The linear stacking slowness operator for a non-zero near offset

John Toldi

Abstract

All previous derivations of a linear relation between interval slowness anomalies and the corresponding anomalies in stacking slowness have assumed a near offset equal to the geophone interval. In common field practices, the distance from the shot to the nearest geophone is several geophone spacings. This practice changes the exact form of the operator relating interval slowness to stacking slowness. Because the near offset is typically only a small fraction of the full cable length, the effect is a subtle one. The generalized stacking slowness operator also provides the means to study the effects on stacking slownesses of using only a limited range of offsets.

Introduction

A careful derivation of the linear relation between interval slowness anomalies and the corresponding anomalies in stacking slowness must take into account the exact recording geometry. In particular, the near offset (the distance from the shot to the nearest geophone) is typically several times the regular offset interval. All previous derivations of such a linear relation (Loinger, 1981, Rocca and Toldi, 1982, and Toldi, 1983), have assumed a near offset equal to the regular offset interval. One might expect that the correction for an arbitrary near offset would be a windowing of the spatial impulse response. As shown in this paper, the shape of the impulse response itself changes, in addition to the expected windowing. In the following section, a revised derivation is presented. For clarity, I present a complete derivation, including some parts from Rocca and Toldi (1982).

Derivation of the linear equation

The derivation of the linear equation relating interval slowness anomalies to stacking slowness anomalies consists of two parts. The first comes from least squares analysis. With an assumed background slowness distribution, the traveltimes to a particular reflector can be fit in a least-squares sense to a line in $t^2 - x^2$ space. Then, to determine how the slope of that line (the stacking slowness squared, = w_s^2) would change if one of the traveltimes (actually t_i^2) were changed by a small amount, a standard result from least squares analysis is used,

$$\Delta\, slope \;=\; 2w_s \Delta w_s \qquad (1)$$

$$= \frac{\left(x_i^2 - \overline{x^2}\right)}{\sum_{i=1}^{n}\left(x_i^2 - \overline{x^2}\right)^2} 2t_i \Delta t_i$$

with the definition,

$$\overline{x^2} = \frac{1}{n}\sum_{i=1}^{n} x_i^2 .$$

The right side of equation (1) can be cast into a useful form, with x_0= near offset:

$$\frac{1}{n}\sum_{i=1}^{n} x_i^2 = \frac{1}{n\Delta x}\sum_{i=1}^{n} x_i^2 \Delta x$$

$$\approx \frac{1}{L-x_0}\left(\frac{x^3}{3}\right)_{x_0}^{L} = \frac{L^2}{3}\left(\frac{L^3-x_0^3}{L^2(L-x_0)}\right) = \frac{L^2}{3} b$$

Note that for $x_0 = 0$,

$$b = \left(\frac{L^3-x_0^3}{L^2(L-x_0)}\right)_{x_0=0} = 1$$

Similarly,

$$\sum_{i=1}^{n}\left(x_i^2 - \overline{x^2}\right)^2 = \sum_{i=1}^{n}\left(x_i^2 - \frac{bL^2}{3}\right)^2 = \sum_{i=1}^{n} x_i^4 - \frac{nb^2L^4}{9}$$

$$= \frac{n}{L-x_0}\left(\frac{L^5}{5} - \frac{x_0^5}{5}\right) - \frac{nb^2L^4}{9}$$

$$= nL^2\left(\frac{L^3}{5(L-x_0)} - \frac{x_0^5}{5(L-x_0)L^2} - \frac{b^2L^2}{9}\right) = nL^2\, a$$

Note once again that for $x_0 = 0$,

$$nL^2\, a \;=\; nL^2\left[\frac{L^3}{5(L-x_0)} - \frac{x_0^{\,5}}{5(L-x_0)L^2} - \frac{b^2L^2}{9}\right]_{x_0=0} \;=\; \frac{4}{45}nL^4,$$

exactly as in Rocca and Toldi (1982). This can be substituted into equation (1) to get:

$$2w_s\Delta w_s \;=\; \frac{\left[x_i^{\,2} - \frac{bL^2}{3}\right]}{anL^2}2t_i\Delta t_i \tag{2}$$

In the second part of the derivation, expressions for t_i and Δt_i are found by making certain assumptions on the type of medium allowed. Because these quantities (t_i and Δt_i) depend only on the geometry of the raypaths, they are derived exactly as in Rocca and Toldi (1982).

t_i is simply the unperturbed traveltime, i.e., the traveltime for the i^{th} offset through the background medium. For a constant background slowness w, with the reflector at depth z, we have

$$t_i \;=\; w\left[x_i^{\,2} + 4z^2\right]^{1/2} \tag{3}$$

Clearly this constant background slowness is also the unperturbed stacking slowness of equation (1).

