The Computation of Synthetic Vertical Seismic Profiles

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Introduction

The vertical seismic profile (VSP) has become an effective tool in geophysical work. It permits the direct measurement of seismic velocities, wave attenuation, reflection coefficients, study of seismic multiples, and the determination of geology at an increased range from the borehole. Many papers have been published on VSP's as theory has developed, but little attention has been given to this subject by the Stanford Exploration Project. For this reason, I have chosen to study VSP's.

The proper tools are necessary for the study of a new subject. One of the most useful tools in VSP work is a set of programs that generates synthetic vertical seismic profiles (SVSP's). With these programs, one may test different processing methods on a VSP without any noise. For example, a velocity filter can be tested on a SVSP first to insure that the filter works. If the filter worked on a SVSP but not a VSP, then the filter was not designed well enough to handle noise. I have developed a series of programs for constructing SVSP's for the use of testing new programs.

Basic Equations

There have been several papers devoted to generating synthetic seismic profiles (see Claerbout, 1976; and Robinson and Treitel, 1980). This basic model assumes that the seismic wave is a normal incidence plane wave striking flat geological reflectors. A second assumption about the model is that the reflection and transmission amplitudes of the seismic wave at a reflector is governed by the equations:

$$R_u = r \, I_d \tag{1a}$$

$$T_b = t \, I_d . \tag{1b}$$

In equations (1a) and (1b), I_d is the amplitude of a normal incident wave. In equation (1a), R_u is the amplitude of the reflected wave and r is the reflection coefficient of the interface. In equation (1b), T_b is the amplitude of the transmitted wave and t is the transmission coefficient of the interface. The coefficients are derived from the formulas:

$$r = \frac{\rho_t v_t - \rho_i v_i}{\rho_i v_i + \rho_t v_t}, \tag{2a}$$

$$t = \frac{2 \rho_t v_t}{\rho_i v_i + \rho_t v_t}. \tag{2b}$$

In the above equations, ρ is the density, v is the velocity, and the subscripts i and t refer to layers in which the wave is incident and transmitted respectively. Attenuation within the layer is assumed to be zero so that there is no loss of energy as the wave travels through a layer. A close approximation of the actual seismic response of a flat layered earth can be calculated with these assumptions.

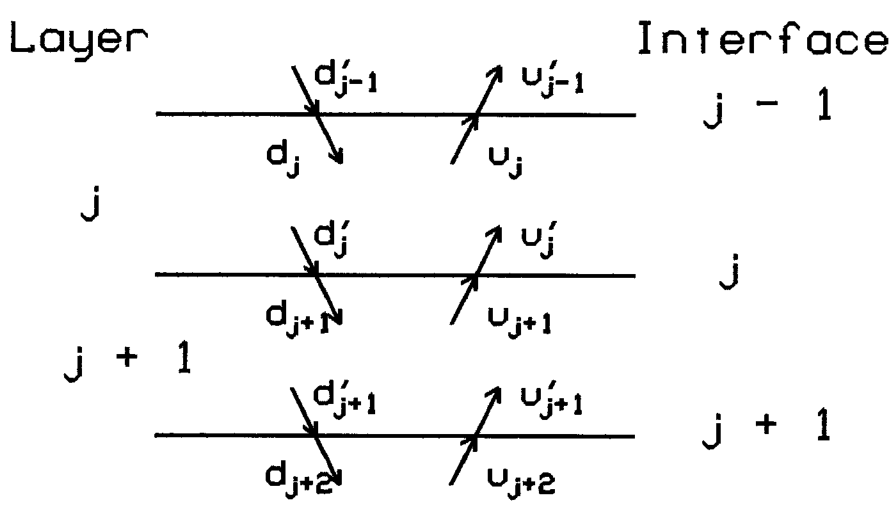

FIG. 1. Model of a layered earth for generating SVSP's.

