CDP dispersal observed

Shuki Ronen

Introduction

The common *mid* point (CMP) is directly above the common *depth* point (CDP), only in the special case of zero dip. In general the CDP is moving up dip when the offset increases. This simple geometric phenomena is observed in seismic data and shown by Figure 1: At high offsets the CMP is moving down dip relative to the CDP which is more or less labeled by the pinch-out. Actually, it is a little more complicated; the observed pinch-out is not exactly the physical pinch-out, because we look at unmigrated data. There is, however, a simple relation between the observed and the physical pinch-out. This relation and other geometrical results will be derived by straight forward geometry. They will be used by a primitive but fast synthetic generator. The dispersal is strongly affected by velocity variations; the dispersal is 0.8 km if the velocity is constant, the observed 2.5 km is explained by velocity variations.

From 3-D to 2-D

A reflector dipping with true dip θ, in the azimuth φ, can be described by a unit normal vector $\vec{n} = \hat{x}\sin\theta\cos\varphi + \hat{y}\sin\theta\sin\varphi + \hat{z}\cos\theta$ and a constant d. Where $\hat{x}$, $\hat{y}$ and $\hat{z}$ are orthonormal basis. If $\vec{r}$ is such that $\vec{n}\cdot\vec{r} = d$ then $\vec{r}$ is on the reflector. If the seismic line is along the x axis then, from Figure 2, since $\cos(90-\alpha) = \vec{n}\cdot\hat{x}$ we have for the apparent dip α:

$$\sin\alpha = \sin\theta\cos\varphi \tag{1}$$

Since all the reflection points are on the same line, the 3-D problem degenerates to a 2-D problem with effective dip α.