To determine Δt_i, we assume that the effect of a localized interval slowness anomaly (Δw_{in}) is a delay of the rays passing through it; that is, the rays stay on the paths determined by the background slowness distribution. For a constant background slowness, the raypaths are straight lines, and the perturbation due to an interval slowness anomaly of thickness Δz_{an}, is therefore

$$\Delta t_i \;=\; \frac{\Delta z_{an}\,\Delta w_{in}}{2z}\left[x_i^{\,2} + 4z^2\right]^{1/2} \tag{4}$$

Note that although equations 3 and 4 were derived with a constant background slowness, modifying them for a depth variable background slowness simply requires replacing the straight raypaths with curved raypaths.

Finally, we can assemble equations 1, 3 and 4 to get

$$\Delta w_s \;=\; \begin{cases} \dfrac{2\Delta z_{an}\,z}{anL^2}\left[1 + \dfrac{x_i^2}{4z^2}\right]\left[x_i^2 - \dfrac{bL^2}{3}\right]\Delta w_{in} \;, & \text{for } x_0 < x_i < L \\ 0 \;, & \text{for } x_i < x_0 \end{cases} \tag{5}$$

A bit more algebra, and the definition $c = \frac{L^2}{4z^2}$, yields

$$\Delta w_s = \begin{cases} \frac{2\Delta z_{an} b}{3an}\left[c\left[\frac{x_i^2}{L^2}\right] + 1\right]\left[\frac{3}{b}\left[\frac{x_i^2}{L^2}\right] - 1\right]\Delta w_{in} & \text{for } x_0 < x_i < L \\ 0 & \text{for } x_i < x_0 \end{cases} \tag{6}$$

So far we have determined the stacking slowness response for one reflector, to an impulse of anomalous interval slowness encountered at a particular offset. Alternately, we could localize the interval slowness anomaly at depth z_{an} beneath a particular midpoint, and then determine the stacking slowness response as a function of midpoint. That is, as the midpoint y approaches that of the anomaly, y_{an}, the anomaly will be first traversed by the raypaths of the long offsets, then the short offsets and ultimately pass out through the long offsets again. The midpoint response can thus be determined by making a change of coordinates (see Figure 1):

$$L' = \frac{(z - z_{an})}{z} L$$

$$x = \frac{2z}{z - z_{an}}(y - y_{an}) = \frac{2L}{L'}(y - y_{an})$$

$$\Delta y = \frac{(L - x_0)}{2n}\left[\frac{z - z_{an}}{z}\right] \rightarrow \frac{1}{n} = \frac{2(L - x_0)L'}{L}\Delta y$$

And finally,

$$\Delta w_s(z, y, y_{an}) = \frac{4zb}{3a}\left[\frac{L}{L - x_0}\right]\left[c\left[\frac{2(y - y_{an})}{L'}\right]^2 + 1\right]\left[\frac{3}{b}\left[\frac{2(y - y_{an})}{L'}\right]^2 - 1\right] \tag{7}$$

$$\times \; \Delta w_{in}(y_{an}, z_{an})\Delta z_{an}\Delta y_{an} ,$$

$$\text{for } \frac{x_0 L'}{2L} < |y - y_{an}| < \frac{L'}{2}$$

$$= 0 , \qquad \text{for } |y - y_{an}| < \frac{x_0 L'}{2L}$$

Figure 2 is a plot of the impulse response as a function of midpoint. Two curves are shown, one with $x_0/L = 0$, the other with $x_0/L = .1$, a more typical value for most field data configurations. The first and most straightforward change in the operator, is the