The standard method for calculating normal seismic profiles finds the response of a flat layered earth through one equation:

$$U'_0(z) = -z^k \frac{q_k(z)}{p_k(z^{-1})} D'_0(z). \tag{3}$$

In equation (3), $U'_0(z)$ is the response of the earth at the surface of the earth and $D'_0(z)$ is the input signal that is goes into the earth. $q_k(z)$ and $p_k(z)$ are polynomials of the unit delay operator z, and are derived from the reflection coefficients in the model. k is the number of interfaces in the model. Equation (3) works well for the total response of the earth at the surface. However, this approach does not allow one to calculate intermediate values at different levels of the earth. I have developed an approach to correct this situation.

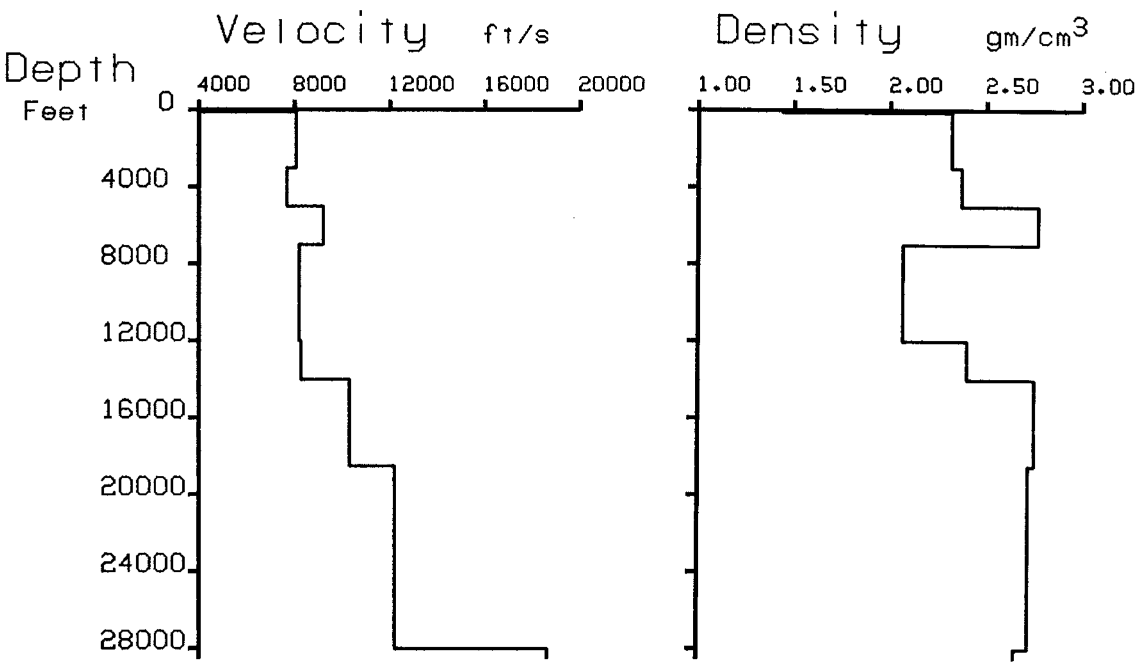

FIG. 2. A simplified version of the earth that was used for the construction of SVSP's for this article.

The basic equations which describe the response of a series of seismic interfaces are derived from the model of the earth shown in figure 1. This model is based on the assumptions that the traveltime from the top of a layer to the bottom is unity and that it is represented by the delay operator z. Suppose that a geophone had been lowered down to the level of interface j and waves are propagating up and down, then the amplitudes of downgoing waves are represented by d, and the amplitudes of upcoming waves are u. The amplitude that the geophone records, a_j, at the j interface is

$$a_j(z) \;=\; d'_j(z) + u_{j+1}(z). \tag{4}$$

The equations for d'_j and u_{j+1} are

$$d'_j(z) \;=\; z\,(1 + c_{j-1})\, d'_{j-1}(z) - z\; c_{j-1}\; u_j(z) \tag{5a}$$

$$u_{j+1}(z) \;=\; z\,(1 - c_{j+1})\, u_{j+2}(z) + z\; c_{j+1}\; d'_{j+1}(z). \tag{5b}$$

