5.1 Linear Moveout and Snell Waves

Historically, we often regard echo delay as somewhat synonymous with depth. When it becomes necessary to think about angles, it is natural that we measure them by their departure from the vertical ray. Actually, the best seismic data is generally that recorded at a substantial angle from the vertical. Most of our data is certainly not from vertical rays. Usually we don't even record the data which travels near the vertical ray. In this chapter we develop a pattern of thinking which is oriented about a selected *non*-vertical ray. Rotation of coordinates is not the method of choice. Rotation would lose the fact that the plane on which we make our measurements is the plane given by the very simple equation $z = 0$. Rotation would also make a mess out of the simple seismic velocity function $v(z)$ making it a strongly two dimensional function $v'(x',z')$.

In this chapter we will learn the basic concepts of linear moveout (LMO). This will offer a deeper understanding of offset than the view presented in Chapter 3. While not fully incorporated in the modern production environment, this deeper view is of special interest to researchers. This view offers better understanding of velocity estimation. It also offers an understanding of multiple reflections, a subject untouched in Chapter 3.

Interpretation of Stepout as Snell's Parameter

A basic seismic measurement is that of *stepout*. Consider two geophones separated by a distance Δx which record an echo. One geophone receives the echo at a time Δt later than the other. The *stepout* of the echo is defined to be $\Delta t / \Delta x$. It has units of inverse velocity and may be given in units of milliseconds per meter or seconds per kilometer. Figure 1 depicts a plane wave incident on the earth's surface. From the geometry, you can see that the stepout is a function of the seismic velocity and the angle of the wave, namely

$$\frac{dt}{dx} \;=\; \frac{1}{\textit{horiz. speed at } z = 0} \;=\; \frac{\sin\vartheta}{v} \qquad (1)$$

We will see that the stepout seen on the surface must equal the stepout seen at any depth. Think about an airplane flying horizontally above the earth at a constant speed (inverse stepout). The airplane goes from $x = -\infty$ to $x = +\infty$. Imagine an earth of plane horizontal layers. In this model there is nothing to distinguish any point on the x-axis from any other point on the x-axis. Consider a

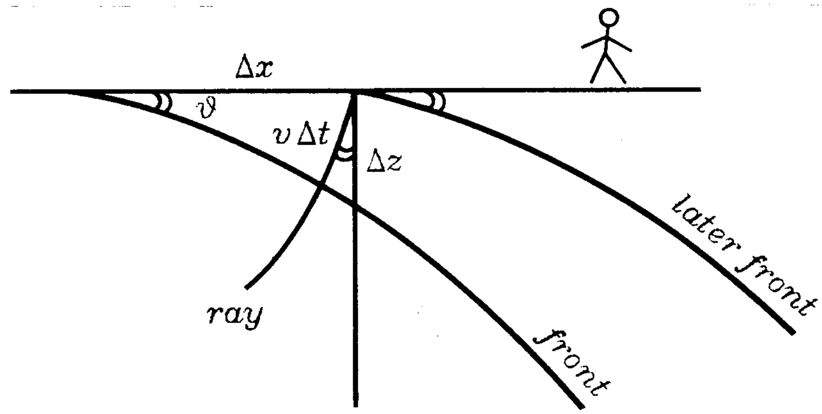

FIG. 1. Wave front arrival at earth's surface showing that observation of dt/dx gives the ratio $dt/dx = (\sin\vartheta)/v$.

snapshot (x,z) picture of the wavefronts in the vicinity of the airplane. There may be reflections, refractions, and multiples. Whatever the picture is, it moves along with the airplane. The top of the picture and the bottom of the picture both move along at the same rate even if the earth velocity increases with depth. So dt/dx is a constant function of z.

