5.2 Interval Velocity by Linear Moveout Methods

Linear moveout forms the basis for a simple, graphical method for finding seismic velocity. The method is particularly attractive for the analysis of data which is no longer in a computer, but just exists on a piece of paper. Additionally, the method offers a number of insights beyond the usual computerized hyperbola scan. It will help us rid ourselves of the notion that angles should be measured from the vertical ray. Non-zero Snell parameter can be the "default".

Ultimately this method leads to a definition for *velocity spectrum*, a plane in which the layout of the data itself tells you the seismic velocity.

Graphical Method for Interval Velocity Measurement

Consider a point source. The wavefront after a time t is a circle of radius $v\,t$ and is given by

$$v^2 t^2 \;=\; x^2 + z^2 \tag{1}$$

Letting $f = g - s$ denote the lateral source-receiver offset and z_s denote the depth to an image source under a horizontal plane layer we have

$$v^2 t^2 \;=\; f^2 + (z - z_s)^2 \tag{2}$$

We make our measurements at the earth's surface where $z=0$. Differentiating (2) with respect to t we obtain

$$v^2\, 2\, t \;=\; 2\, f\, \frac{df}{dt} \tag{3}$$

$$v^2 \;=\; \frac{f}{t}\,\frac{df}{dt} \;=\; \frac{f}{pt} \tag{4}$$

Figure 1 shows that the three parameters required by (4) to compute the material velocity are readily measured on a common midpoint gather.

Of course, we can measure some kind of velocity by means of equation (4) even if the earth does not have the assumed constant velocity. The question then becomes, what does the measurement mean? In the case of a stratified medium $v(z)$ we can quickly establish the answer to be the familiar RMS, or root-mean-square velocity. To do so, first note that the bit of energy arriving at the point of tangency has throughout its entire trip into the earth been propagating with a constant Snell's parameter p. The best way to

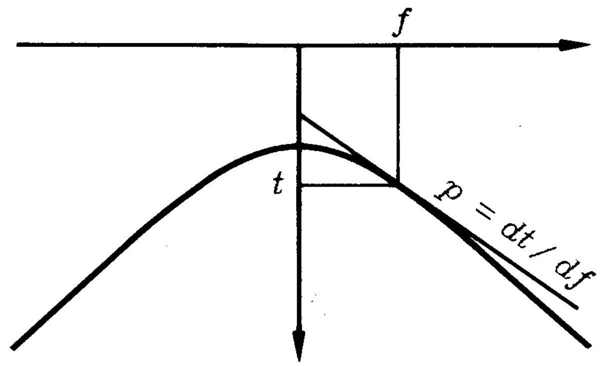

FIG. 1. (Gonzalez) A straight line, drawn tangent to hyperbolic observations. The slope p of the line is arbitrary and it may be chosen so that the tangency occurs at a place of good signal-to-noise ratio.

specify velocity in a stratified earth is to give it as some function $v(z)$. Another way is to pick a Snell's parameter p and start descending into the earth on a ray with this p. As the ray goes into the earth from the surface $z=0$ at $t=0$, the ray would be moving with a speed of, say, $v'(p,t)$. It is an elementary exercise to compute $v'(p,t)$ from $v(z)$ and vice versa. So, when convenient, we may refer to the velocity as some function $v'(p,t)$. The horizontal distance f which a ray will travel in time t is given by the time integral of the horizontal component of velocity, namely

$$f = \int_0^t v'(p,t) \sin\vartheta \, dt \tag{5}$$

Replacing $\sin\vartheta$ by pv and taking the constant p out of the integral yields

$$f = p \int_0^t v^2 \, dt \tag{6}$$

Inserting (6) into (4) we get

$$v^2_{measured} = \frac{f}{p\,t} = \frac{1}{t} \int_0^t v^2 \, dt \tag{7}$$

which justifies the assertion that

$$v_{measured} = v_{root\text{-}mean\text{-}square} = v_{RMS} \tag{8}$$

Equation (7) is exact. It does not involve a "small offset" assumption or a "straight ray" assumption.

