The Separation of Events on Vertical Seismic Profiles

Ronald Ullmann

Introduction

The art of seismic interpretation involves identifying patterns on a seismic section and matching them to the subsurface geometry that produced the pattern. For example, a long, horizontal line on a seismic section obviously corresponds to a flat reflector in the earth. An upside-down hyperbola indicates a point diffractor in the earth. A simple fault shows up as a long, horizontal line with a break in it. The correspondence between the pattern on the seismic section and the subsurface geometry is usually one-to-one. Similarly, a point diffractor will always produce a hyperbola and nothing else. An anticline may produce a hyperbola-shaped pattern, but there are other features of an anticline pattern that distinguish it from a point diffractor. This one-to-one correspondence allows an interpreter to be confident about the interpretation. However, this one-to-one correspondence does not apply to vertical seismic profiles (VSP's) in the same way as it does to seismic sections. This is due in part to the limited lateral coverage of a VSP. By increasing the number of shot offsets, a better one-to-one correspondence is made.

In this article, I will examine the VSP response to various simple geometries within the earth. I will also investigate several simple methods of identifying the different patterns and separating these patterns. In this article, I will refer to lines as either upgoing or downgoing. This represents the slope of the line on a VSP. If a line is downgoing, it has the same slope as the first arrival event. An upgoing line has the slope of an upgoing reflection from a flat reflector.

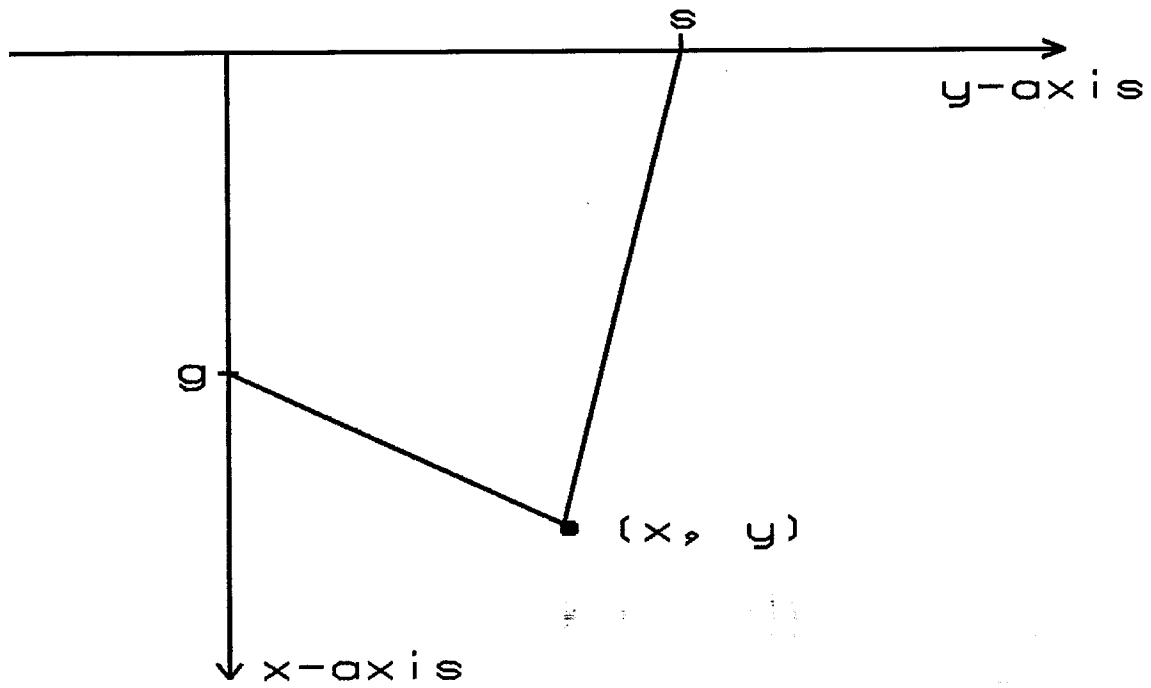

FIG. 1. Model of the earth. A point diffractor is located at the coordinates (x,y). The shot is at s and the geophone is at g. The earth has a constant velocity v.