We have restricted the velocity $v(z)$ to be a function of depth only. Such an earth is said to be ***stratified***. In a stratified earth model the stepout is a most informative measurement. By itself it doesn't tell us the angle $\vartheta(z)$ of the rays nor does it tell us the seismic velocity $v(z)$. But this simple surface measurement does tell us the ratio (1). And the ratio will be the same at all depths. The stepout is also called the *Snell parameter*, because its depth invariance is basically a statement of Snell's law. Snell's parameter, generally denoted p is defined by

$$p = \frac{\sin\vartheta(z)}{v(z)} \tag{2}$$

We have deduced the fact that the parameter p in equation (2) does not depend on depth z. The method of deduction is very general. It applies to both transmitted rays and reflected rays. It doesn't matter if the wave is up-going or down-going, primary or a multiple, or even if some legs of the journey are by shear waves. For any ray traveling from a source to a receiver in a stratified media, the Snell parameter p is a constant function of time as well as depth.

The inverse to Snell's parameter p is known as the ***horizontal phase velocity***. For a vertically incident plane wave this velocity is infinity. Less steep angles have slower velocities. We rarely consider *anisotropic* media (wave speed depending on direction). In that case dt/dx is constant with depth, but it does not have the Snell interpretation (1).

Definition of Snell Wave

Consider the wave generated by a moving volume of high pressure such as surrounds the nose of a supersonic jet plane. To get the wave to penetrate into the earth, the airplane speed should exceed that of the seismic waves in the earth. Unfortunately, this requirement seems to be unrealistic for cases of practical interest, but that doesn't detract from the conceptual model. [Actually, for two-dimensional (x,z) analysis, we would need a line source along the third dimension y]. Anyway, in a stratified medium with some $v(z)$ the airplane initiates a ray at every point along the x-axis. Each ray has the same Snell parameter. For a constant velocity medium, the wave is nothing other than a plane wave. In a stratified medium $v(z)$ the wavefronts become curved and are no longer planar. Such wavefronts are so central to applied seismogram analysis in petroleum prospecting that they require a name. To prevent us from inaccurately referring to these wavefronts as non-vertically-incident plane waves, I propose to call them *Snell waves*. A Snell wave is nothing more than a plane wave which enters a stratified medium and becomes curved. In optics a beam of light is said to have an "angular spectrum." We seismologists worry a lot about the velocity increasing with depth. So instead of an angular spectrum, we have the "dip spectrum" by which we really mean the spectrum of Snell parameters.

Lateral Invariance

The nice thing about a vertically incident source of plane waves $p = 0$ in a horizontally stratified medium is that the ensuing wave field will be laterally invariant. In other words, an observation or a theory for a wave field would be of the form $P(t) \times const(x)$. Snell waves for any particular non-zero p value are also laterally invariant. That is, with

$$t' = t - p\,x \tag{3a}$$

$$x' = x \tag{3b}$$

lateral invariance is given by the statement

$$P(x,t) = P'(t') \times const(x') \tag{4}$$

Obviously when an apparently two-dimensional problem can be reduced to one dimension, great conceptual advantages result, to say nothing of advantages of sampling and computation. Before proceeding, study equation (4) until you realize why the wave field can vary with x but be a constant function of x' when (3b) says $x = x'$.

You might notice that the coordinate system (3) is a retarded coordinate system, not a moving coordinate system. Moving coordinate systems work out badly in solid earth geophysics. The velocity function is never time variable in the earth, but it becomes time variable in a moving coordinate system. This adds a whole dimension to the computational complexity. We are intending to solve problems with real data for which the model velocity is a function of all space dimensions. But we will solve these problems in a coordinate system which has a reference velocity which is a function of depth only.

Coordinates for Snell Waves

The Snell wave has three intrinsic planes, which are suggestive of a coordinate system. First are the layer planes of constant z which include the earth surface. Second is the plane of rays. Third is the plane of the wavefront. The planes become curved when velocity varies with depth.