Next let us consider the so-called *interval* velocity. Figure 2 shows hyperboloidal arrivals from two flat layers where a straight line of slope p has been constructed to have the same slope p. Then the tangencies are measured to have locations (f_1, t_1) and (f_2, t_2). Combining (6) with (4), using the subscript i to denote the i-th tangency (f_i, t_i), we have

$$f_i \frac{df}{dt} = \int_0^{t_i} v^2 \, dt \tag{9}$$

Assume that the velocity between successive events is a constant $v_{interval}$ and subtract (9) with $i+1$ from (9) with i to get

$$(f_{i+1} - f_i) \frac{df}{dt} = (t_{i+1} - t_i) \, v_{interval}^2 \tag{10}$$

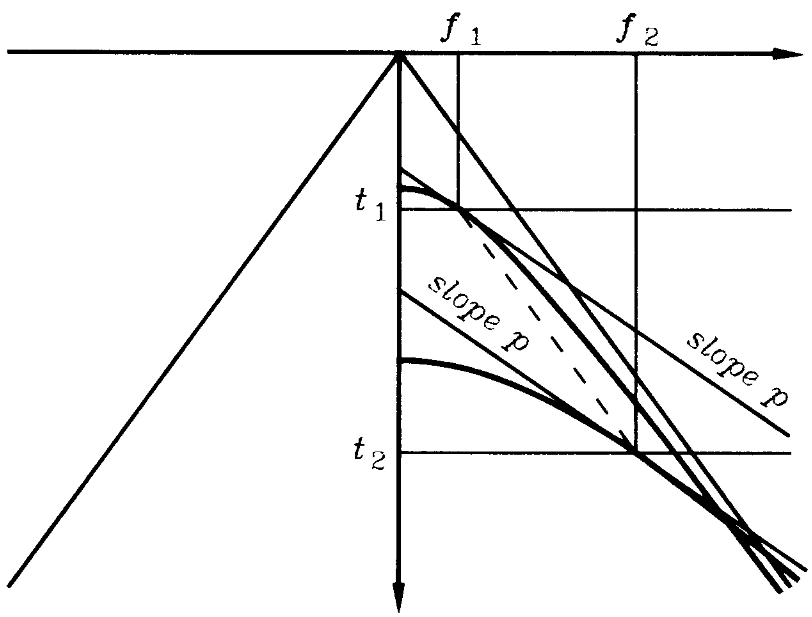

FIG. 2. (Gonzalez) Construction of two parallel lines on a common midpoint gather tangent to reflections from two plane layers.

Solving for the interval velocity we get

$$v_{interval}^2 = \frac{f_{i+1} - f_i}{t_{i+1} - t_i} \frac{df}{dt} \tag{11}$$

So the velocity of the material between the i-th and the $i+1$-st reflectors can be measured directly by the square root of the product of the two slopes in (11), which are the dashed and solid straight lines in figure 2. The advantage of manually placing straight lines on the data, over automated analysis, is that you can graphically visualize the sensitivity of the measurement to noise, and you can select the best offsets on the data at which to make the measurement.

When doing this routinely one quickly discovers that the major part of the effort is in accurately constructing two lines which are tangent to the events. When this happens, it is convenient to replot the data with linear moveout $t' = t - pf$. After replotting, the sloped lines have become horizontal so that any of the many timing lines can be used. Locating tangencies is now a question of finding the tops of convex events. This is depicted in figure 3.