When equations (5a) and (5b) are combined, the amplitude observed by the geophone at each interface is

$$a_j(z) \;=\; (1 + c_{j-1})\, d_{j-1}(z) - c_{j-1}\, u_j(z) + (1 - c_{j+1})\, u_{j+2}(z) + c_{j+1}\, d'_{j+1}(z). \tag{6}$$

Equations (5a) and (5b) may be used to calculate the up-going and down-going amplitudes for each layer. At the surface of the earth, equation (5a) becomes

$$d'_0 \;=\; s(z) \tag{7}$$

where $s(z)$ is the source function of the seismic source. If there are n interfaces, then equation (5b) becomes

$$u_n(z) \;=\; z\; c_n\; d'_n(z). \tag{8}$$

The above equation assumes that there is no seismic energy entering the model from the bottom. Before the source waveform enters the model, the amplitudes at all the interfaces are set to zero and represents the system at rest. For each time unit, the amplitudes at each layer are calculated from the top of the model to the bottom. The amplitudes at each interface are then summed to produce an amplitude for each level. This model produces a seismic section which is a transpose of field sections recorded by standard techniques. A full time trace is recorded in the field at each level before the geophone is moved. For this model, every level records an amplitude at one time where a geophone is located. This process is repeated for each time unit.

The ratfor subroutine *vertpr*, which is listed in the Appendix, performs this algorithm. The source signal as a function of time is contained in the array *signal* and is *ninput* time units long. The variable *ngeop* is the number of geophones, and *dg* is the vertical spacing between geophones. *refco* is an array containing the reflection coefficients for the *nlay* interfaces where *dx* is the thickness of each layer. *ntime* is the length of output signal that the subroutine produces. *outsig* is an output subroutine that is dependent of the computer system. Figure 2 shows the structure of the earth that was used in this article for the construction of the SVSP's. Figure 3 shows an example of the transposed output of this subroutine.