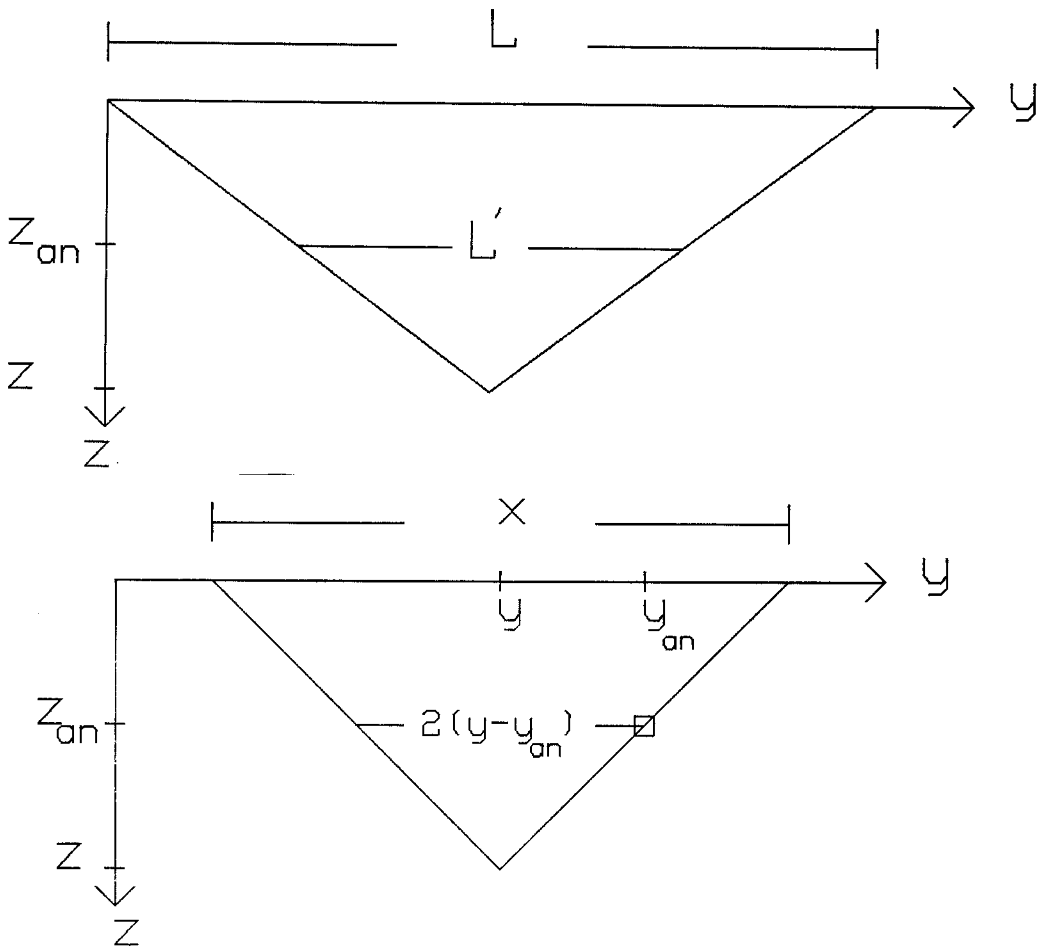

FIG. 1. Geometry to relate offset response to midpoint response. a) definition of effective cable length L', b) geometry for an offset less than the full cable length

windowing which occurs as the near offset increases. Secondly, the location of the zero crossing, i.e. the offset at which there is no response, shifts outward towards the end of the cable. Because this zero crossing is the balance point for the line in $x^2 - t^2$ space, it would naturally shift out towards the end of the cable. Furthermore, the impulse response becomes sharper--the peaks become higher and the troughs become deeper--as the near offset increases.

One final, less obvious point, is that the area under each of the two curves is the same. That is, the stacking slowness response to a constant change of interval slowness throughout a thin layer of thickness Δz_{an} is independent of the distance to the near offset. Integrating equation (7) over all midpoints produces an expected result:

$$area = \frac{\Delta z_{an}}{z},$$

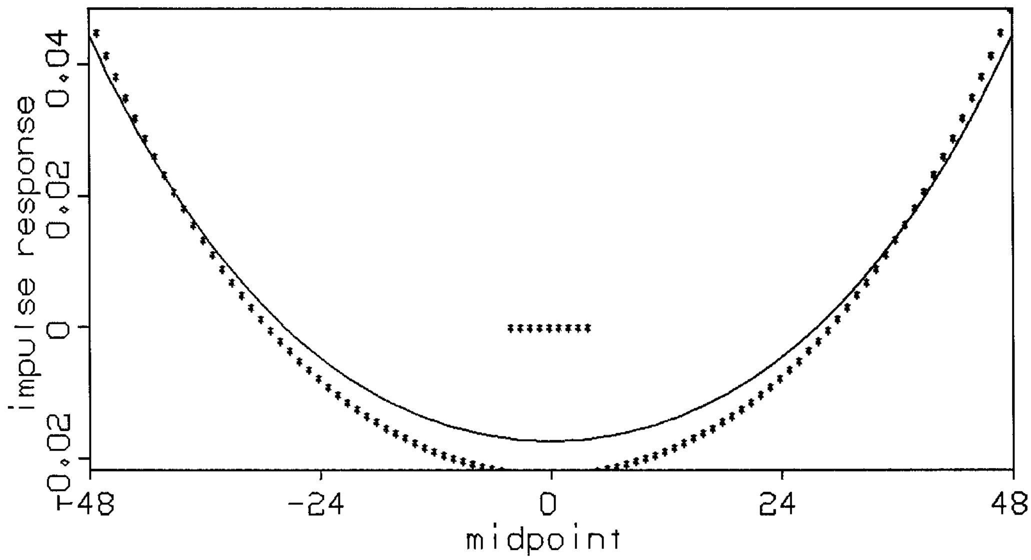

FIG. 2. Impulse response as a function of midpoint. The solid line corresponds to $x_0/L = 0$, dotted line corresponds to $x_0/L = .1$.

i.e. the ratio of the anomalous thickness to the total thickness involved in determining the stacking slowness.