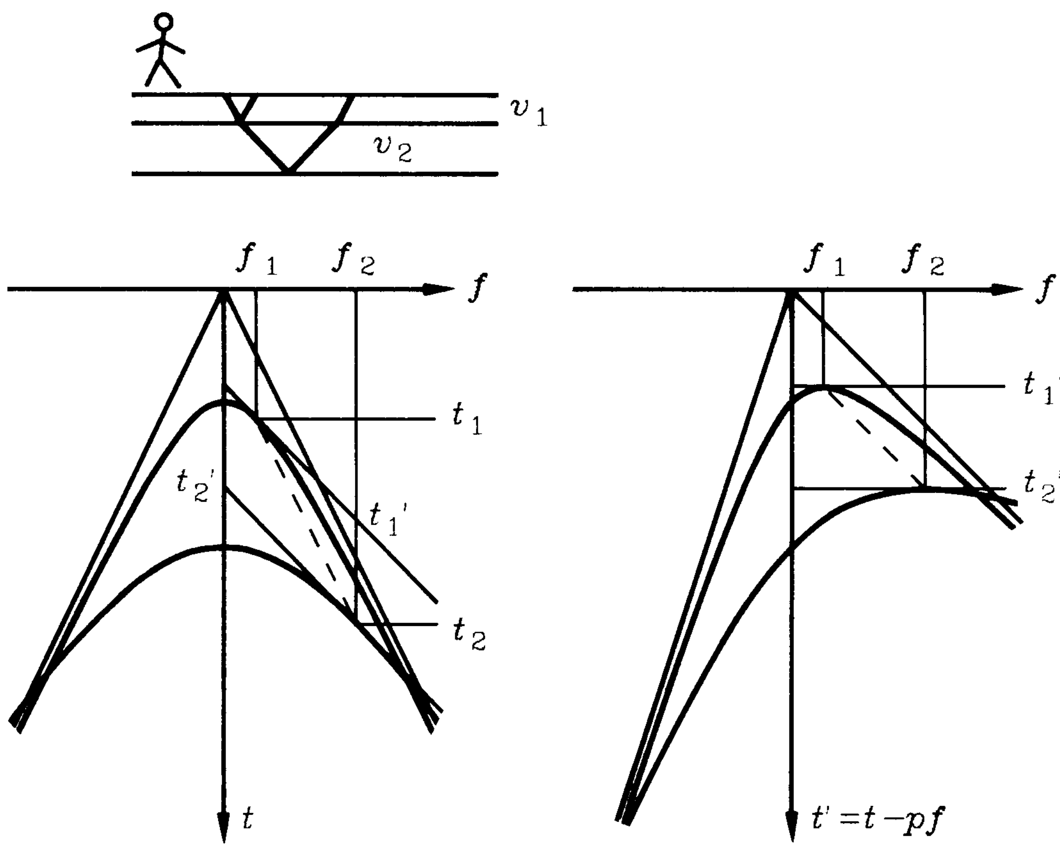

FIG. 3. (Gonzalez) Linear moveout converts the task of identifying tangencies to constructed parallel lines, into the task of locating tops of convex events.

In terms of the time t', equation (11) becomes

$$v_{interval}^2 = \frac{1}{\frac{\Delta t}{\Delta f}} \frac{1}{p} = \frac{1}{\frac{\Delta t'}{\Delta f} + p} \frac{1}{p} \tag{12}$$

Common Midpoint Snell Coordinates

Common *midpoint* slanted wave analysis is a more conservative approach to seismic data analysis than the Snell wave approach. The advantage of common midpoint analysis is that the effects of earth dip tend to show up mainly on the midpoint axis, and the effect of seismic velocity shows up on the offset axis.

The disadvantage of midpoint analysis is that it is non-physical. When you do a slant stack at common geophone, you are modeling a physical situation and you expect to be able to write a differential equation to describe the stack, no matter what ensues, multiple reflection or lateral velocity variation. A common *midpoint* slant stack does not model anything which is physically realizable. Nothing says that a partial differential equation exists to extrapolate such a stack. This doesn't mean that there is necessarily anything wrong with a common midpoint *coordinate* system. But it does make you respect the Snell stack approach even though it has not made much progress in the industrial world.

(Someone implementing common midpoint slant stack would immediately notice that it is easier than slant stack on common geophone data. This is because at a common midpoint, the tops of hyperboloids must be at zero offset, the location of the Fresnel zone is predictable and interpolation and missing data problems are much alleviated.)

Seismic data is collected in time, geophone, shot, and depth coordinates (t,g,s,z). We will be defining a new four component system. First of all, we will want to define midpoint in the usual way

$$y(t,g,s,z) = \frac{g+s}{2} \tag{13}$$

Next we will want to define a travel-time depth. This will be done by using the vertical phase velocity in a borehole. Keeping as much as possible with conventional notation, we will use the two way travel time.

$$\tau(t,g,s,z) = 2\,z\,\frac{\cos\vartheta}{v} \tag{14}$$

Next we will define the surface offset h'. This will not be the old definition of offset. We plan to use this coordinate system to downward continue shots and geophones. Instead of thinking of them as going straight down, we would like to think of them as going down along the ray. This can be achieved if we define h' as follows

$$h'(t,g,s,z) \quad = \quad \frac{g-s}{2} + z \tan\vartheta \tag{15}$$

With this new definition of h' we can see that for constant h' the separation of the shot and geophone decreases with the depth of downward continuation of the experiment.