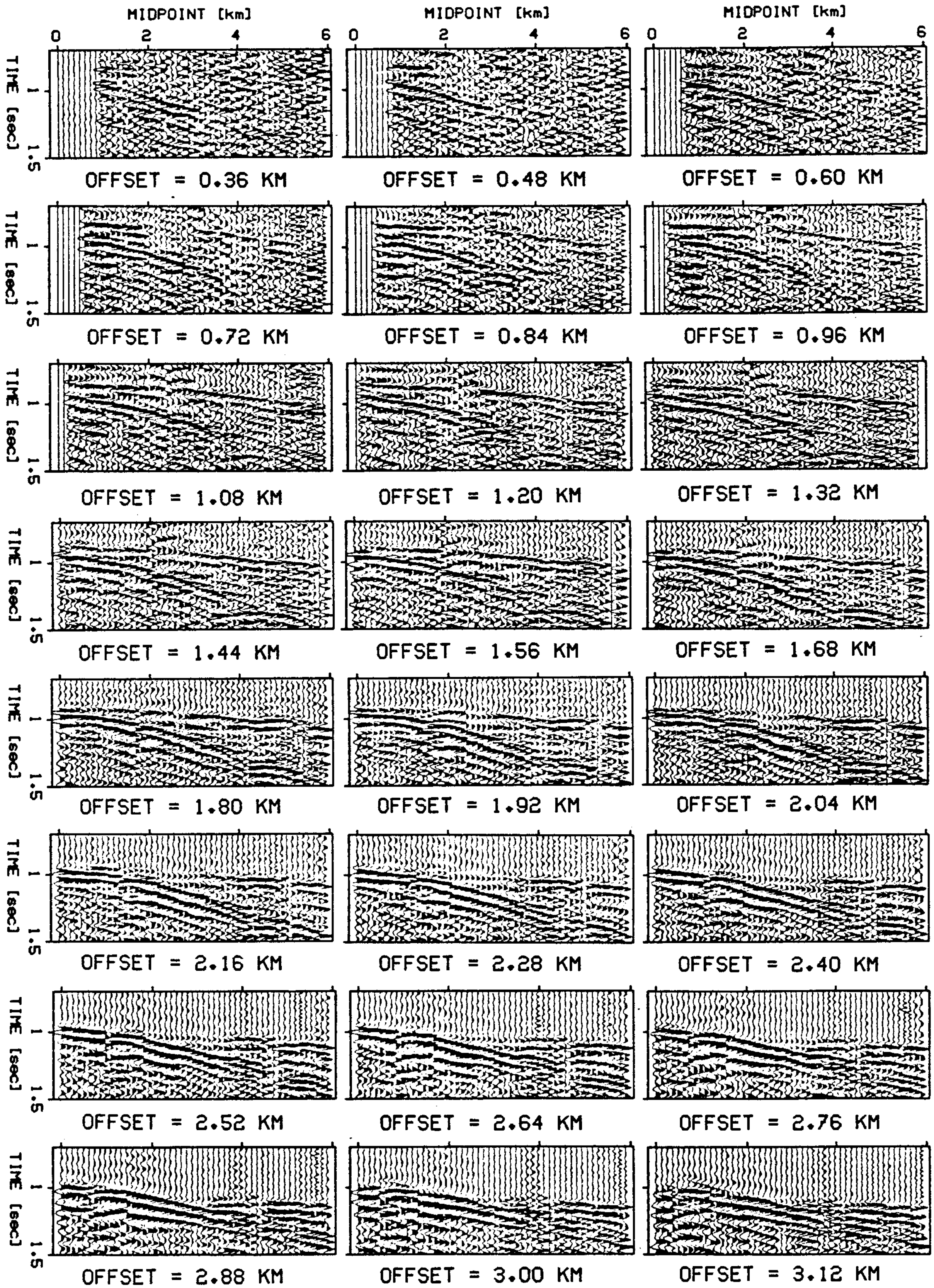

FIG. 1. Pinch-out dispersion ≈ 2.5 km. Note also the surface consistent statics.

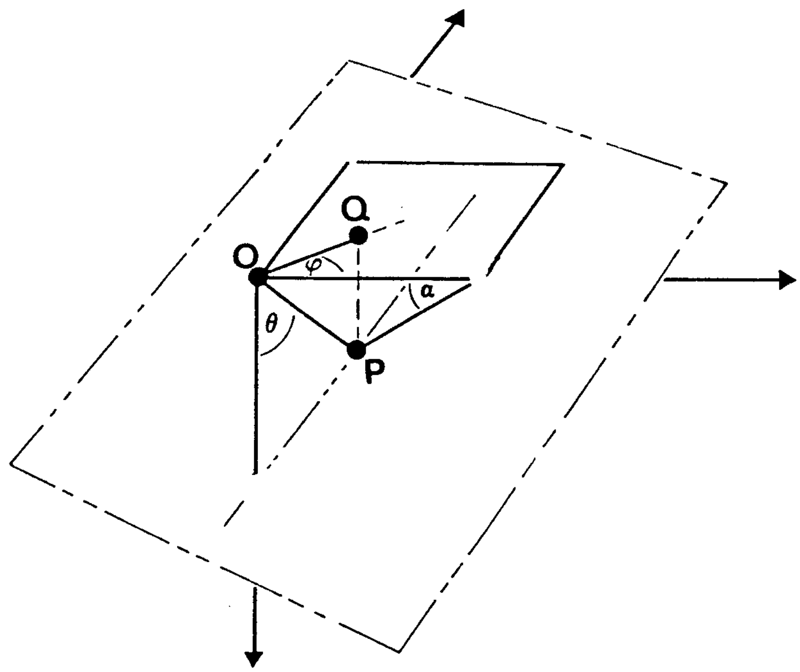

FIG. 2. Geometry of a general dipping bed.