Model

In order to simplify this discussion, the model in figure 1 assumes that the earth has a constant velocity. Figure 1 shows the basic coordinate axis. I have defined the axis so that the shot is on the y axis and the geophone is on the x axis. The shot can be located anywhere on the y axis, the geophone can only be on the non-negative x axis, and any subsurface feature can only be on the positive side of the y axis. The traveltime of a wave that leaves the shot, reflects off the point diffractor, and hits the geophone is given by

$$t = \frac{1}{v}\sqrt{x^2 + (s - y)^2} + \frac{1}{v}\sqrt{(g - x)^2 + y^2} \quad . \tag{1}$$

The traveltime curve given by equation (1) is a hyperbola in a geophone - time plane with a peak at $x = g$. Writing equation (1) in the standard formula of a hyperbola, it becomes

$$\frac{(t\ v - t_s\ v)^2}{y^2} - \frac{(g - x)^2}{y^2} = 1 \tag{2a}$$

where t_s is the traveltime between the shot and the diffractor and is given by

$$t_s = \frac{1}{v}\sqrt{x^2 + (s - y)^2} \ .$$

Equation (1) also forms a hyperbola in a shot - time plane with a peak at $y = s$. The equation for the shot - time hyperbola is

$$\frac{(t\ v - t_g\ v)^2}{x^2} - \frac{(s - y)^2}{x^2} = 1 \qquad \text{(2b)}$$

where t_g is given by

$$t_g = \frac{1}{v}\sqrt{(g - x)^2 + y^2} \ .$$

Equation (1) becomes an ellipse in shot - geophone planes at constant traveltimes. The foci of this ellipse are at the geophone and at the shot. Equation (1) can be rewritten to get the equation for this ellipse:

$$\frac{\left[x\ g - y\ s + \frac{1}{2}(s^2 - g^2)\right]^2}{v^2\ t^2} + \frac{(x\ s - y\ g)^2}{v^2\ t^2 - (g^2 + s^2)} = \frac{s^2 + g^2}{4} \ . \qquad \text{(2c)}$$

The above equations describe a hyperboloid of two sheets in time, geophone, and shot space. Plane cuts parallel to geophone and shot axis are ellipses, and plane cuts parallel to the time axis are hyperbolas.

Figure 2 shows a rough drawing of how a hyperboloid of two sheets appears. A good way to visualize this figure is to imagine two bowls that are centered on time axis and opened away from each other. The vertices of this shape occur above $y = s$ and $x = g$. The asymptotic shape for this figure is a double cone with its apex on the shot - geophone plane at (x,y). One cone opens upward and the other cone opens downward. The slope of the side of the cone is equal to the reciprocal of the velocity. The bottom hyperboloid will not be considered since it is in the negative time space.

The surface of the cone represents the first arrival of the wave from the shot to the geophone. Therefore, the hyperbolic response of a point diffractor in geophone - time space approaches the traveltime of the direct arrival as the geophone goes deeper. Figure 3 shows the traveltimes of a VSP when a point diffractor is present. The traveltime curve corresponding to the point diffractor appears to approach the direct arrival curve on the downgoing side. However, there is a minimum difference between the two curves. This difference can be derived from equations (1) and the traveltime equation for the direct arrival:

$$t = \frac{1}{v}\sqrt{s^2 + g^2} \ . \qquad \text{(3)}$$

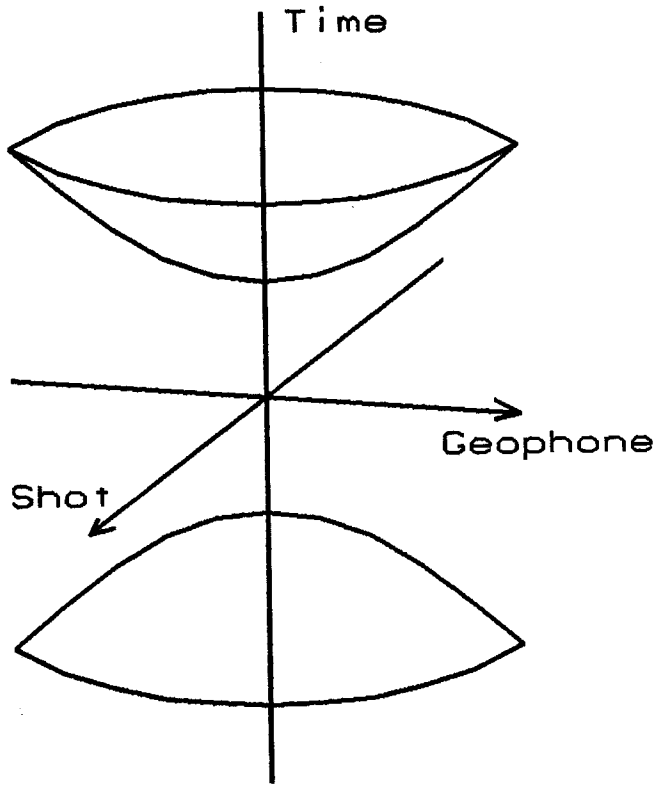

FIG. 2. A hyperboloid of two sheets. This is the shape of the response of point diffractors in time - geophone - shot space.

Equation (3) is a hyperbola with the asymptotic equations

$$t = \pm \frac{g}{v}. \tag{4}$$

The asymptotic equations for equation (1) in geophone - time space is

$$t = t_s - \frac{x}{v} + \frac{g}{v} \tag{5a}$$

$$t = t_s + \frac{x}{v} - \frac{g}{v}. \tag{5b}$$

Equation (5a) is the asymptote that appears as a downgoing wave. Equation (5b) is the asymptote that appears as a upgoing wave. The time difference between the two equations is $t_s - \frac{x}{v}$. Therefore, the ability to separate the diffracting pattern from the first arrival is dependent on the traveltime of a wave from the shot to the diffractor minus the traveltime from the surface to the diffractor. This means that if the shot is right above the diffractor,

