip.

One can generalize the near-surface velocity estimation to encompass non-vertically propagating rays by applying a simplified version of seismic tomography. Exactly as it is in statics, the estimation is broken into two parts: picking of time shifts, and inversion of these time shifts to model parameters. In statics one picks time shifts by cross correlating large time segments of the data; for tomography the time shifts must be picked from each reflector independently. As a consequence, the program must work on many small time windows of data.

The second part of the estimation process inverts the time shifts to model parameters, according to a linear theory. The model used is a simplified version of that of the full seismic tomography problem: velocity variations are restricted to the near-surface. The linearization is precisely the one that I used in deriving equations 13 and 20, that is that traveltimes can be computed along raypaths determined by some background velocity distribution.

Shuki Ronen and Dan Rothman discuss elsewhere in this report an alternate approach to the statics problem. Rather than picking times and formally inverting for the model estimates, they estimate the statics by performing repeated forward modeling, and then look for the set of estimates that maximizes the power in a common midpoint stack. That is, they correct the data using an estimate of the velocity, then see if this correction improves the stack. This approach allows the velocities to be estimated directly from the data, rather than through the intermediary of picked traveltimes. In the remainder of this section I will examine how offset-dependent near surface corrections could be incorporated into the stack-optimization approach.

The stack optimization approach to near-surface velocity analysis requires a fast forward modeling procedure. And, one must be able to easily test whether a correction has improved the stack. Thus the estimation should be done after NMO: one would not want to require NMO after each try at a static shift. The emphasis in estimation must thus be on speed and simplicity, rather than on absolute accuracy. A single trace process is preferable to a multi-trace process. Similarly, a time-independent process is preferable to a time-dependent process.

Static shifts are, of course, time-independent, single-trace processes. So, the simplest approach would be to estimate with static shifts, but use in the estimation only those parts

of the data where the vertical propagation model is satisfied: reflections from deep, relatively flat reflectors. In most cases this is possible. Only in those cases such as data with a high velocity near-surface or no deep reflectors, then, is there reason to not use static shifts in the estimation.

A wave-theory approach has one advantage: it requires no knowledge of the reflector geometry in order to make accurate corrections. However, as part of an estimation procedure, this advantage is overshadowed by its chief disadvantage: it is a multi-trace process. Furthermore, it is difficult to interchange with NMO. Although one should theoretically not interchange the order of near-surface corrections with NMO (because they don't commute), one could approximate this by using stretched time in the finite difference operator. The result is that one must convolve with a time-dependent operator.

The simplified ray approach of equation 20 seems to be a better candidate: it is a single trace process, albeit a time-dependent one. Although interchanging it with NMO would introduce some error in the corrections (once again the operations don't commute), the essential nature of the corrections would remain the same: single trace, time-dependent corrections. By considering data within time windows, one could reduce the corrections to offset dependent time shifts.

	Practicality		Accuracy	
	single trace	time independent	non-vertical raypaths	dipping reflectors
statics	yes	yes	no	no
rays	yes/no	no	yes	no
wave eq.	no	yes	yes	yes

TABLE 4. Comparison of near-surface correction methods

Conclusions

Static shifts form the basis for the conventional method of near-surface velocity correction. In this paper, I have looked at correction methods that relax the underlying assumption of vertical propagation in the near surface. Figure 4 summarizes the tradeoffs between accuracy and practicality for the different correction methods.

Because they are the only single-trace time-independent process, statics are certainly the most practical of the methods. On the other hand, they are accurate only when the vertical raypath assumption is valid. This assumption is often a good one. There are important cases, however, such as permafrost or correction for a relatively thick near surface, where the assumption is clearly not valid. In these cases, one of the other methods should be considered.

Both the wave equation and the ray methods presented in this paper are more accurate than static shifts. The wave equation method is particularly accurate, in that it requires no assumed model below the near surface. Both methods are less practical than static shifts, the wave-equation method by being a multi-trace process, the ray method by being a time dependent process. Because time stretching is easier to accomplish than multi-trace convolution, the ray method seems to have the edge in practicality over the wave equation method.

In the final part of the paper, I looked at near-surface velocity estimation. In estimation, accuracy must be sacrificed for practicality. By virtue of being the only single-trace, time-independent corrections, static shifts are overwhelmingly the best choice. Only when the vertical raypath assumption is violated in all parts of the data should other estimation methods be tried. Because it is a single-trace process, the simplified ray method seems to be the best alternative for estimation procedures.

REFERENCES

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