Define the *LMO time* as the travel time in the point source experiment less the linear moveout. So at any depth we can compute the LMO time by $t - p(g-s)$. As we defined h' to be the *surface* half offset, we will now define t' to be the *surface* LMO time. From the LMO time of a buried experiment we find the LMO time at the surface by adding in the travel time depth.

$$t' \quad = \quad t - p(g-s) + \tau \tag{16}$$

You might like to think of this as a slant on the old time retardation for up coming waves, say $t' = t_{LMO} + (z_{slant}/v)$. Formally we have

$$t'(t,g,s,z) \quad = \quad t - p(g-s) + 2\, z\, \frac{\cos\vartheta}{v} \tag{17}$$

Figure 4 is a geometrical representation of these concepts.

From the geometry of figure 4 we can deduce that a measurement of a reflection at some particular value of (h',t') directly determines the velocity. Write an equation for the reflector depth

$$\left(\frac{t'}{2} + p\, h' \right) \cos\vartheta \quad = \quad \text{reflector depth} \quad = \quad \frac{h'}{\tan\vartheta} \tag{18}$$

Using Snell's law to eliminate angles and solve for velocity we immediately get

$$v^2 \quad = \quad \frac{1}{p} \, \frac{1}{p + \dfrac{t'}{2h'}} \tag{19}$$

Comparing back to equation (12) we can see that we are on the track.

Gathering the above definitions into a group, and allowing for depth variable velocity by replacing z by the integral over z, we have