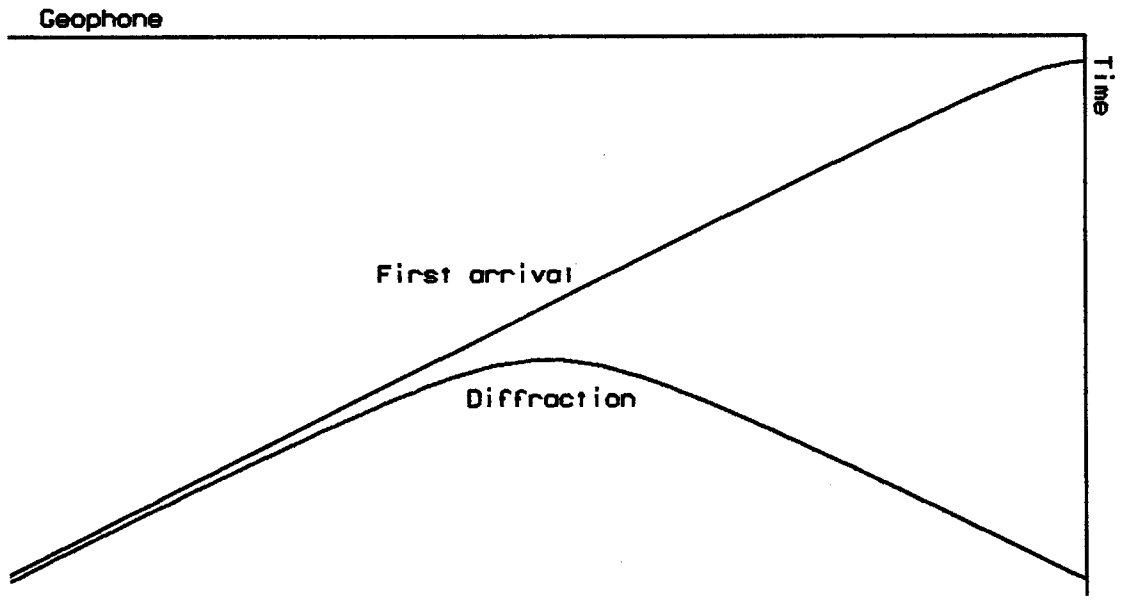

FIG. 3. Traveltime curves for a point diffractor and the first arrival. The geophone depth increases to the left and time increases downward. This is the standard set of axis for a VSP.

then the downgoing limb of the diffractor's response will asymptotically approach the first arrival curve. If there is a large horizontal offset between the shot and the diffractor, than the two curves will remain separate. Figure 4 shows the response of two diffractors. The first diffractor is almost under the shot while the second diffractor is further away. The difference between the two can be seen in figure 4.

The upgoing limb of the diffractor's hyperbola appears as an upgoing event on a VSP. It also appears to be the response of a flat reflector, which is given by

$$t = \frac{1}{v}\sqrt{s^2 + (2\,d - g)^2} \tag{6}$$

where d is the depth of the reflector. The upgoing asymptote of a flat reflector is