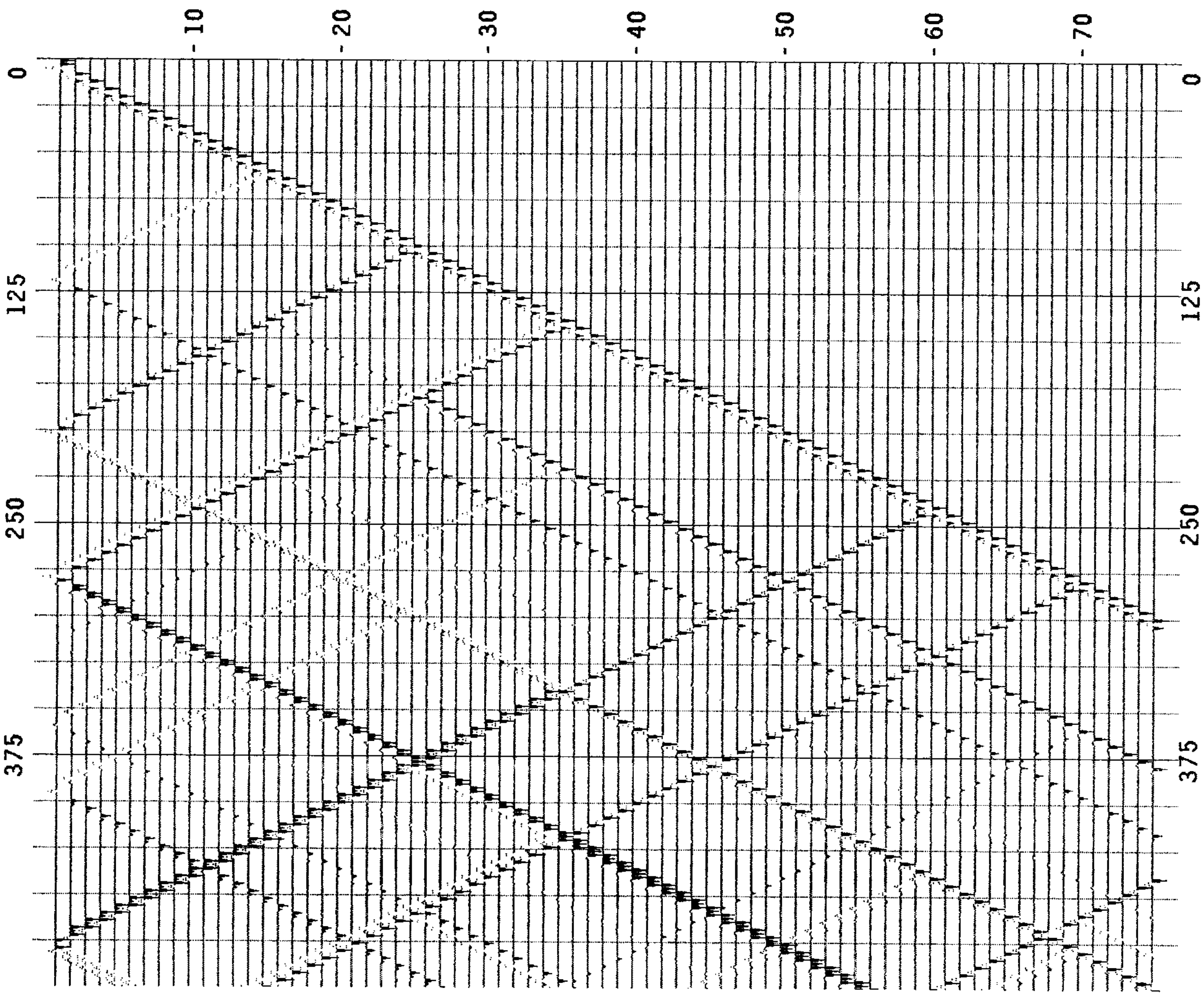

FIG. 3. Synthetic vertical seismic profile of the model in figure 2. This SVSP was calculated at constant time increments.

Thus far, the basic problem with this model is that it assumes the geophone is placed at intervals corresponding to constant traveltime. In the field, the geophone is placed at constant intervals of vertical distance. To solve this problem, it is necessary to plot the trace that is closest to the actual position of the geophone. The plot in figure 4 shows a SVSP which was made with this technique.

