

**MIGRATION OF REFLECTION SEISMIC DATA
IN ANGLE-MIDPOINT COORDINATES**

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By

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Migration of Reflection Seismic Data in Angle-Midpoint Coordinates

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Abstract

The earth's subsurface may be imaged by migrating reflection seismic data in *angle-midpoint sections*. An angle-midpoint trace is the part of a *common midpoint gather* (the set of all seismic recordings with the same source-receiver midpoint) with the same time-offset slope. An angle-midpoint section is the collection of such traces from a row of common midpoint gathers.

Migration in angle-midpoint coordinates has theoretical and practical advantages over most existing migration methods. These include improvements in sub-surface images and velocity analysis. The most widely used migration method is to migrate the hyperbolic stacks of common midpoint gathers. Although this method is very practical, the migration equations are inaccurate for steep dips, wide offsets, and reflectors of different dips in the same location. The alternative of migrating *shot profiles* (recordings spaced at various distances from the same shot) is theoretically sound, but is hurt by aliasing and truncation. The other method of migrating *constant offset sections* (one offset from each profile) is more workable, but makes dip, angle, or velocity approximations in its migration equations. The angle-midpoint migration equations make none of these approximations and have workable implementations.

There are a couple of mathematical reasons why angle-midpoint coordinates are accurate. First, point diffractions are always hyperbolas on an angle-midpoint section. Simple and accurate migration responses can be designed for such impulse responses. Second, an angle-midpoint section is defined as the seismic data with the same offset-time slope. This allows for the straight forward and accurate

implementation of the migration equation for unstacked data that is written in terms of those slopes.

There are three ways of transforming a into angle-midpoint coordinates:

- (1) *Slant stacking* sums across the common midpoint gather along a slanted linear trajectory.
- (2) *Radial traces* are extracted from the common midpoint gather along diagonal lines.
- (3) *Snell traces* are extracted from the common midpoint gather along the line formed by the part of each reflection hyperbola that has the same slope.

The migration equations in each case have different accuracy and practical limitations. The migration equations for slant stacks are the most accurate. However, the construction of slant stacks suffers from aliasing and truncation problems. In contrast, the migration equations for radial traces assume constant velocity, but the transformation into radial traces is very workable. A *partial migration* scheme using radial traces is presented in this thesis. Partial migration is applied before conventional processing and is velocity insensitive. Snell trace migration is a hybrid of the other two methods: almost as practical as radial traces and almost as accurate as slant stacks. Slant stacks and Snell traces seem to give the best results. Radial traces are the easiest to implement.

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