The following equations define *Snell wave coordinates*.

$$z'(z,x,t) \quad = \quad z \, \frac{\cos\vartheta}{v} \tag{5a}$$

$$x'(z,x,t) \quad = \quad z \, \tan\vartheta \; + \; x \tag{5b}$$

$$t'(z,x,t) \quad = \quad z \, \frac{\cos\vartheta}{v} \; - \; x \, \frac{\sin\vartheta}{v} \; + \; t \tag{5c}$$

Equation (5a) is just a definition of a travel time depth by the vertical phase velocity seen in a bore hole. Interfaces within the earth are just planes of constant z'.

Setting x' defined by equation (5b) equal to a constant, say x_0 we get an equation for a ray, namely $(x - x_0)/z = -\tan\vartheta$. Different values of x_0 are different rays.

Setting t' defined by equation (5c) equal to a constant, we get the equation for a moving wave front. To see this, set $t' = t_0$ and note that at constant x one sees the borehole speed, and at constant z one sees the airplane speed.

Mathematically, one equation in three unknowns defines a plane. So if you set the left side of any of the equations (5a,b,c) to a constant, you have an equation defining a plane in (z,x,t)-space. To get some practice, let us look at the intersection of two planes. Staying on a wavefront requires $dt' = 0$. Using equation (5c) we have

$$dt' \;=\; 0 \;=\; \frac{\cos\vartheta}{v}\,dz \;-\; \frac{\sin\vartheta}{v}\,dx \;+\; dt \tag{6}$$

Combining the constant wavefront equation $dt' = 0$ along with the constant depth equation $dz' = dz = 0$ we get a familiar relationship

$$\frac{dt}{dx} \;=\; p \tag{7}$$

When coordinate planes are non-orthogonal, the coordinate system is called ***affine***. With affine coordinates, such as these, there is no problem of computational tractability, but there is often a human confusion problem. For example in displaying movies of marine field data, one sees a sequence of (h,t)-planes. Successive planes are successive shot points. So the data are displayed in (s,h) while people tend to think in (y,h) or (s,g). With affine coordinates I find it easiest to forget about the coordinate axis, and think about the plane to which it is perpendicular. The shot axis $\mathbf{s}$ can be thought of as a plane of constant geophone, say cg. So I think of the marine data movie as being in (cs,ch,ct)-space. In this movie, another plane, really a family of planes, the planes of constant midpoints cy, sweep across the screen, along with the "texture" of the data.

To define Snell coordinates when the velocity is depth variable, it is only necessary to interpret (5) carefully. First, all angles must be expressed in terms of p by the Snell substitution $\sin\vartheta = pv(z)$. Then z must everywhere be replaced by the integral with respect to z.

Snell Waves in Fourier Space

The chain rule for partial differentiation says that

$$\begin{bmatrix} \partial_t \\ \partial_x \\ \partial_z \end{bmatrix} \;=\; \begin{bmatrix} t'_t & x'_t & z'_t \\ t'_x & x'_x & z'_x \\ t'_z & x'_z & z'_z \end{bmatrix} \begin{bmatrix} \partial_{t'} \\ \partial_{x'} \\ \partial_{z'} \end{bmatrix} \qquad \text{(8a,b,c)}$$

In Fourier space, equations (8a) and (8b) may be interpreted as

$$-i\,\omega \;=\; -i\,\omega' \qquad (9a)$$

$$i\,k_x \;=\; +p\,\omega' \;+\; i\,k'_x \qquad (9b)$$

Of particular interest will be the energy which is flat after linear moveout (constant with x'). For such energy $\partial / \partial x' = i\,k'_x = 0$. Combining (9a) and (9b) we get a familiar equation

$$p \;=\; \frac{k}{\omega} \qquad (10)$$

Snell Wave Information in Field Data

The superposition principle establishes the idea of creating an impulse function by a superposition of sinusoids of all frequencies. A three-dimensional generalization of this idea is the creation of a point source by means of superposition of plane waves going in all directions. Likewise, a plane wave can be thought of as a superposition of many Huygens secondary point sources. A Snell wave can be simulated by an appropriate superposition, called slant stack, of conventional exploration data. The simple process of propagation spreads out a point disturbance to where, from a distance, the waves appear to be nearly plane waves or Snell waves. Little bits of real data where the arrivals appear to be planar can be analyzed as if they were Snell waves.