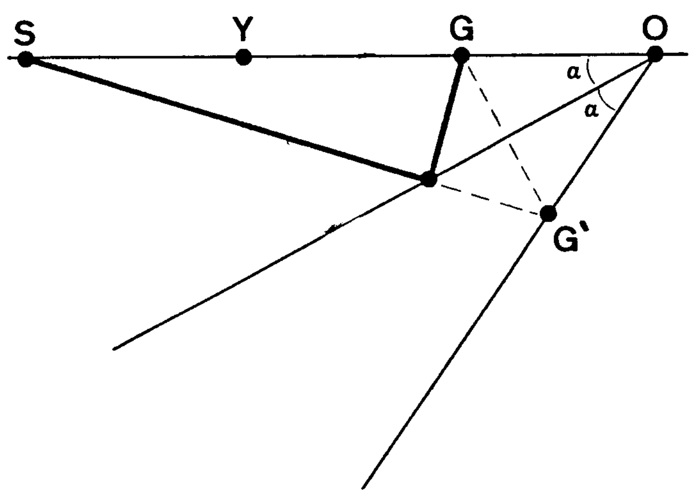

FIG. 3. Reflection from a dipping bed.

The reflection

Referring to Figure 3, G' is the image geophone. Using the cosine theorem in triangle $\Delta G'SO$, we have

$$(G'S)^2 = (OG')^2 + (OS)^2 - 2(OG')(OS)\cos 2\alpha \tag{2}$$

The travel time $Vt = G'S$. If $y = OY$, we have $OG' = y - h$ and $OS = y + h$, hence

$$(Vt)^2 = (y-h)^2 + (y+h)^2 - 2(y-h)(y+h)\cos 2\alpha$$

which is simplified to

$$\left(\frac{Vt}{2}\right)^2 = y^2 \sin^2\alpha + h^2 \cos^2\alpha \tag{3}$$

Or, in 3-D

$$\left(\frac{Vt}{2}\right)^2 = y^2 \sin^2\theta \cos^2\varphi + h^2 \left(1 - \sin^2\theta \cos^2\varphi\right) \tag{4}$$

The problem with measuring distances from point O is that point O does not exist when $\alpha = 0$, i.e. either the reflector is flat or the seismic line is in the strike direction. To avoid this problem substitute $y \sin\alpha = Vt_0/2$ and obtain

$$\left(\frac{Vt}{2}\right)^2 = \left(\frac{Vt_0}{2}\right)^2 + h^2 \cos^2\alpha$$

which is the well known result that $V_{NMO} = V/\cos\alpha$. Going to 3-D, using equation (1) we have Levin's result (F.K. Levin, 1971).

$$\left(\frac{Vt}{2}\right)^2 = \left(\frac{Vt_0}{2}\right)^2 + h^2 \left(1 - \sin^2\theta \cos^2\varphi\right) \tag{5}$$

The refraction

Refraction occurs at high offsets when there is a sharp contrast of velocities; when the angle of incidence is the critical angle and beyond there is total reflection. The critical angle γ is $\sin\gamma = V_1/V_2$, and by Snell law the angle of the transmitted wave is $90°$; along the interface.

The travel time of the refraction on a flat reflector is from Figure 4,

$$t = \frac{2z/\cos\gamma}{V_1} + \frac{2h - 2z\tan\gamma}{V_2} = \frac{2h}{V_2} + \frac{2z}{V_1}\cos\gamma \tag{6}$$

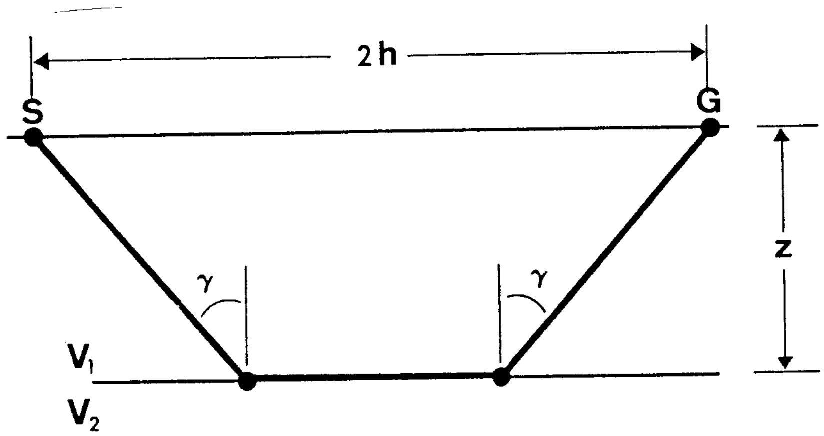

FIG. 4. Refraction from a flat reflector.