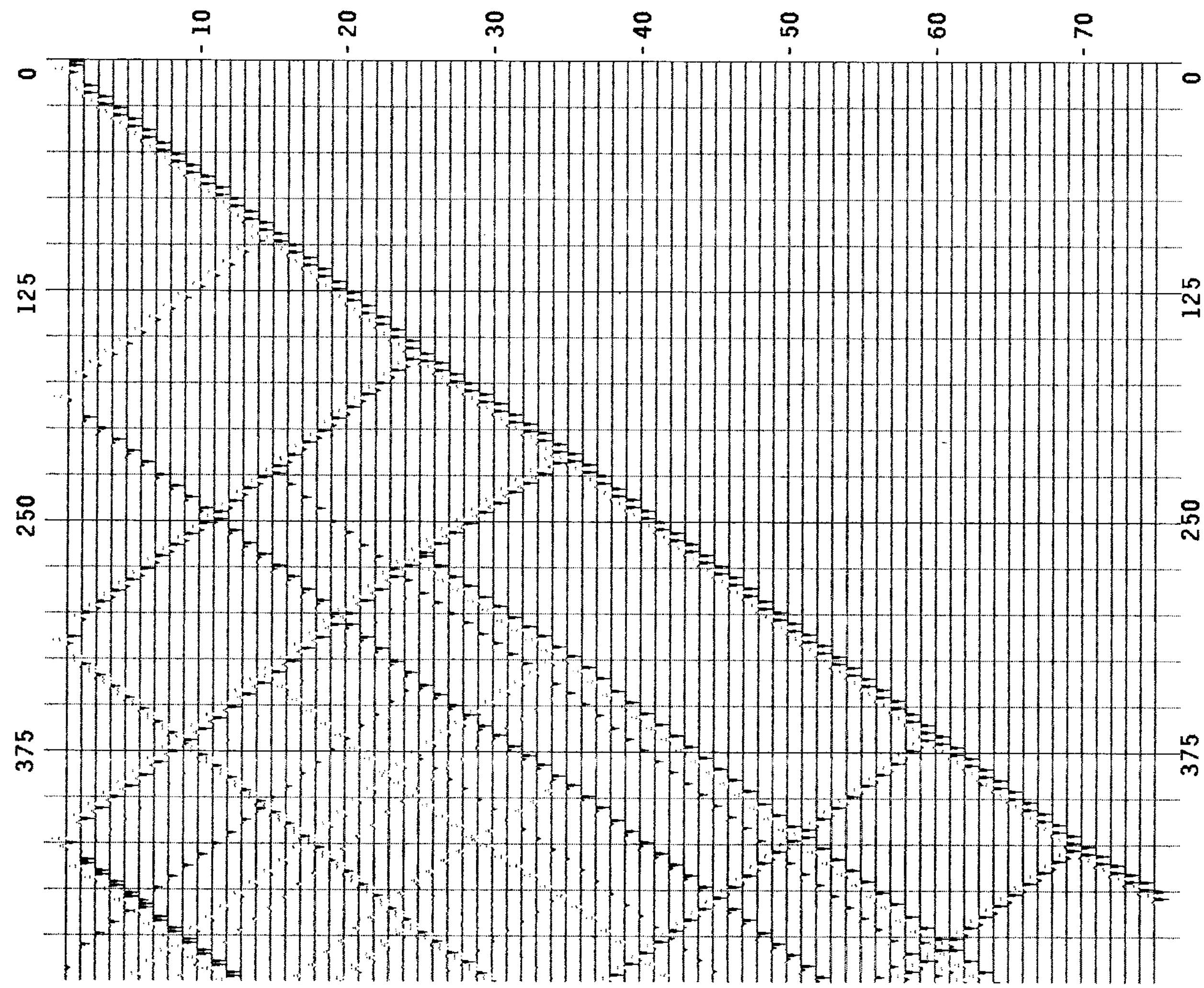

FIG. 4. Synthetic vertical seismic profile after adjusting traces for the constant distances between geophone locations.

Profiles Without Multiples

Occasionally it is desirable to study a SVSP without having to deal with multiples of downgoing waves in the profile. For the purposes of this article, a multiple is defined to be an upcoming wave which is reflected downward from any interface. On a VSP, a multiple appears as a downgoing event which parallels the first arrival event. On actual VSP data, multiples are so strong that they tend to obscure the upcoming events. The downgoing events can be filtered out on a VSP by some form of dip filter. The multiples could be filtered on a SVSP, but it would be easier not to calculate the multiples in the first place. This can be easily done.

