Principle of reverse time migration

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Introduction

At the Migration Research Workshop of the 52^{nd} SEG meeting in Dallas (1982), attendees were made privy to an elegant method of wave-equation migration, known as reverse-time migration, presented by Dan Whitmore of Amoco. Shortly thereafter McMechan (1982, 1983) published a virtually identical algorithm under the tag "boundary value migration". This, along with closely related work by Baysal, Sherwood and Kosloff (1983a,b) done independently at the University of Houston's Seismic Acoustic Laboratory, has sparked much discussion.

In the May 1983 issue of Geophysics, Loewenthal and Mufti published quite a lengthy short note detailing a migration algorithm they developed as early as 1977 also referred to as "reversed time migration."

This last algorithm is quite different from any of the former and this difference has created confusion as to how reverse-time migration should be properly viewed.

In this short note, which I have submitted to Geophysics, I endeavor to explain the principle of reverse-time migration and how it is applied in practice in the above algorithms.

Exploding reflectors model

Conventional methods of migrating stacked data rely on the assumption that the input time section fits the *exploding reflectors* model. This model assumes that all energy present at time t on the stacked data arises from reflection at time $\frac{1}{2}t$ and so the strength of the reflection coefficient can be determined by propagating that energy back into the earth to half its arrival time or, equivalently, propagating the energy back to time zero at half the earth velocity.

Time reversed waves

All the above cited researchers use the exploding reflectors model and quite rightly remark that solutions of the scalar wave equation[1]

$$P_{xx} + P_{zz} = \frac{1}{c^2(x,z)} P_{tt} \tag{1}$$

generally used to propagate this data can be driven either forward or backward in time by simply reversing the time axis. That is if $P(x,z,t)$ is one solution to equation (1), then so is $P(x,z,t_0 - t)$ for any fixed constant t_0.

Reverse time propagation

Since the whole point of migration with the exploding reflectors model is to move energy back in time to its reflection (explosion) time, it is natural to use the scalar wave equation (1) to take successive steps backwards in time until the desired initial time is reached.

Historically this was *not* done. (Timoshin (1970), who applied the Kirchhoff integral to imaging multi-offset data, is a notable exception.) Instead the complete time history recorded at the surface, presumably represented by the stacked data, was used to predict the corresponding time history at successive *depth* increments. One of the principle developers and proponents of this downward extrapolation has been Jon Claerbout at Stanford University.

That approach treated the time section as a natural *boundary* condition at the surface of the earth. The stacked data prescribed values along the $z = 0$ edge of a t,x,z halfspace which are then extrapolated to the interior $z < 0$. The key idea of reverse time migration is that the same data can alternatively be thought of as a *source* function. Time reversing each receiver turns it into a loudspeaker broadcasting recorded energy (i.e. a time reversed trace) back into the earth.

Equating boundary values and sources is not new. Any text on partial differential equations (or mechanics) discusses transformation between homogeneous and inhomogeneous problems. Courant and Hilbert (1953 p.277) for example, point out that the linear problem $L[u] = 0$ with boundary values $u = f$ can be reformulated in terms of $v = u - f$, as $L[v] = -L[f]$ with boundary condition $v = 0$ where f is suitably extended to the interior of the domain (in our case with zeros) thus transforming boundary values into a source term.

[1] or any other scalar or elastic equation not involving time dependent coefficients or time derivatives of odd order.

Here Loewenthal and Mufti go astray. Instead of recognizing the time section as a source function, they scale time by velocity to produce an equivalent coordinate with units of length (e.g. vertical depth conversion) and then treat that data as a snapshot ("blurred depth section") of the wavefield throughout the subsurface as it might have appeared at the latest recording time. They then further blur the picture by not extrapolating back to $t = 0$ but instead imaging at $t = z/c$. Skipping past their explanations and simply looking at their equations one readily finds that, for constant velocity c, their algorithm migrates with the dispersion relation

$$c\,k_z - \omega \;=\; \omega - \sqrt{\omega^2 + c^2 k_x^2} \tag{2}$$

agreeing only to second order (15°) with

$$c\,k_z - \omega \;=\; \sqrt{\omega^2 - c^2 k_x^2} - \omega \tag{3}$$

used for conventional migration.

Instabilities

A classic problem with wave extrapolation is the presence of paired exponentially growing and decaying nonpropagating solutions known as evanescent waves. These are, of course, no problem in nature, which automatically picks the one decaying away from its source, but can be a problem for numerical algorithms where noise or round off error can unintentionally introduce rapidly growing, meaningless solutions.

Here reverse-time extrapolation appears to have the advantage over conventional downward continuation. In the conventional approach we use equation (3) to extrapolate exponential solutions $exp(\,i\,k_z z\,)$; evanescent waves arise from the argument of the square root being negative. Reverse time propagation employs (3) to generate exponential solutions $exp(\,i\,\omega\,t\,)$ with

$$\omega \;=\; \pm c\sqrt{k_x^2 + k_z^2} \tag{4}$$

which guarantees the solutions do not change amplitude in the direction of propagation.