This result can be used to find the travel time of a refraction from a dipping plane. Referring to Figure 5, using equation (6), the travel time of the refraction is

$$t = \frac{SF}{V_2} + \frac{2z}{V_1}\cos\gamma - \frac{GF}{V_1}$$

Simple geometry provides:

$$GE = 2h \sin\alpha$$

$$SE = 2h \cos\alpha$$

$$EF = GE \tan\gamma = 2h \sin\alpha \tan\gamma$$

which are used for SF, z and GF:

$$SF = SE + EF = 2h\,(\cos\alpha + \sin\alpha \tan\gamma\,)$$

$$GF = GE / \cos\gamma = 2h \sin\alpha \,/ \cos\gamma$$

$$z = OS \sin\alpha$$

Therefore we have for the travel time of the refraction

$$t = \frac{2h}{V_2}(\cos\alpha + \sin\alpha \tan\gamma\,) + 2\,\frac{OS}{V_1}\sin\alpha\cos\gamma - \frac{2h}{V_1}\,\frac{\sin\alpha}{\cos\gamma}$$

If we substitute $OS = y + h$ we find after using some trigonometric identities that

$$\frac{V_1 t}{2} = y \sin\alpha\cos\gamma + h \cos\alpha\sin\gamma \qquad (7)$$

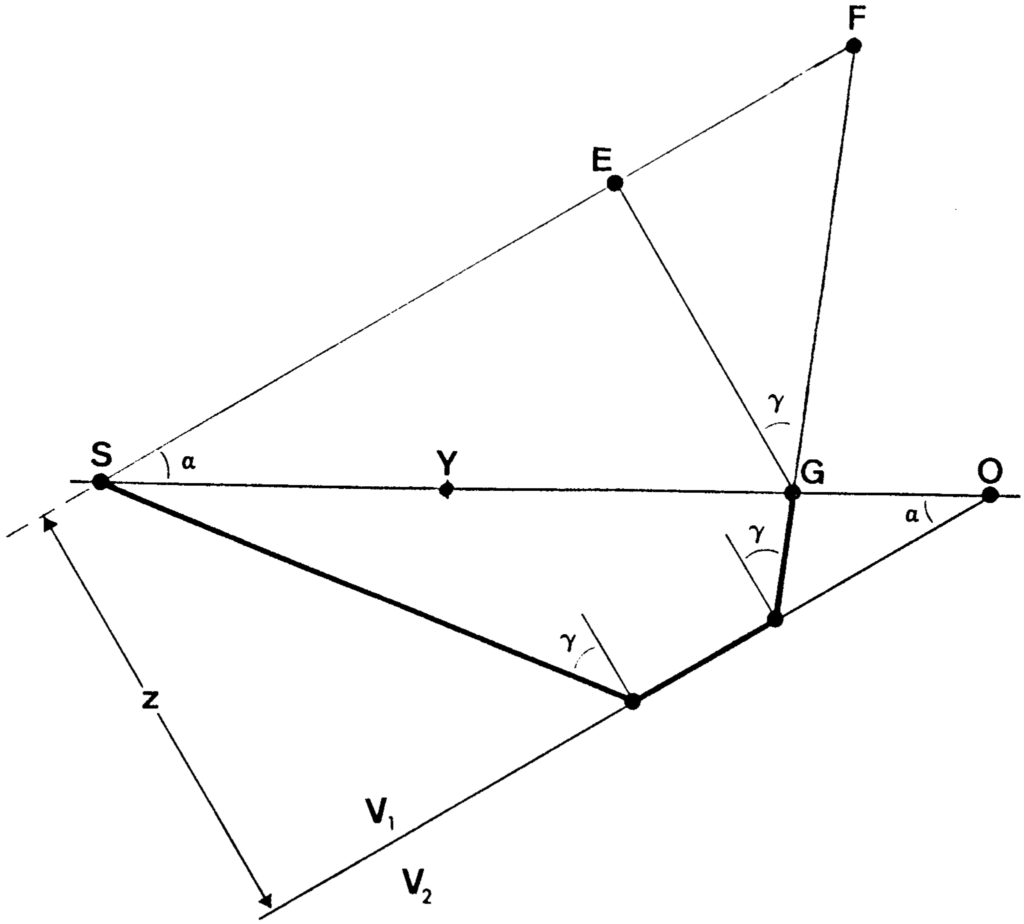

FIG. 5. Refraction from a dipping reflector.

Alternatively substitute $V_1 t_0 = y \sin\alpha$ to obtain

$$\frac{V_1 t}{2} = \frac{V_1 t_0}{2} \cos\gamma + h \cos\alpha \sin\gamma \tag{8}$$

where t_0 is the arrival time of the *reflection* at zero offset.

Going to 3-D, using $\sin\alpha = \sin\theta \cos\varphi$, we have