Looking on profiles and gathers for events of some particular stepout p amounts to scanning hyperbolic events trying to pick the places where they are tangent to a straight line of slope p. The search and the analysis will be facilitated if the data is replotted with *linear moveout*. That is, energy located at offset $f = g - s$ and time t in the (f, t)-plane is moved to offset f and time $t' = t - pf$ in the (f, t')-plane. This is depicted in figure 2. The linear moveout converts all events stepping out at a rate p in (f, t)-space to "horizontal" events in (f, t')-space. The presence of horizontal timing lines facilitates search, identification, and measurement of the locations of the events.

Just as waves have a Fourier spectrum, they have a dip (or stepout) spectrum. A single shot point generates Snell waves for each value of p. Filtration methods can be used to eliminate all but a small bandwidth of waves about some particular p. Start with data at the earth surface $z = 0$. Associate x with the geophone axis, say $x = g$. The Snell transformation is $z' = 0$, $x' = 0$ and $t' = t - px$. Converting the data to the Snell frame just amounts to applying linear moveout. The components in the data divergence correction. In other words, slant stacking takes us from two dimensions to one, but a $t^{1/2}$ remains to correct the conical wavefront of three dimensions to the plane wave of two.

We can sum up by saying that slant stack or dip filtering along the geophone axis can be used to extract an up-coming Snell wave component from any upcoming wave field.

Since we can't find an airplane to go fast enough to make a satisfactory down-going Snell wave we can consider simulating one with conventional, point-source data. Basically we must superpose all the shots with a delay phased to match the speed of the desired airplane. Actually we do not have line sources out of the plane of the survey so the wavefronts we would actually simulate would be conical with the apex of the cone at the moving source. The major difference between the two cases is like a cylindrical-divergence amplitude correction. A minor difference predicted by wave theory would be a short wavelet with a little color and phase shift.

Integration over a time shifted shot axis is a form of dip filtering in (s,t)-space. In summary, a *downgoing* Snell wave is achieved by dip filtering in *shot* space whereas an *upcoming* Snell wave is achieved by dip filtering in *geophone* space.

Muting and Data Recording

The basic goal of muting is to remove horizontally moving energy. Such energy is unrelated to the earth image we seek. At the present time most people who record and process data apply a muting function (a weighting function) which zeros data beyond some value (approximately) of $(g-s)/t$. There is no question that this removes *some* horizontally moving energy. But more could be done. Horizontally moving energy can often be found inside the mute zone. The way to get rid of it is with a dip filter instead of a weighting function. This couldn't be done before modern high density recording because slow moving noises were often aliased on the geophone cable. Think of the data in terms of the emergent angle or the Snell parameter dt/dg. If the emergent angle isn't small enough, the waves couldn't have come from the exploration target. We would like to apply such dip filtering in shot space as well as geophone space, but that won't be so easy in practice. Don't fall into the trap of thinking that you should do this dip filtering on a common *mid*point gather. That would not reject backscattered ground roll because it has no moveout.

Marine side-scatter is frequently so strong as to be poorly suppressed by conventional processing. The reason is that out of plane scatterers often give hyperbolic arrivals, which have steep dip, hence have sediment rather than water stacking velocities. What is needed is two dip filters. One to reject waves leaving the shots at non-penetrating angles, and the other to reject waves arriving at the geophones at non-penetrating angles. Present day field arrays filter on the basis of spatial frequency. We would be left with more high frequency energy in our data if we would build recording equipment which used dip filters instead of spatial frequency filters. The causal dip filters described in an earlier chapter might work nicely.

