Residual migration after migration with non-constant velocity

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Introduction

Residual migration is only wave-theoretically accurate when the initial migration is performed with a constant velocity (Rothman, Levin, and Rocca, 1983). Errors occurring in a residual migration after migration with $v(z)$ are expected to be related to similar errors in two-pass 3-D migration (Gibson, Larner, and Levin, 1983). Unlike two-pass 3-D migration, however, the errors that would result from a residual migration are, in principle, *correctable*. This insight can be gained from the following simple, geometrical analysis.

Summation paths and points of tangency

Preliminary, inaccurate migration is performed when the initial diffraction hyperbola

$$t^2 = \tau^2 + \frac{4x^2}{v^2(\tau)} \tag{1}$$

is migrated using the summation paths

$$t^2 = \tau_\varepsilon^2 + \frac{4y^2}{v_m^2(\tau_\varepsilon)} \tag{2}$$

where

τ, x = apex time and offsets of the diffraction

τ_ε, y = apex time and offsets of the summation path

v = true velocity, a function of τ

v_m = initial, incorrect velocity, a function of τ_ε

t = time along the diffraction or summation flanks .

The residual, unfocused energy resulting from this initial migration can be found by solving for its flank time τ_ε as a function of offset $z = x - y$ and true apex time τ. Eliminating t^2 by combining equations (1) and (2), we find an equation specifying the intersection of the summation paths and the diffraction; thus

$$\tau_\varepsilon^2 \ = \ \tau^2 + \frac{x^2}{v^2(\tau)} - \frac{y^2}{v_m^2(\tau_\varepsilon)} \ . \tag{3}$$

The most significant contributions to the summations are made when this intersection is a point of tangency, which occurs when dt/dx and dt/dy are equal. Thus, equating the derivatives of (1) and (2), we have

$$y \ = \ \frac{v_m^2(\tau_\varepsilon)}{v^2(\tau)} x \ , \tag{4}$$

describing how far laterally the energy at point (t, x) migrates. Substituting this into equation (3) gives

$$\tau_\varepsilon^2 \ = \ \tau^2 + \frac{x^2}{v^2(\tau)} \left[1 - \frac{v_m^2(\tau_\varepsilon)}{v^2(\tau)} \right] \tag{5}$$

which tells us how far vertically the same energy moves. Finally, relating the (partially) migrated energy to the apex of the original diffraction hyperbola, we make the substitution $z = x - y = x\,[\,1 - v_m^2(\tau_\varepsilon) / v^2(\tau)\,]$ to get

$$\tau_\varepsilon^2 \ = \ \tau^2 + \frac{z^2}{v^2(\tau) - v_m^2(\tau_\varepsilon)} \ , \tag{6}$$

describing the shape of the remaining, unfocused energy after preliminary migration. It is hyperbolic only if v_m is constant. If v_m is not constant, the flank times τ_ε cannot be solved for explicitly (since v_m is also a function of τ_ε), but the correct times may be calculated by iteration. The resulting residual summation paths would not be hyperbolic, so residual migration by conventional frequency domain or finite difference methods would be inappropriate. A Kirchhoff summation algorithm, however, may be modified to incorporate the non-hyperbolic summation paths.

This geometric analysis provides guidelines for variable velocity residual migration, but it does not provide a wave-theoretical justification. Nothing has been said about amplitude variation along the residual "diffraction." More importantly, if v_m does vary with τ_ε, then the solution to equation (6) may be *non-unique*, making residual migration an even trickier affair, though probably not impossible.

Synthetic illustration

The synthetic data shown in Figures 1a-d depict the differences between initial migration with constant and non-constant velocities. Figure 1a is a diffraction; the velocity is 2000 m/sec, the trace spacing is 4 m, and the sampling interval is 4 msec. When undermigrated with v=1800 m/sec, a negative 10% error, the result is the substantially smaller hyperbolic diffraction in Figure 1b. Overmigration with a constant positive 10% velocity error yields the elliptical smile in Figure 1c. When a depth-variable velocity is used, however, the result is neither an hyperbola nor an ellipse, but rather the curve described by equation (6). This is shown in Figure 1d, for which the migration velocity was 10% too low at the apex of the diffraction and linearly increasing to a velocity 10% too high at the bottom of the section. The rounding of the edges of this residual "diffraction" is a manifestation of the non-unique solutions to equation (6).

Two-step 3-D vs. residual

When velocity varies in two-step 3-D migration, errors occur because the diffraction flanks for the second, orthogonal migrations are created from hyperbolic summations with velocities not equal to the velocity describing the actual hyperboloidal surface (Gibson et al, 1983). These errors, though slight, cannot be corrected by migration in the orthogonal direction because needed, out of plane information from the first direction of migration is not available. In residual migration, however, there is no actual perpendicular direction; errors in initial migration should be correctable because the same data is always present, whether or not it is fully focused.

ACKNOWLEDGMENTS

Discussions with Stew Levin were helpful and instructive.