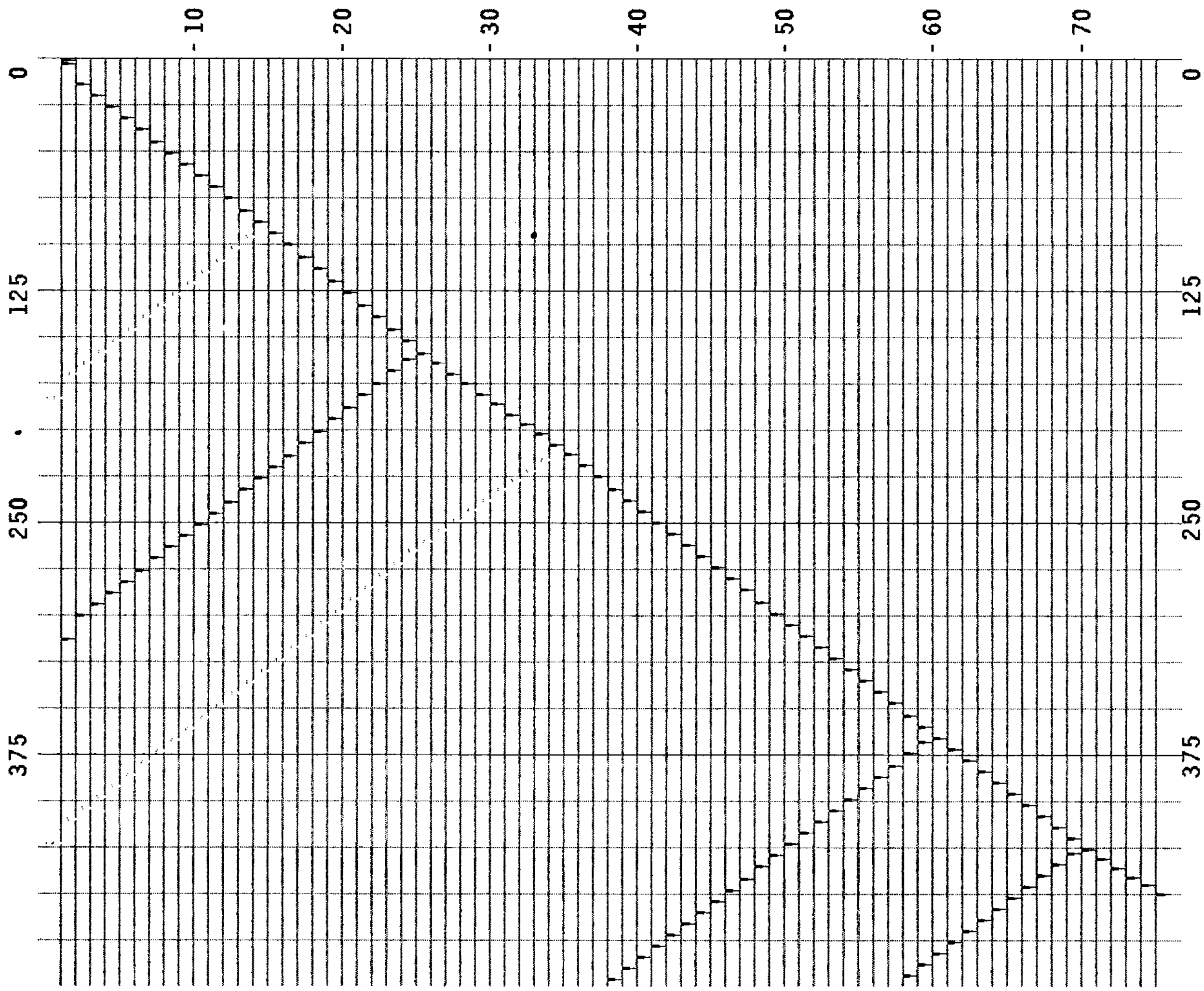

FIG. 5. Synthetic vertical seismic profile without multiple events.

The model in figure 1 shows that a multiple consists of an upgoing wave that has been reflected downward. In equation (5a), this corresponds to the $-z\ c_{j-1}\ u_j(z)$ term. By eliminating this term from equation (5a), the source of all multiples is eliminated. Rewriting equations (5a) and (5b), we get the multiple-free equations:

$$d'_j(z) = z\ (1 + c_{j-1})\ d'_{j-1}(z) \quad (9a)$$

$$u_{j+1}(z) = z\ (1 - c_{j+1})\ u_{j+2}(z) + z\ c_{j+1}\ d'_{j+1}(z)\ . \quad (9b)$$

The plot in figure 5 shows the results of applying the above equations to the model used for figure 4.

It is possible to calculate a SVSP with only the primary events and the first order multiples. For more information, see Wyatt (1981).