Synthesizing the Snell Wave Experiment

Let us synthesize a downgoing Snell wave with field data, then imagine how the up-coming wave will look and how it carries us information about the subsurface.

Slant stack will take survey line data $P(s,g,t)$, which is a function of shot location s, geophone location g, and traveltime t, and sum over the shot dimension synthesizing the upcoming wave $U(g,t)$ which should have been recorded from a downgoing Snell wave. It is remarkable that this is the case even though there may be lateral velocity variation and multiple reflections. The summation process is quite confusing because there are three different kinds of time:

t = traveltime in the point source field experiments.

t' = $t - p(g-s)$ is interpretation time. The shallowest reflectors are seen just after $t' = 0$.

t_{pseudo} = time in the Snell pseudo-experiment of a moving source.

Time in the pseudo-experiment in a horizontally layered earth has the peculiar characteristic that the further you move out the geophone axis, the later the echoes will arrive. So we transform directly from the field experiment time t to interpretation time t' by

$$t' \;=\; t_{pseudo} - p\,x \;=\; t - p(g-s)$$

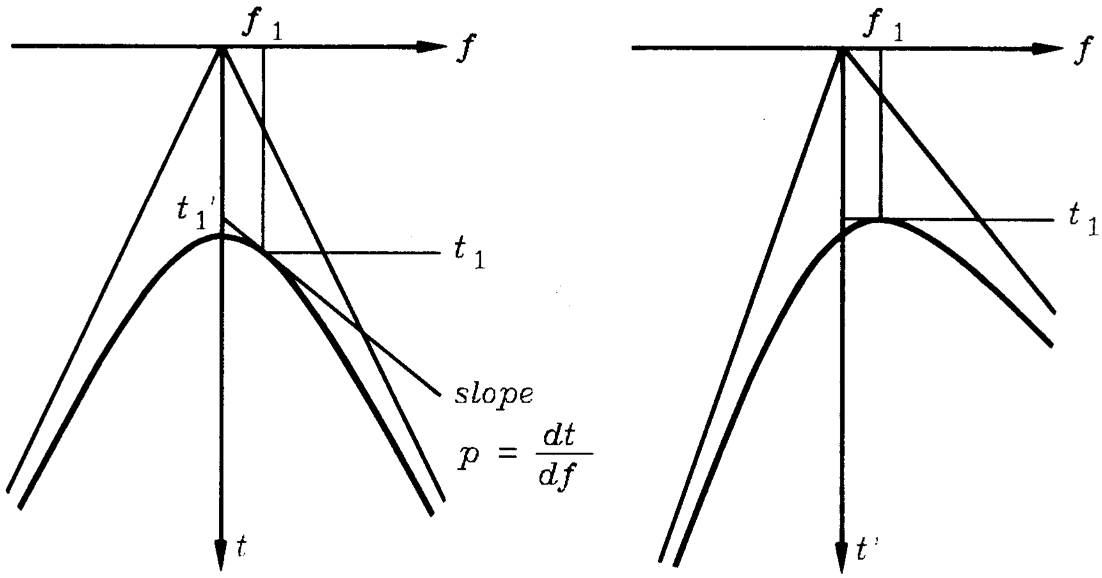

FIG. 2. (Gonzalez) Linear moveout converts the task of identifying tangencies to constructed parallel lines, to the task of locating tops of convex events.

which have Snell parameter near p are now slowly variable along the x'-axis. To extract them you apply a low-pass filter on the x'-axis, and do so for each value of t'.