REFERENCES

Gibson, Larner, and Levin, 1983, Efficient 3-D migration in two steps, Geophysical Prospecting 31, p. 1-33.

Rothman, Levin, and Rocca, 1983, Residual migration, SEP-35, p. 153-171.

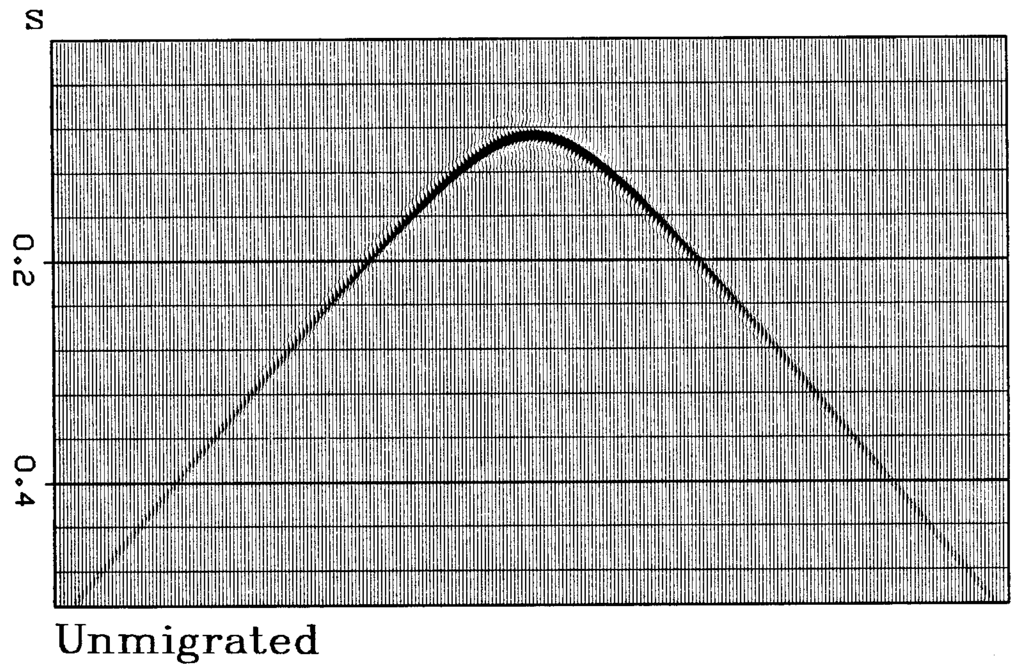

FIG. 1a. An unmigrated diffraction. The constant velocity is 2000 m/sec, the trace spacing is 4 m, and the sampling interval is 4 msec.

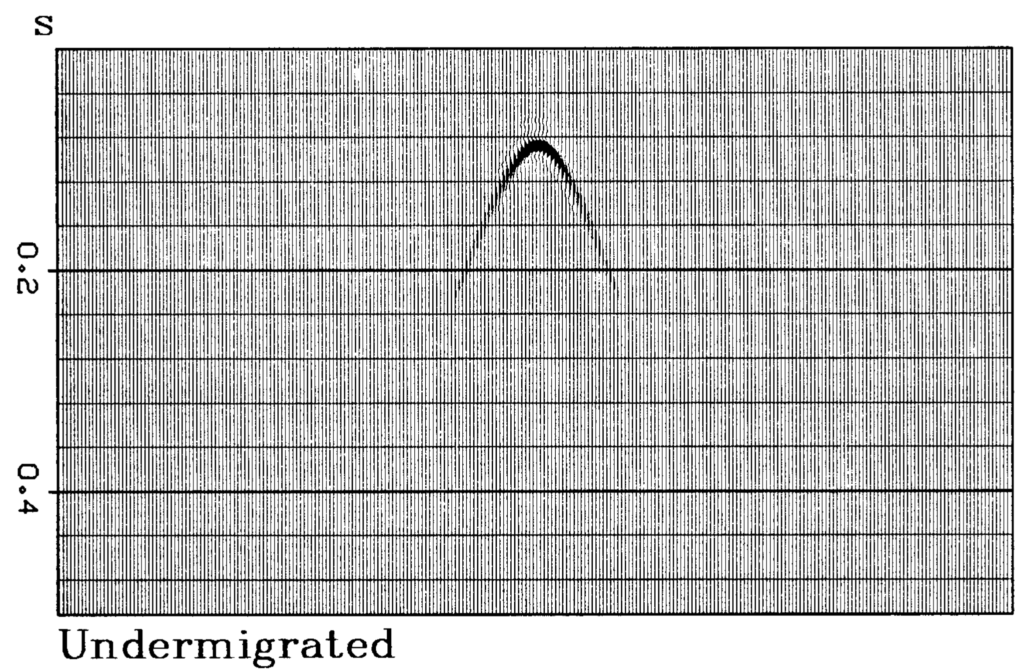

FIG. 1b. Incomplete migration of the diffraction in (1a). The migration velocity was 10% too low.