$$\frac{V_1 t}{2} = \frac{V_1 t_0}{2} \cos\gamma + h \ (1 - \sin^2\theta \cos^2\varphi)^{1/2} \sin\gamma \tag{9}$$

To find the critical refraction, consider Figure 6:

$$\beta = 180 - \gamma - (90 - \alpha) = 90 - (\gamma + \alpha)$$

Using the sine theorem at ΔASG:

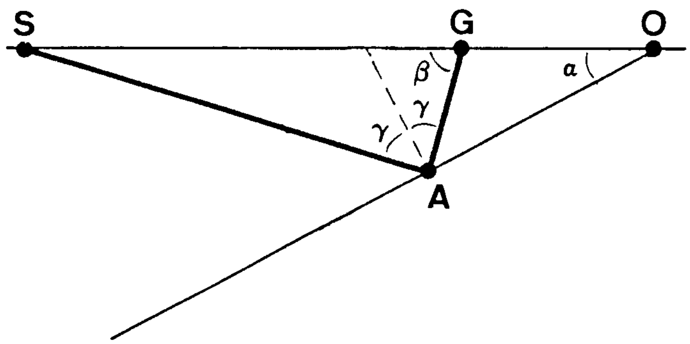

FIG. 6. Critical refraction.

$$\frac{2h}{\sin 2\gamma} = \frac{AG}{\sin\beta} = \frac{AG}{\cos(\gamma+\alpha)}$$

Using the sine theorem at ΔAGO:

$$\frac{OG}{\cos\gamma} = \frac{AG}{\sin\alpha} \quad => \quad \frac{y-h}{\cos\gamma} = \frac{2h\cos(\gamma+\alpha)}{\sin\alpha\sin 2\gamma}$$

Simplifying, the critical midpoint is

$$h < y \tan\alpha \tan\gamma \tag{10}$$

The meaning of (10) is that on a constant offset section, the refraction will be seen only for y smaller then some critical midpoint, or more familiarly, on a common midpoint gather the refraction will be seen only for h bigger than some critical offset depending on the midpoint.

Going to 3-D, the critical point is:

$$y < h \frac{(1 - \sin^2\theta\cos^2\varphi)^{1/2}}{\sin\theta\cos\varphi\tan\gamma} \tag{11}$$

It is straight forward to show that the critical refraction occurs when the reflection [Equation (3)] is tangent to the refraction [Equation (7)], as can be seen at Figure 9.

Reflection point dispersal

On a common midpoint gather the reflection point is moving up dip with increasing offsets.