The procedure of slant stacking is first to do linear moveout with $t' = t - px$, then to sum over x'. In other words, you can slant stack in either of two ways: 1) sum along slanted lines in (t,x)-space; or 2) do linear moveout $t' = t - px$ and then sum over x' at constant t'. In either case, the entire gather $P(x,t)$ gets converted to a single trace which is a function of t'. Let us think about what this trace actually is. We will assume that the sum over observed offsets is an adequate representation of integration over all offsets. The (slanted) integral over offset will receive its major contribution from where the path of integration becomes *tangent* to the hyperboloidal arrivals. On the other hand, if rays carry a wavelet with no zero-frequency component, and if the arrival time curve crosses the integration curve at any fixed angle, then the contribution to the integral vanishes.

The strength of an arrival depends on the length of the zone of tangency. The *Fresnel* definition of the length of the zone of tangency is based on a half-wavelength condition. In an earth of constant velocity (but many flat layers) the width of the tangency zone would broaden with time as the hyperbolas flatten. This increase goes as $t^{1/2}$, which accounts for half the spherical-

Figure 3 depicts a downgoing Snell wave.

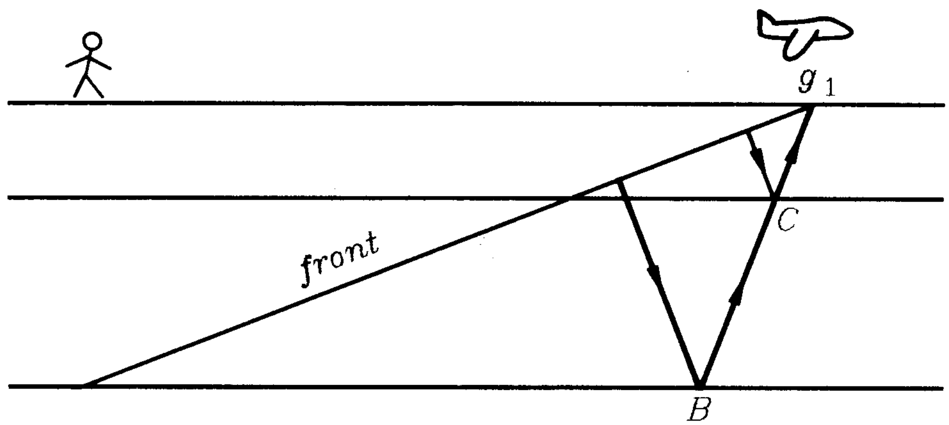

FIG. 3. Wavefront of a Snell wave which reflects from two layers carrying information back up to g_1

Figure 4 shows a hypothetical common geophone gather which could be summed to simulate the Snell wave seen at location g_1 in figure 3. The lateral offset of B from C is identical in figure 3 with both places in figure 4, Repeating the summation for all geophones we get the synthesized up-coming wave from a down-going Snell wave.

The variable t' may be referred to as an interpretation coordinate because shallow reflectors are seen just after $t' = 0$, and horizontal beds give echoes which arrive without horizontal stepout, unlike the pseudo-Snell wave. For horizontal beds there is no detection of lateral location, unless we allow an abrupt lateral change in reflection coefficient. On figure 3 the information about the reflection strength at B is actually recorded rightward at C instead of being seen at A where it would be on conventional stack. Moving received data to an appropriate lateral location is thus an additional requirement for full interpretation.

Figure 5 shows the same two flat layers as figures 3 and 4, but additionally there are anomalous reflection coefficients at the points A, B, and C. Point A is directly above B. The path of the wave reflected at B leads directly to C and thence to g_1. Subsequent frames show the diffraction hyperbolas associated with these three points. Notice that the pseudo-Snell waves reflecting from the flat layers step out at a rate p. Hyperbolas from the scatters A, B, and C come tangent to the Snell waves at points a, b, and c. Notice that b and c lie directly under g_1 because