This raises a question. The same surface data is used to determine the interior values of the wavefield in both methods. Where did the evanescent energy go? One answer is provided by the Kirchhoff integral (Schneider 1978). For constant velocity this may be written as a three-dimensional convolution (Schneider equation 7)

$$P(x,y,z,t) \;=\; P(x,y,z_0,t) * \frac{1}{2\pi}\frac{\partial}{\partial z_0}\left[\frac{\delta\left(t \pm \dfrac{r}{c}\right)}{r}\right] \tag{5}$$

which, though treated there as a downward extrapolation formula, can, as discussed by Timoshin, be viewed equally well as a reverse-time propagation formula. Berkhout and Van Wulfften Palthe (1980) compute the triple Fourier transform

$$\pm\, exp(\ -i \mid z \mid \sqrt{\omega^2 / c^2 - k_x^2 - k_y^2}\ ) \tag{6}$$

over x, y and t of the convolutional kernel verifying that it contains both propagating and evanescent components. This argues that the wavefield will contain the same evanescent components regardless of whether downward extrapolation or reverse time propagation is employed.

John Toldi (private communication) points out that the evanescent waves arise mathematically from monofrequency plane waves on which Fourier analysis is based. When impulsive sources are used (e.g. the delta function in eq. 5), the only amplitude changes are spherical spreading and reflection and transmission effects at interfaces. Evanescent waves are then associated with post-critical incidence. In this case the incident wave arrives at the interface *too slow* to produce a coherent wave front on the other side of the interface. Huygen's construction simply produces a bunch of expanding, incoherently interfering waves.

Multiples and full wave-equation solutions

One nice feature of reverse-time migration is that it allows synthetic generation programs to be used directly for the purpose of migration. This includes both one-way (primaries only) and two-way (primaries + multiples) algorithms. Indeed, Amoco presented an example of reverse-time migration imaging a reflection from the underside of a rim syncline (presumably waves refracted along rays bottoming out although multiples may also have been involved) using a full wave equation finite difference (forward-time) modeling program. As such, this allows full wave equation processing without the dip limitations imposed by Kosloff and Baysal (1983) to avoid evanescent instabilities.

There are still limitations. First, the exploding reflectors model only predicts a limited class of multiples and does not handle unsymmetric raypaths such as triangular refracted paths and most interbed multiples. Furthermore, even for unstacked data, we really need boundary values for the bottom (and sides) of the depth section we wish to image. Without these transmitted waves, multiples (and primaries) will split further at each reflector rather than coalesce into the simpler events from which they originally arose. Also, as Kosloff and Baysal point out, the imaging condition should be more sophisticated (e.g. selecting only downgoing energy or requiring time coincidence of up and downgoing energy) to avoid imaging most of these spurious multiples. One practical alternative artificially adjusts densities to match impedances (velocity × density) at all interfaces, making them transparent to normally incident waves (Baysal, Sherwood, and Kosloff 1983b).

Another perspective

Ozdogan Yilmaz (private communication) points out that Claerbout (1976 p.245), without realizing its interpretation, actually outlines an algorithm for reverse-time migration while discussing finite differences for the 15° wave equation. In that work downward continuation corresponds to z –outer ordering of computations while reverse-time propagation arises from t –outer ordering. Because his differential equation

$$P_{zt} \;=\; -\frac{v}{2} P_{xx} \tag{7}$$

contains z and t derivatives symmetrically, computational ordering was simply a matter of convenience. As seen in our earlier discussion of instability, this may not be true for higher order approximations to the wave equation and stability might depend upon the direction of recursion on the differencing grid.

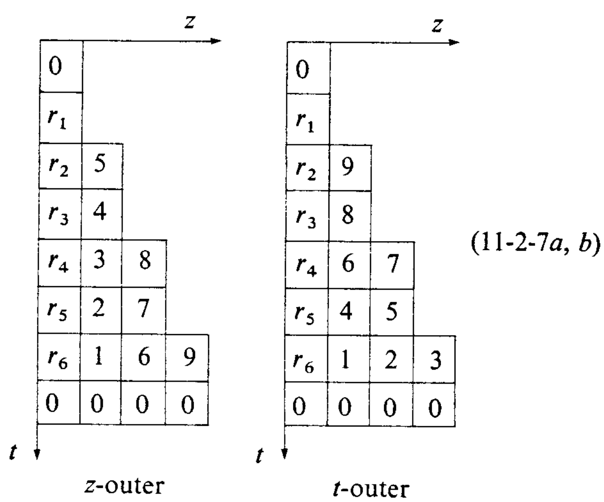

FIG. 1.. Schematic z –outer and t –outer 15° migrations. Both advance the stacked time section, r_i, to the same migrated, diagonal section in the indicated orders. Figure reproduced from Claerbout (1976).

I remark that reverse time migration is more directly implemented not as t –outer marching but simply by filling in successive planes parallel to the diagonal, migrated image. I will discuss this in detail in a future report.

Summary

Reverse-time migration achieves solution of linear scalar or elastic wave equations by treating a wavefield recorded at the surface of the earth as a time dependent secondary source distribution rather than a boundary condition in space. It also simplifies imaging of some classes of multiple energy. Numerical algorithms based on this method should not produce solutions that grow exponentially in the direction of propagation but can still correctly propagate evanescent energy. Finally, the simple t –outer and z -outer algorithms of Claerbout provide one bridge between reverse-time and conventional migration schemes.

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