$$t = \frac{2\,d}{v} - \frac{g}{v}. \tag{7}$$

The slope of equations (5b) and (7) is $\frac{1}{v}$. Therefore, if

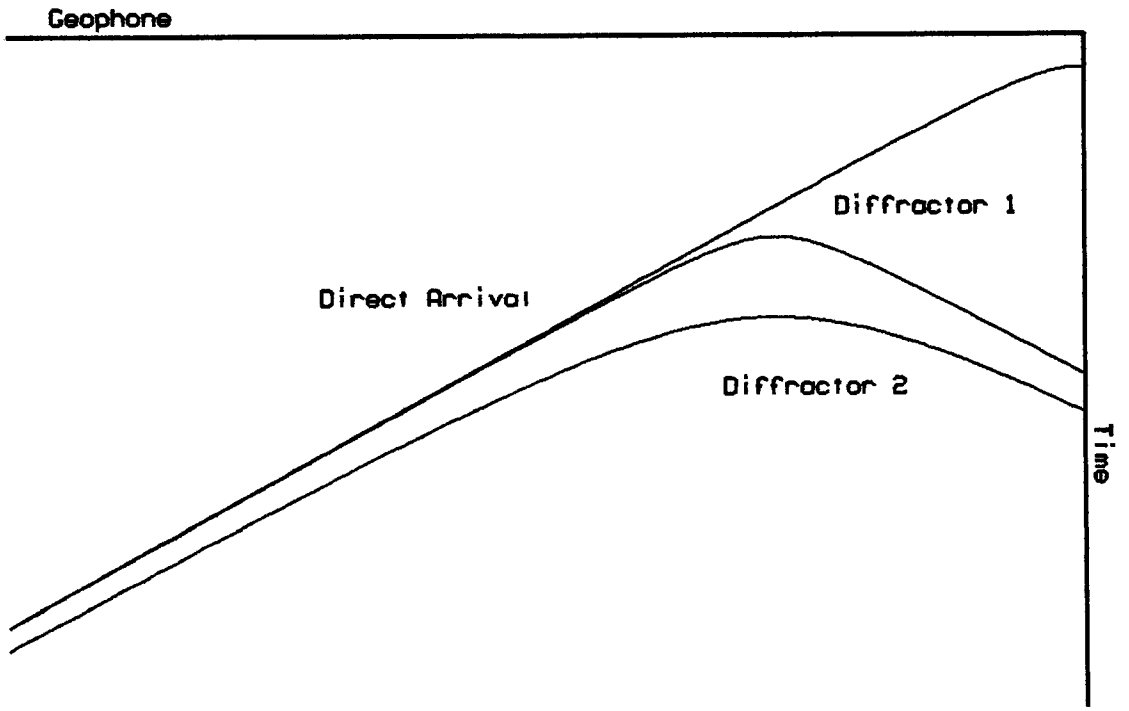

FIG. 4. The traveltime curves for two different diffractors. The first diffractor is directly under the shot, and the second diffractor is further away horizontally from the shot.

$$\frac{2\,d}{v} = t_s + \frac{x}{v},$$

then the diffractor's response will be indistinguishable from a flat layered response when the geophone is close to the surface.

The only place on a VSP where the diffractor's response will stand out is when the geophone is near the depth of the diffractor. This geophone depth corresponds to the area about the vertex of the hyperbola, which occurs as $x = g$. If the vertex is too close to the direct arrival, the diffractor's response will appear to be the response of a flat layer. The response of a flat reflector has its vertex at $g = 2d$, which places the vertex before the direct arrival on a VSP. This is similar to having a downgoing wave picking up a reflection as it passes down through the reflector to the geophone. For this reason, the traveltime curve for a flat reflector is not defined for $g > d$. The upgoing branch of the reflector's hyperbola will show very little curvature when it intersects the direct arrival. The diffractor's response will have more curvature. This difference is the best clue to the subsurface geometry which caused the pattern.

Lateral Coverage

Suppose there is some flat reflector at a depth d. Any reflections from a shot to a geophone reflects off the reflector in a region between the well and half the distance between the shot and the well. Figure 5 shows the geometry of this situation. As the geophone moves up from the layer, the concentration of reflections near $\frac{s}{2}$ becomes greater. If the shot offset is small, this concentration becomes greater. At a high density of rays, the reflector can almost be treated as a point diffractor. For this reason, the upgoing response of both geometries is similar. A diffractor that is directly under the shot and at the same level as the reflector will have identical asymptotes to the reflector's response. A multiple-offset VSP is needed to pick out these type of events. For example, a point diffractor in the form of a fault could exist near the well. If the shot were over the edge of the fault, then the edge would not be very visible.