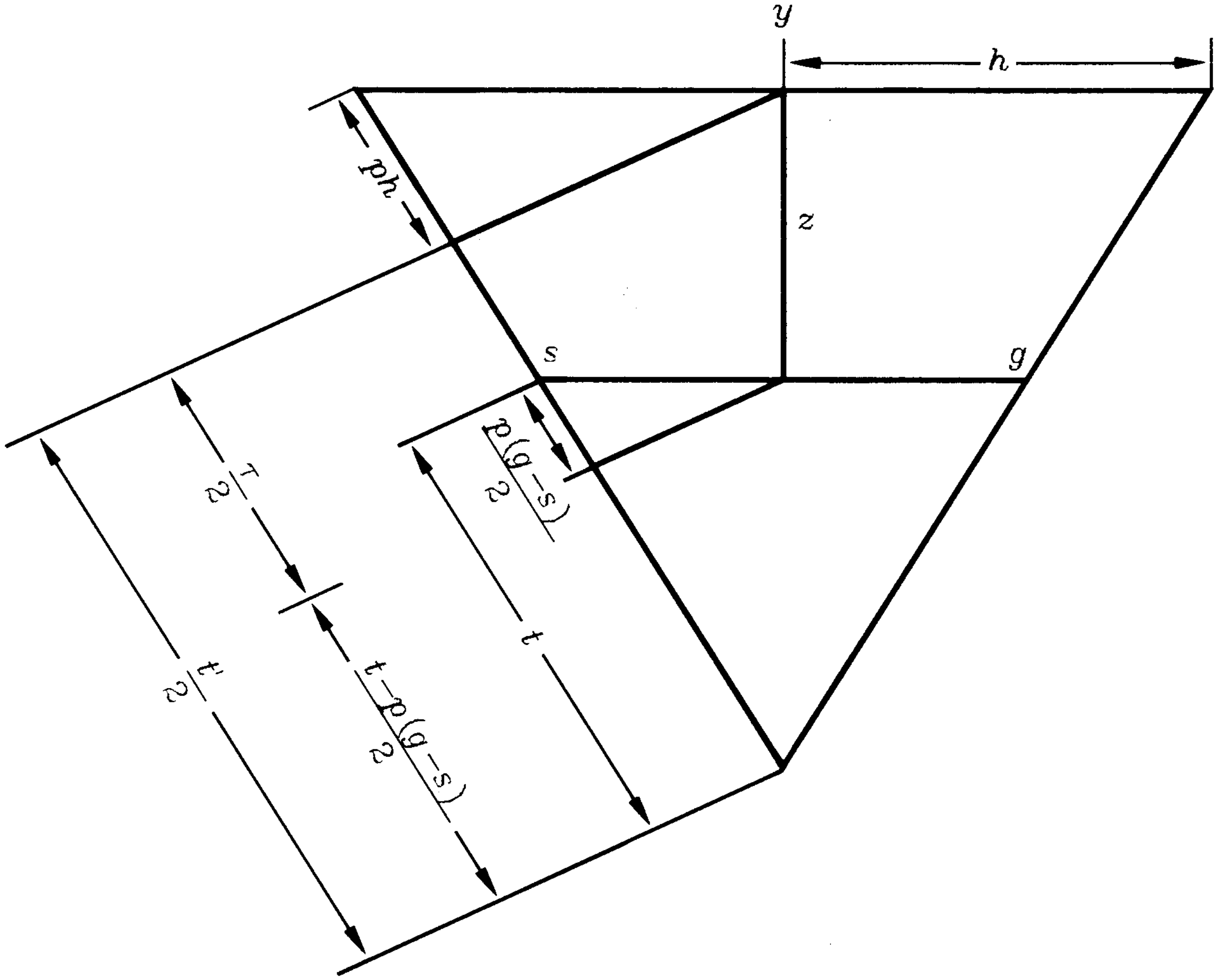

FIG. 4. The common midpoint linear moveout geometry.

$$t'(t,g,s,z) = t - p(g-s) + 2\int_0^z \frac{\cos\vartheta}{v}\,dz \tag{20a}$$

$$y(t,g,s,z) = \frac{g+s}{2} \tag{20b}$$

$$h'(t,g,s,z) = \frac{g-s}{2} + \int_0^z \tan\vartheta\,dz \tag{20c}$$

$$\tau(t,g,s,z) = 2\int_0^z \frac{\cos\vartheta}{v}\,dz \tag{20d}$$

Before these equations are actually used, all of the trigonometric functions are eliminated by Snell's law for stratified media, which says that $\sin\vartheta(z) = p\,v(z)$, where Snell's parameter p is a numerical constant throughout the analysis.

The equation for interval velocity determination (12) again arises by combining dt'/dz from (20a) and dh'/dz from (20c)

$$\frac{dt'}{dh'} = \frac{2\cos\vartheta}{v\,\tan\vartheta} \tag{21}$$

Eliminating the trig functions with $p\,v = \sin\vartheta$ allows us to solve for the interval velocity.

$$v^2 = \frac{1}{p}\,\frac{1}{p + \frac{1}{2}\frac{dt'}{dh'}} \tag{22}$$

At the earth's surface $z = 0$, seismic survey data can be put into the coordinate frame (20) merely by numerical choice of p and doing the linear moveout. No knowledge of velocity $v(z)$ is required so far. Then you look at the data for some tops of the skewed hyperbolas. Finding some, equation (12), (19) or (22) gives you a velocity which you may use to begin downward continuation.

Waves can be described in either (t,g,s,z) physical coordinates or the newly defined coordinates (t',y,h',τ). In physical coordinates we are familiar with the idea that reflectors exist wherever echoes arrive at zero traveltime, namely

$$t = 0 \qquad \text{and} \qquad g = s \tag{23a,b}$$

We would like to express these conditions in the Snell coordinates. Inserting (23) into (20a) and (20d) we get what programmers call the stopping condition

$$t' = \tau \tag{24}$$

This is the depth at which the velocity information should be best focused in the (h',t')-plane. Next we need some downward continuation equations.

Differential Equations and Fourier Transforms

The chain rule for partial differentiation gives

$$\begin{bmatrix} \partial_t \\ \partial_g \\ \partial_s \\ \partial_z \end{bmatrix} = \begin{bmatrix} t'_t & y_t & h'_t & \tau_t \\ t'_g & y_g & h'_g & \tau_g \\ t'_s & y_s & h'_s & \tau_s \\ t'_z & y_z & h'_z & \tau_z \end{bmatrix} \begin{bmatrix} \partial_{t'} \\ \partial_y \\ \partial_{h'} \\ \partial_\tau \end{bmatrix} \tag{25}$$

In our usual notation, time derivative ∂_t has the Fourier representation $-i\omega$. Likewise, $\partial_{t'}$ and the spatial derivatives $(\partial_y,\partial_{h'},\partial_\tau,\partial_g,\partial_s,\partial_z)$ are associated with $i(k_y,k_{h'},k_\tau,k_g,k_s,k_z)$. Using these Fourier variables in the vectors of (25) and differentiating (20) to find the indicated elements in the matrix of (25), we have

$$\begin{bmatrix} -\omega \\ k_g \\ k_s \\ k_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -p & 1/2 & 1/2 & 0 \\ p & 1/2 & -1/2 & 0 \\ \frac{2\cos\vartheta}{v} & 0 & \tan\vartheta & \frac{2\cos\vartheta}{v} \end{bmatrix} \begin{bmatrix} -\omega' \\ k_y \\ k_{h'} \\ k_\tau \end{bmatrix} \qquad (26a,b,c,d)$$