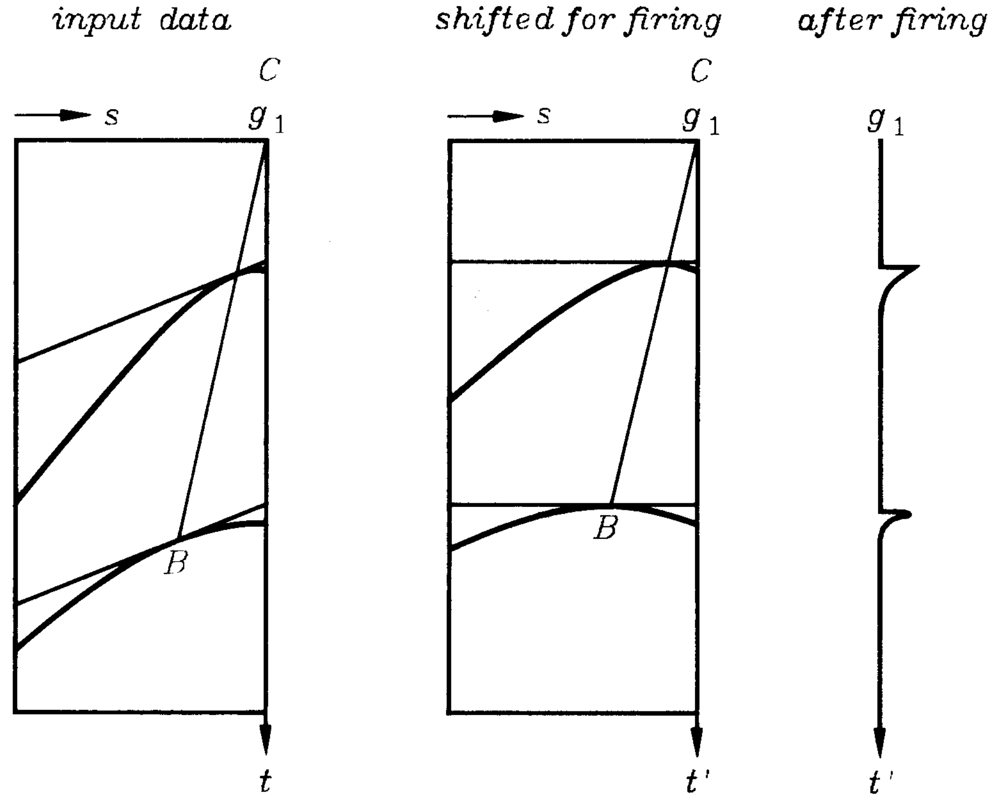

FIG. 4. (Gonzalez) Left shows a common geophone gather at g_1 over two flat reflectors. Center shows the data shifted by linear moveout in preparation for generation of the synthetic Snell wave by summation over shots. Right shows the Snell wave trace recorded at geophone g_1. A Snell Wave seismic section consists of many side-by-side traces like g_1.

all are aligned by a ray path with Snell parameter p. The points A, B, and C locate the tops of the hyperbolas since the earliest arrival must be directly above the point scatterer, no matter what the incident wave field. Converting to the interpretation coordinate t' in the next frame offers the major advantage that arrivals from horizontal layers become horizontal. But the hyperboloids have become skewed. Limiting our attention to the arrivals with little stepout we see that our information about the anomalous reflection coefficients is found entirely in the vicinities of a, b, and c, which originally lay on hyperbola flanks. These points do not have the correct geometrical location, namely that of A, B and C, until we laterally shift information to the left, say $g' = g - f(t')$. Then a lies above b. The correct amount of shift $f(t')$ is a subject relating to to velocity analysis. The next section