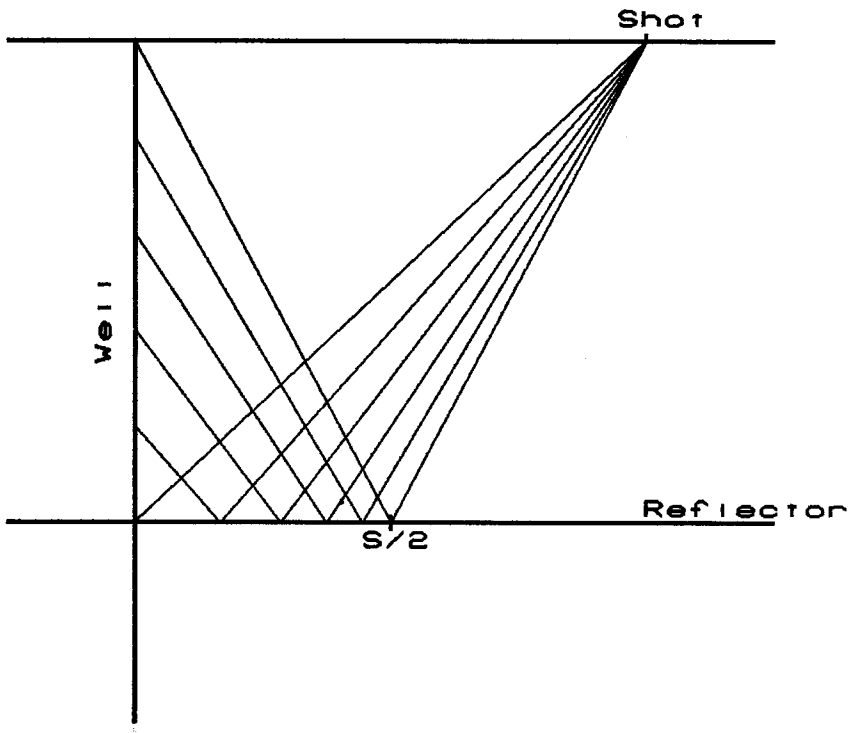

FIG. 5. A model of the earth showing the raypaths of a wave that reflect off a flat reflector. Notice the increasing density of raypaths toward the point $\frac{s}{2}$.

Viewing the Data in Three Dimensions

In order to look at a VSP data set in three dimensions, there must be multiple shot and geophone positions. The problem in this situation is how to determine whether or not a pattern was caused by a point diffractor. Before examining the data in different dimensions, it is helpful to have a visual idea of the main events. Each type of response in a VSP can be pictured as a different bowl that opens upward along the positive time axis in shot - geophone - time space. The largest bowl belongs to the direct arrival and is centered at the origin. The diffraction response forms a smaller bowl which is completely within the first arrival bowl. The diffraction bowl is not centered about the origin and is raised above the shot - geophone plane by the time

$$t = \frac{1}{v}(x + y) \ .$$

The reflected response forms a bowl that has its vertex outside the first arrival bowl on the geophone axis at $g = 2d$. It intersects the first arrival bowl along a hyperbola in shot - time space, which has the equation:

$$t = \frac{1}{v}\sqrt{s^2 + d^2} \ . \tag{8}$$

Each diffractor and flat reflector forms a bowl, which appears on VSP's as events.

One method of finding point diffractor patterns is to examine geophone - time sections for curvature of upgoing waves near the first arrival. The user can put a series of sections together in a movie and watch each event as it changes. For increasing shot offsets, the first arrival event will start to flatten and move down the time axis. The response of the flat reflector will also move down the time axis. If the point diffractor is on the same side of the well as the shot is, its response will move up the time axis until the shot is above the diffractor. As the shot moves outward, the diffractor response will move down with the rest of the events. If the point diffractor is on the opposite side of the well, then its response goes down without going up. Therefore, it is important to shoot on both sides of the well in order to receive more helpful information. The upward motion of the diffractor's response should stand out if all other events are moving down.

The VSP data can be arranged in shot - time sections. The main events are still hyperbolas. Figure 6 shows what the three main events look like in a shot - time plane. The sections can be placed in a movie so that each frame corresponds to a different geophone depth. As the geophone goes down, the first arrival hyperbola moves down. The responses of the diffractor and reflector move up until the geophone is at the same depth as the diffractor or reflector. Then the responses start to move down. This method of examining the