Let S be the sine of the take-off angle at the source and G be the sine of the emergent angle at the geophone. If velocity v is known, these angles are directly measurable as stepouts on common geophone gathers and common shot gathers. Likewise, on a constant offset section or a slant stack observed stepouts relate to a sine like quantity Y, and on a linearly moved out common midpoint gather stepouts measure a sine-like quantity H'. The precise definitions of these sine-like quantities are given by

$$S = \frac{vk_s}{\omega} \qquad G = \frac{vk_g}{\omega} \qquad (27a,b)$$

$$Y = \frac{vk_y}{2\omega} \qquad H' = \frac{vk_{h'}}{2\omega} \qquad (27c,d)$$

With these definitions (26b) and (26c) become

$$G = pv + Y + H' = Y + (H' + pv) \qquad (28a)$$

$$S = -pv + Y - H' = Y - (H' + pv) \qquad (28b)$$

We see that the familiar offset stepout angle H is related to the LMO residual stepout angle H' by $H' = H - pv$. Setting H' equal to zero means setting $k_{h'}$ equal to zero, indicating integration over h', which in turn means slant stacking data with slant angle p. Small values of H'/v or $k_{h'}/\omega$ refer to stepouts near to p.

Processing Possibilities

The double square root equation is

$$\frac{k_z}{\omega} = -\frac{1}{v}\left(\sqrt{1 - S^2} + \sqrt{1 - G^2}\right) \qquad (29)$$

Using the substitutions (26a,d), and (27a,b) we discover that in the retarded Snell coordinates the double square root equation becomes

$$\frac{k_\tau}{\omega} = 1 - \frac{pv}{1-p^2v^2}H' - \frac{1}{2}\left\{\left[1 - \frac{2pv(H'-Y)+(H'-Y)^2}{1-p^2v^2}\right]^{1/2} + \left[1 - \frac{2pv(H'+Y)+(H'+Y)^2}{1-p^2v^2}\right]^{1/2}\right\} \quad (30)$$

Equation (30) is an exact representation of the double square root equation in what is called *retarded Snell midpoint coordinates*.

The coordinate system (20) can describe any wavefield in any media. It is particularly advantageous only in a stratified media of velocity near $v(z)$ for rays which are roughly parallel to any ray with the chosen Snell's parameter p. There is little reason to use these coordinates unless they "fit" the wave being studied. Waves which fit are those which are near our chosen p value. This means that H' doesn't get too big. A variety of simplifying expansions (30) are possible. There are many permutations of magnitude inequalities among the three ingredients pv, H', and Y. The expansion to use depends upon the circumstances. The appropriate expansions and production considerations have not yet been fully delineated. But let us take a look at two possibilities.

First, any data set can be decomposed by stepout into many data sets each with a narrow bandwidth in stepout space, CMP slant stacks for example. For any such data set we might ignore H' altogether. Then (30) reduces to

$$\frac{k_\tau}{\omega} = 1 - \frac{1}{2}\left\{\left[1 - \frac{-2pvY+Y^2}{1-p^2v^2}\right]^{1/2} + \left[1 - \frac{+2pvY+Y^2}{1-p^2v^2}\right]^{1/2}\right\} \quad (31a)$$

or

$$\frac{k_\tau}{\omega} = 1 - \frac{1}{2\sqrt{1-p^2v^2}}\left[\sqrt{1-(Y-pv)^2} + \sqrt{1-(Y+pv)^2}\right] \quad (31b)$$

The above approach is similar to the one employed by Richard Ottolini in his dissertation.

A second possibility is to filter away the portion of the data which is far from the chosen p, then process the rest of the data set at the chosen value of p. Keeping powers up to quadratics in H' and Y we get

$$\frac{k_\tau}{\omega} = \frac{H'^2}{2(1-p^2v^2)} + \frac{Y^2}{2(1-p^2v^2)}. \quad (32)$$

It is no accident that there are no powers of Y and H' less than squares. The coordinate system was designed so that energy near the chosen model $Y=0$ and $H=pv$ should not drift in the (h',t')-plane as the downward continuation proceeds.

The velocity spectrum idea of equation (32) is to use the H' term to focus the data on the (h',t')-plane. After this it should be possible to read interval velocities directly as slopes connecting events on the gathers. This approach was used in the dissertation of Alfonso Gonzales.

Pre-stack partial migration should have a place here somewhere, but so far it hasn't been worked out.