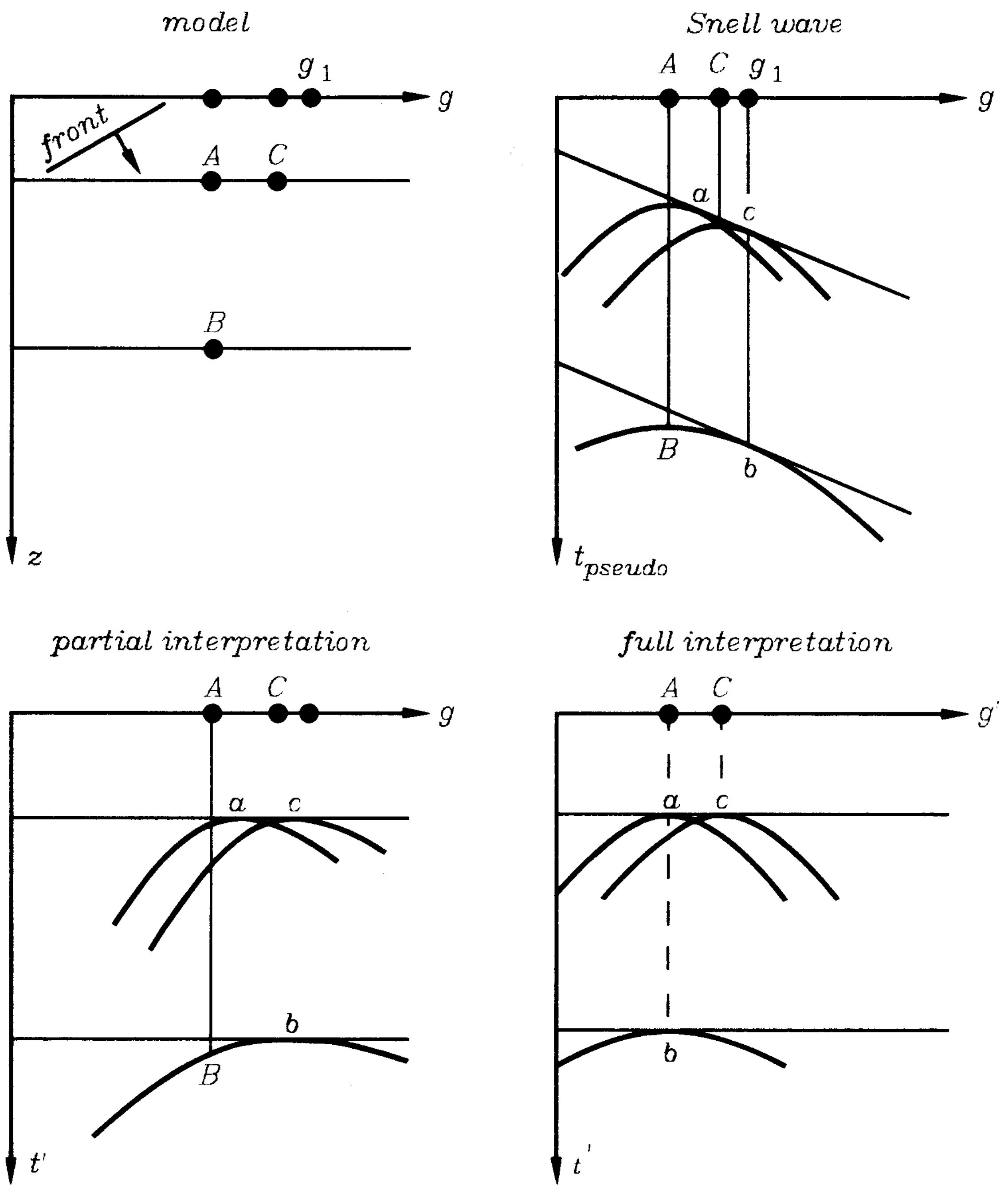

FIG. 5. (Gonzalez) Top left is three point scatterers on two reflectors. Top right is the expected Snell wave. Bottom left is the Snell wave after linear moveout. Bottom right is after transform to full interpretation coordinates. At last a, b, and c are located where A, B, and C began.

works out the velocity analysis.

EXERCISES

1. Equation (5) is for *up-going* Snell waves. What coordinate system would be appropriate for *down-going* Snell waves?