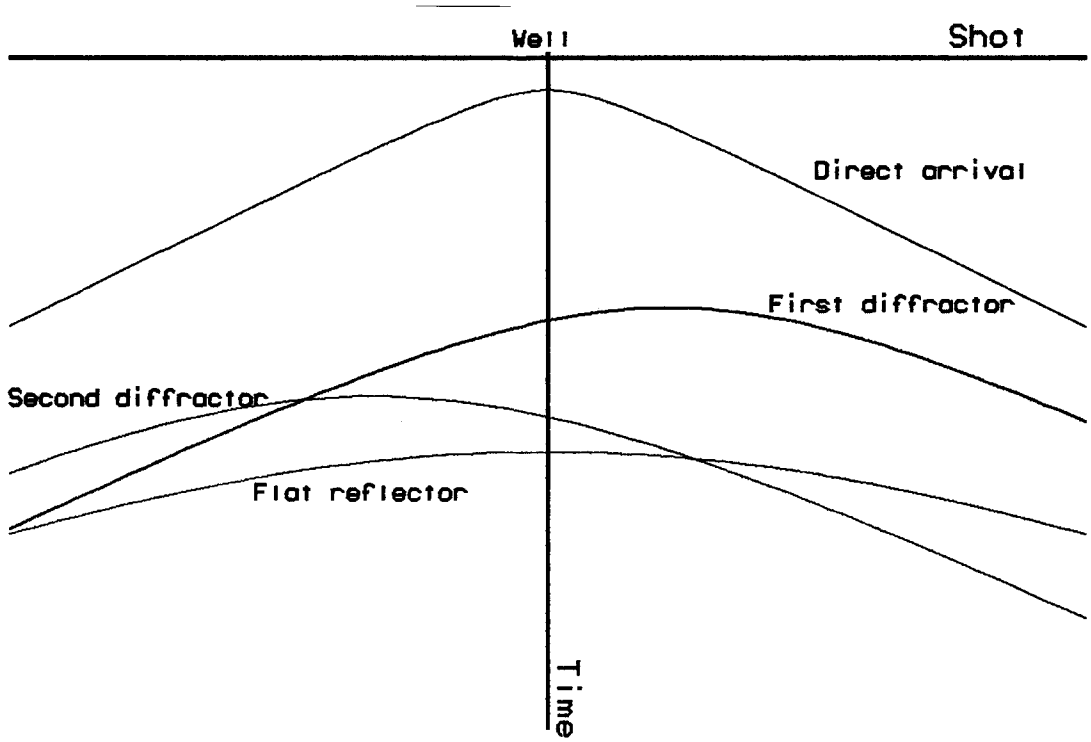

FIG. 6. A shot - time section of a VSP. This draw assumes that the shot was used on both sides of the well. The two diffractors are on opposite sides of the well.

data does not yield a reliable method of spotting diffractors based on the motion of the events. However, the responses of the diffractors are not symmetric with respect to the well, while the reflector's response is symmetric.

The next method involves taking sections parallel to the geophone and shot axis. All of the events appear as ellipses that grow larger as time increases. Figure 7 shows an example of how this type of section may appear. The direct arrival appears as a large semi-circle which is centered about the origin. The diffractor's response appears as an ellipse that is centered at the coordinates of the point diffractor, (x,y). The reflector's response is a semi-circle that is centered outside the circle of the first arrival. At $t = 0$, the first arrival is the only event visible near the origin. As time increases, the first arrival's circle grows larger. As the sections cross the other events, these later events appear on the section and grow larger. The motion of the events is in different directions. The first arrival's response grows away from the origin, the reflector's response grows toward the origin, and the diffractor's response grows in both directions.