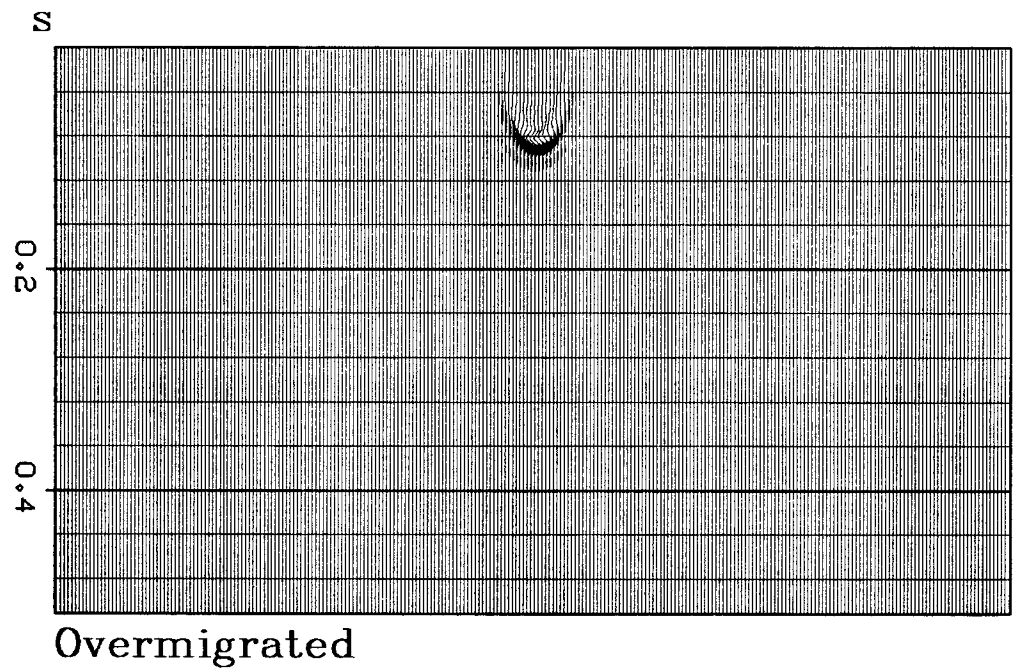

FIG. 1c. Overmigration of the diffraction in (1a). The migration velocity was 10% too high.

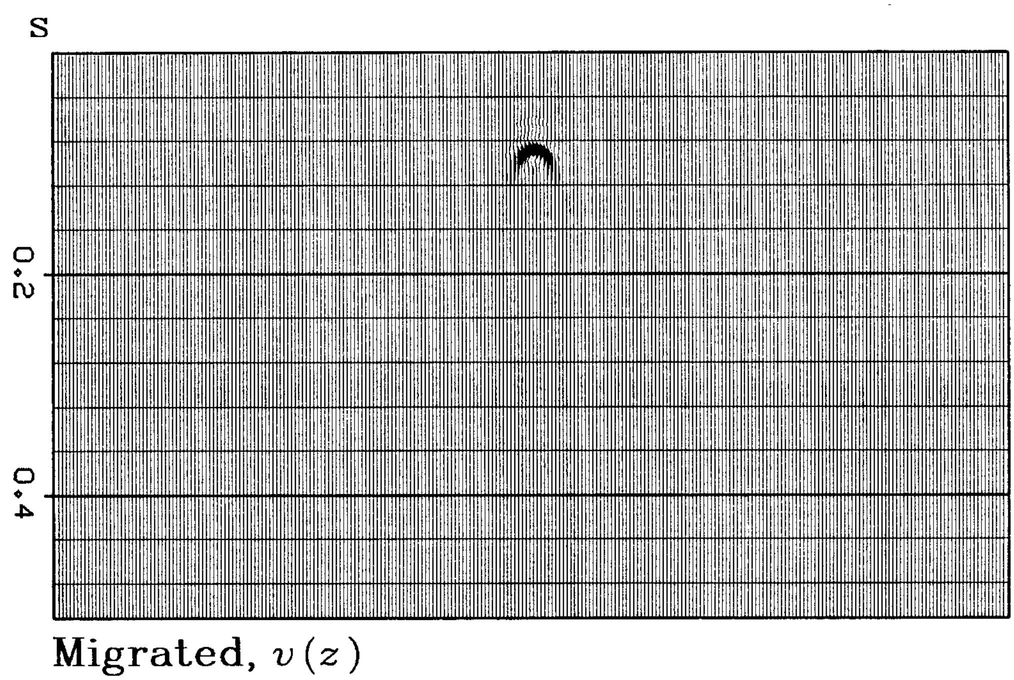

FIG. 1d. A depth-variable migration velocity yields a residual "diffraction" that is neither hyperbolic nor elliptical. This is the result of migrating the diffraction in (1a) with a velocity 10% too low at the apex, but linearly increasing to a velocity 10% too high at the bottom of the section. Note the rounded shape of the flanks.