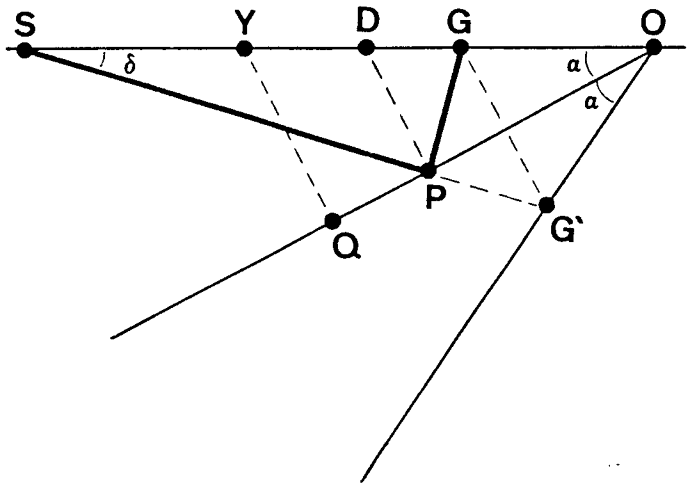

FIG. 7. Reflection point dispersal

Referring to Figure 7, from the sine theorem in $\Delta OG'S$

$$\sin\delta = \frac{g}{Vt}\sin 2\alpha \qquad ; \qquad \cos\delta = \frac{s - g\cos 2\alpha}{Vt}$$

With some trigonometric identities, using $s = y + h$ and $g = y - h$ we find

$$\sin(\alpha+\delta) = \frac{2y}{Vt}\sin\alpha$$

The sine theorem in ΔOPS gives

$$\frac{OP}{\sin\delta} = \frac{s}{\sin(\alpha+\delta)}$$

and after some simplification

$$OP = \frac{sg}{y}\cos\alpha$$

The dispersal on the reflector is, in agreement with Levin (Levin, 1971),

$$QP = \frac{h^2}{y}\cos\alpha \qquad (12)$$

On the surface, since $YD = QP/\cos\alpha$ we have a shift of

$$YD = \frac{h^2}{y} \tag{13}$$

Substituting $y\sin\alpha = Vt_0/2$, the dispersal on the surface is

$$YD = \frac{2h^2\sin\alpha}{Vt_0} \tag{14}$$

When t_0 is the zero offset arrival time of the reflection at common midpoint Y. If we are interested in a certain depth point, P, then the shift of midpoint YD should be expressed in terms of $PD = Vt'_0/2$. Where t'_0 is the zero offset travel time of a reflection from depth point P (at midpoint D). It was shown (Deregowski, 1982) that the change of variables $t_0 \rightarrow t'_0$, gives

$$YD = \frac{1}{2}\left\{\left[\left(\frac{Vt'_0}{2\sin\alpha}\right)^2 + 4h^2\right]^{1/2} - \frac{Vt'_0}{2\sin\alpha}\right\} \tag{15}$$

t'_0 is the zero offset travel time at point D.

Dispersal of a pinch-out

Suppose we have a flat reflector pinched by a dipping reflector. Ignoring truncation hyperbola, the flat event will appear at

$$t_F^2 = t_0^2 + \frac{4h^2}{V^2}$$

The dipping event will appear at

$$t_D^2 = \frac{4}{V^2}\left(y^2\sin^2\alpha + h^2\cos^2\alpha\right) \tag{3}$$

Equating $t_F = t_D$ gives

$$y^2 = \left(\frac{Vt_0}{2\sin\alpha}\right)^2 + h^2$$

Or, using $y_0 = Vt_0/2\sin\alpha$,

$$y^2 = y_0^2 + h^2 \tag{16}$$

The observed dispersal of the pinch-out point is therefore

$$\Delta y_{OBSERVED} = \left[\left(\frac{Vt_0}{2\sin\alpha} \right)^2 + h^2 \right]^{1/2} - \frac{Vt_0}{2\sin\alpha} \tag{17}$$

The physical dispersion is given by equation (15). If we substitute $t'_0 = t_0 / \cos\alpha$ we get the expression

$$\Delta y_{PHYSICAL} = \left[\left(\frac{Vt_0}{2\sin 2\alpha} \right)^2 + h^2 \right]^{1/2} - \frac{Vt_0}{2\sin 2\alpha} \tag{18}$$

Comparing the physical shift to the observed shift we see that the physical is bigger than the observed.