Transfer function

Because the inversion of stacking slowness for interval slowness is carried out in the spatial frequency domain, it is necessary to Fourier transform the impulse response over midpoint. Using the constants a, b, and c, defined in the previous section, and the definition of a normalized wavenumber,

$$k = \frac{K_y L}{2} \frac{(z - z_{an})}{z} = \frac{K_y L'}{2} ,$$

we find the following rather forbidding expression for the transfer function:

$F(z, z_{an}, K_y) =$

$$\frac{4z}{3a}\left(\frac{L}{L-x_0}\right)\frac{1}{k^5}\left\{\begin{array}{l} \left[(3c+3-cb-b)\,k^4 - (36c-2cb+6)\,k^2 + (72c)\right]\sin k \\ + \left[(12c+6-2cb)\,k^3 - (72c)\,k\right]\cos k \\ - \left[(3cu_0^4+(3-cb)u_0^2-b)\,k^4 - (36cu_0^2-2cb+6)\,k^2 + (72c)\right]\sin ku_0 \\ - \left[(12cu_0^3+(6-2cb)u_0)\,k^3 - (72cu_0)\,k\right]\cos ku_0 \end{array}\right\}$$

with

$$u_0 = \frac{x_0}{L}$$

Figure 3 is a graph of the transfer function plotted as a function of normalized wavenumber, shown for the same values of x_0/L as in Figure 2. First, observe that the DC value, i.e. the response at $k = 0$, is the same for the two curves, exactly as expected from the earlier space domain discussion. Although the curves are similar throughout, notice how the exact location of the zero-crossings changes as x_0/L changes. This change will clearly affect the inversion of stacking slownesses for interval slownesses.

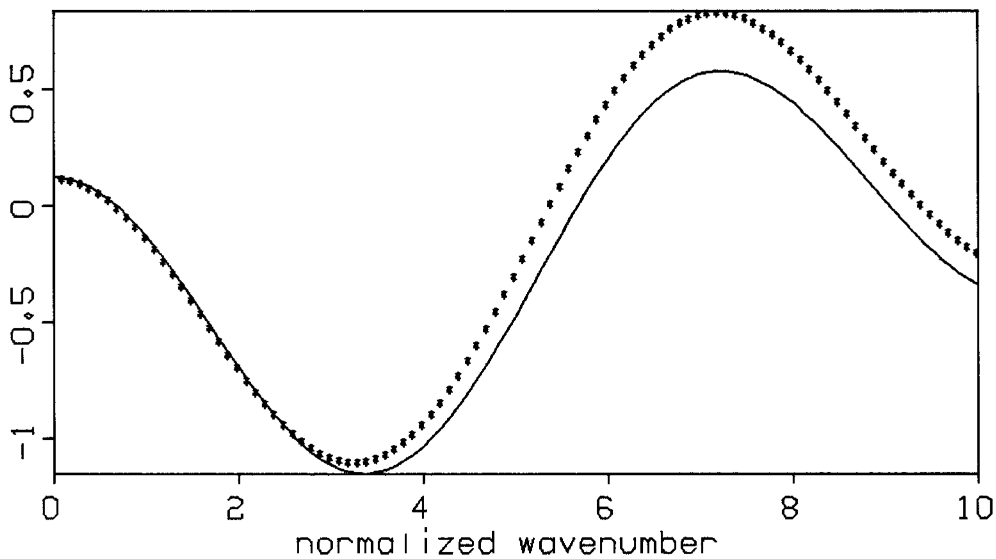

FIG. 3. Transfer function as a function of normalized wavenumber. The solid line corresponds to $x_0/L = 0$, dotted line corresponds to $x_0/L = .1$.

Analysis

The results of the previous section show that the effects of a non-zero near offset on the linear stacking slowness operator are subtle, given the typical field configuration. The generalization of the operator does, however, have further applications. In particular, the new form of the operator allows one to study the forward problem in a new light: one can now study the effects of a particular range of offsets on determining the stacking slowness.

The implications for the inverse problem are equally interesting. By having an operator that is somewhat local in offset space, one can consider inverting the data independently from different ranges of offsets. Clearly there is an inherent danger in such an approach, in that noise problems related to incorrect picks would only grow worse as one considered fewer traces in the semblance measures.

Consider, however, a very mild decomposition of the offsets into two ranges. The transfer functions corresponding to the two offset ranges have their zeroes at different wavenumbers. This difference in the zeroes could compensate for inversion problems arising from datasets with the usable reflectors grouped closely in depth; i.e. datasets for which all of the reflectors have their zeroes in the same place. This point is considered further in another paper in this report (Toldi, 1983), which deals with some of the difficulties associated with implementing the linear stacking slowness theory for a field dataset. The point to emphasize here is that the decomposition follows quite naturally from the generalization of the linear operator presented in this paper.

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