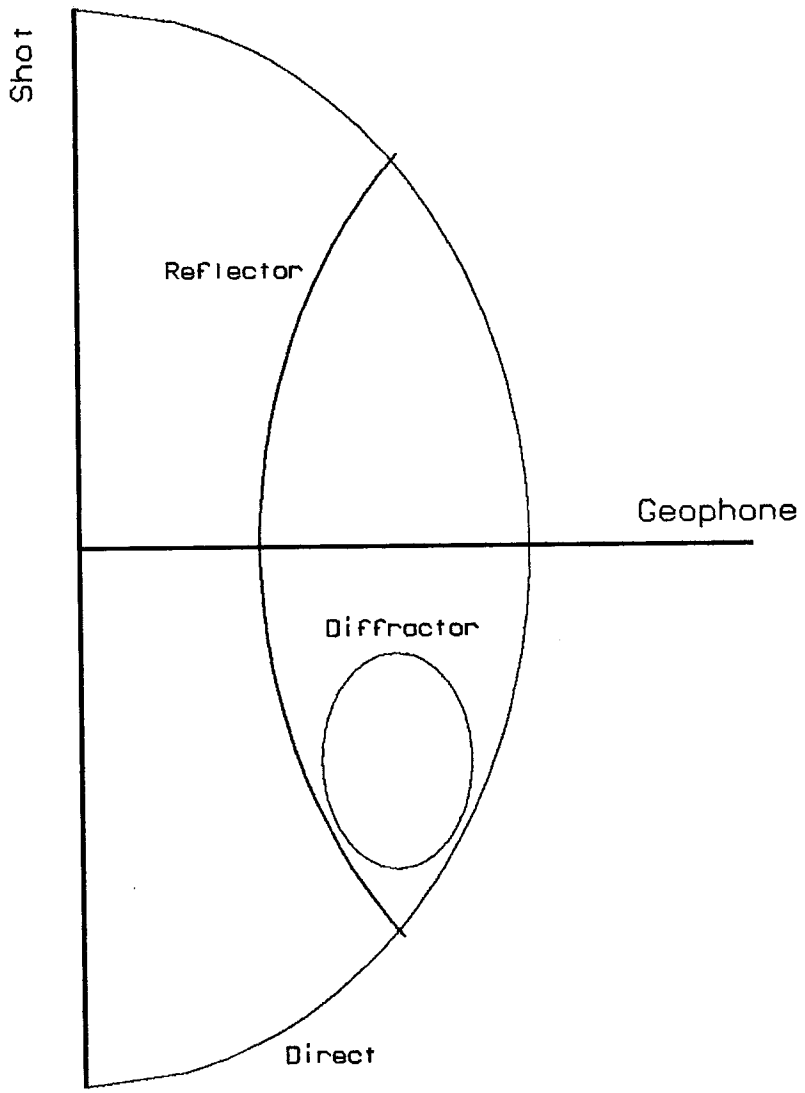

FIG. 7. A shot - geophone section of a VSP. The time axis goes into the paper. This graph shows the geophone and shot positions that would produce a certain traveltime.

The response of multiples appears as smaller bowls which are within the bowls of the source of the multiple. For example, the first order multiple that corresponds to a wave reflects back from the surface appears as a bowl which is within the first arrival's bowl. Since it is centered at the origin, it is roughly parallel to the first arrival. If a section with multiples present is displayed in a shot - geophone movie, the multiples would form a bulls-eye formation. On others type of movies, the multiples have the same motion as the event that caused them. A multiple from a reflector moves up and down with the reflector. The

movie on the SEP-3 videotape includes a few multiples among the primary events. The motion of the multiples can be seen to follow the primary events in all of the different planes.

Conclusions

The geometry of a VSP in shot - geophone - time space is simple when the model is simple also. There is an abundance of useful information available when multiple shot offsets are used. The abundance increases the interpretator's chances of identifying significant patterns on a VSP. While I was making the figures and the movie for this article, I assumed that the shots are as closely spaced as the geophones. Unfortunately, using such a short shot spacing increases the time of making a VSP, and thus makes the VSP more expensive. However, adequate information about the earth is still obtainable from a few shot offsets on both sides of the well. With a some extra offsets, it becomes feasible to image the geology around the well. In order to continue my research, I am interested in obtaining a VSP data set that has multiple shot offsets. I propose to use this data in developing techniques for imaging the area around the well.

There are several methods of examining multiple offset VSP data. Two of the easier methods were covered in this article. The movie approach described above is one available method. Each of the main events has a characteristic motion in three dimensions that appears in a movie. With some practice, an interpretator can tell the difference between a diffractor and a reflector by observing its movement in a movie. A second method is to examine VSP sections taken along different planes. This method involves more skill because the interpretor can examine only one section at a time. Each event produces its own pattern which eventually can be recognized, provided the patterns are separated enough to allow recognition.

The next project is to consider the responses of a realistic model of a layered earth. There were many complications left out of this article, such as the presence of s-waves and sloping reflectors. There is also the possibility of developing a mathematical operation which takes diffractions and reflections and migrates them back to the proper place in space. Due to the simple nature of these methods, it is not yet possible to take into account all the complications of a real earth. With further work, we will develop better operations which help separate the events on vertical seismic profiles.