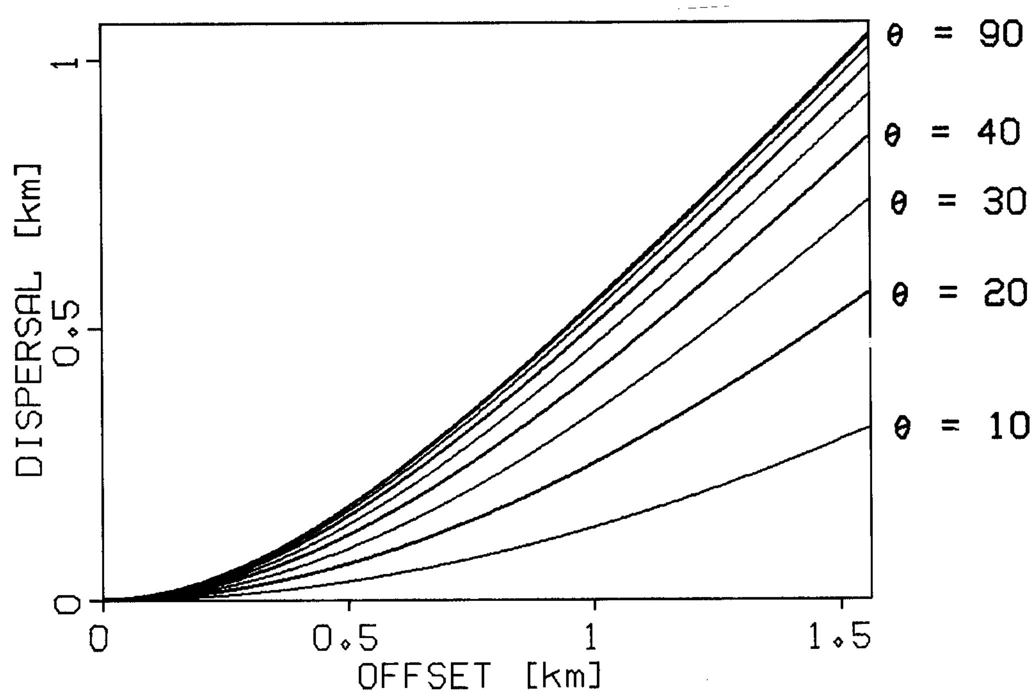

FIG. 8. CMP - CDP dispersal as a function of half offset and reflector dip. [Equation (17)].

The pinch out in practice

Substitute $Vt_0 / 2 = 0.6$ km and $\theta = 15°$, in equation (17) and get

$$\Delta y_{OBSERVED} = 0.8 \text{ km},$$

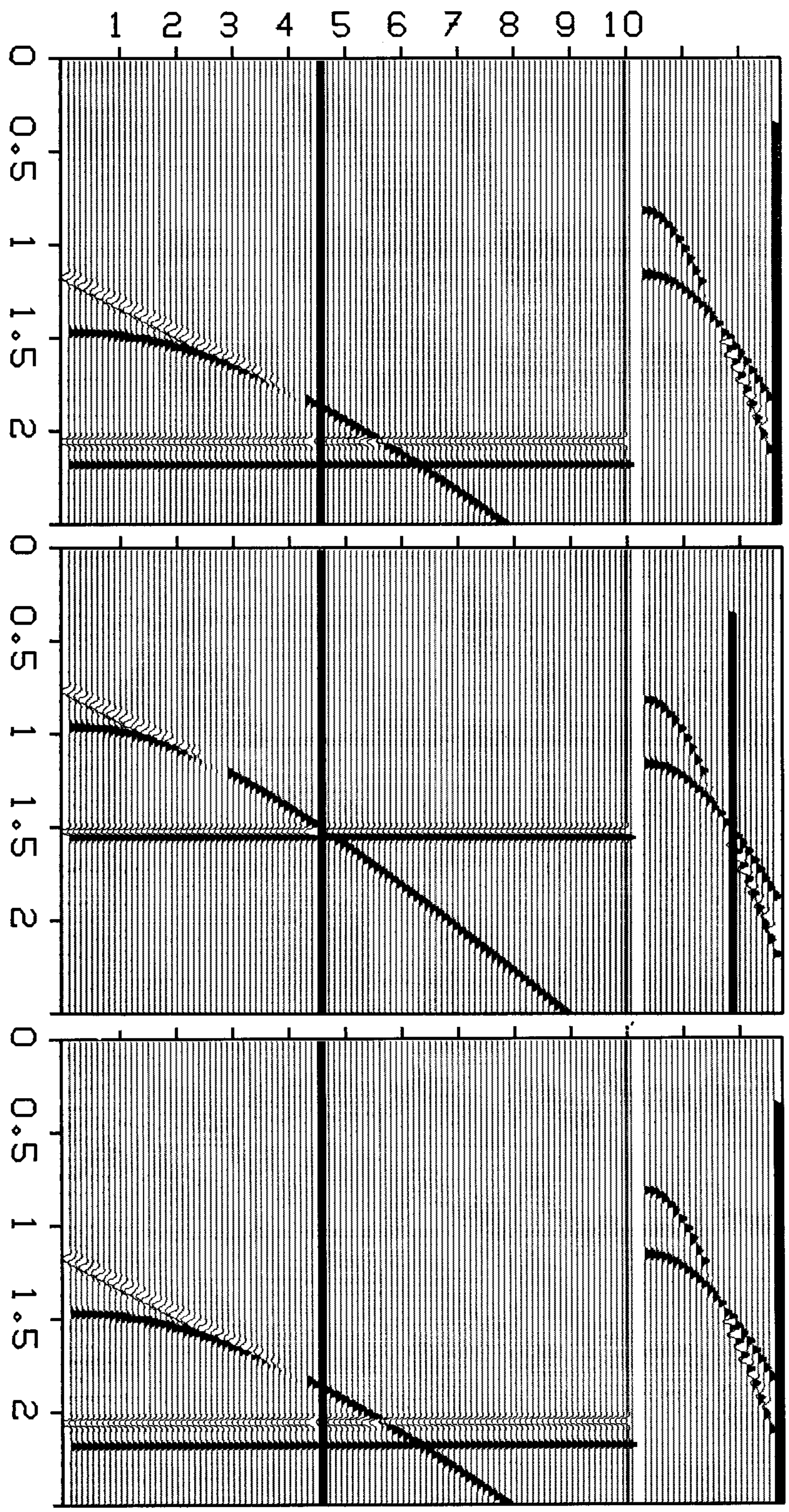

FIG. 9. Synthetic pinch-out dispersion: Each figure shows a common offset section and a CMP gather. The thick vertical lines denote the crossing of the section and the gather. Horizontal scale in km. Vertical scale in seconds. The offset increases from zero above to $h = 1.56$ km in the bottom.

The measured length is, from Figure 1,

$$\Delta y_{OBSERVED} \approx 2.5 \text{ km } (!?)$$

The difference looks big, but can be explained by velocity variation. Figure 8 shows a synthetic, generated with velocity of 1.5 km/sec for the flat reflector and 2.0 km/sec for the dipping reflector. A velocity of 3.0 km/sec was used for the refraction from the dipping reflector.

REFERENCES

Deregowski, S. M. 1982, Dip move out and reflection point dispersal. Geophysical Prospecting, v. 30 p. 318-322.

Levin, F. K., 1971, Apparent velocity from dipping interface reflections. Geophysics v. 36, p. 510-516.