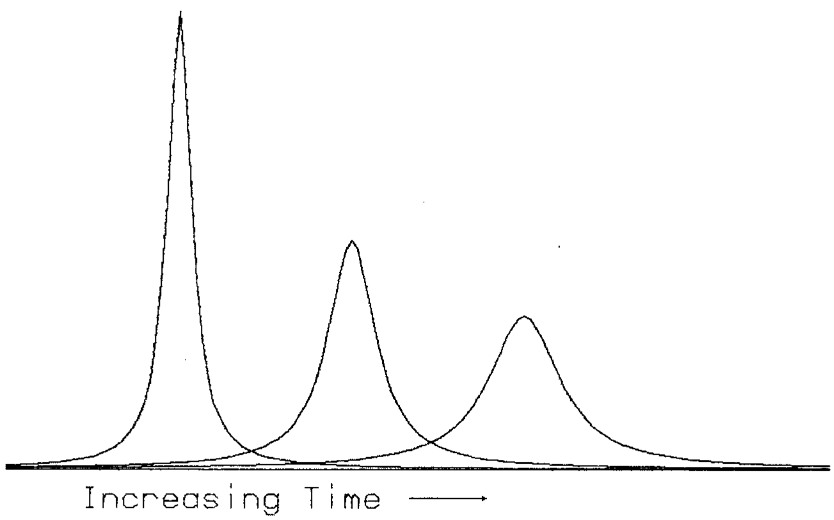

FIG. 6. Graph of the impulse response of an attenuating medium.

Profiles With Attenuation

The plot in figure 4 shows all of the events very clearly because the input signal was an impulse and the shape of the wave remained constant. The record could be convolved with a different waveform to simulate a more realistic input signal, but it would still lack the effect of attenuation. Therefore, the model must be altered to include the effect of attenuation on the seismic wave. The easiest way is to alter equations (5a) and (5b):

$$d'_j(z) = z\,(1 + c_{j-1})\,d'_{j-1}(z) * B_{j-1}(z) - z\,c_{j-1}\,u_j(z) * B_j(z) \qquad (10a)$$

$$u_{j+1}(z) = z\,(1 - c_{j+1})u_{j+2}(z) * B_{j+2}(z) + z\,c_{j+1}\,d'_{j+1}(z) * B_{j+1}(z)\,. \qquad (10b)$$

$B_j(z)$ is the impulse response of layer j if there is attenuation in the layer. The symbol * in the above equations is the convolution operator. The equation for B_j is derived with the assumption that the Fourier transform of the particle velocity of the wave is given by

$$B_j(\omega) = e^{-(i\,\omega\,\tau + \omega\,\tau\,\frac{\alpha_j}{2})} \qquad (11)$$

(see Trorey, 1962). τ is the one-way traveltime across the layer, α_j is the attenuation factor of layer j, and ω is the angular frequency. Applying the inverse Fourier transform to equation (11), B_j is expressed by

$$B_j(t) = \frac{2}{\alpha_j\,\tau\,\pi\left[1 + 4\,\frac{(t - \tau)^2}{(\alpha_j\,\tau)^2}\right]}\,. \qquad (12)$$

Several graphs of equation (12) appear in figure 6.

At present, it takes the computer about two minutes to compute a SVSP similar to the one in figure 4. If equations (10a) and (10b) were implemented on the computer, the computing time of a SVSP would increase considerably. This is due to the many convolutions that would be done at each interface. The computing time can be shortened if all the layers with the same attenuation are grouped together. For a SVSP, the best method to group the layers is to assume that the interval between two geophones has a constant attenuation. Within the interval between the geophones, the response of the layers is calculated using equations (5a) and (5b). When the program reaches an interface corresponding to a geophone, the equations (10a) and (10b) are used. For example, if a geophone interval contained 10 time interfaces, equations (5a) and (5b) would be used on these ten interfaces. At the eleventh interface, which corresponds the lower geophone, the downgoing wave would be convolved with equation (12). The result of this convolution would be recorded by the geophone and sent down to the next interface. In a similar manner, the upgoing waves would be convolved before recording it and sending it up.

Conclusions

There are three readily apparent applications for this modeling. The first application is to produce a noise-free VSP on which to test new processing techniques. This model is fast, easy to make, and produces a realistic VSP to match any unusual flat subsurface geological formations. The limitations of VSP to delineate thin beds may be tested. Dip filters may be tested on SVSP's as well as different stacking techniques to attenuate downgoing events. This SVSP approaches the response of a VSP when the source is close to the top of the well, so it can be used as check for more complicated modeling programs that do account for non-zero offset.

The second application is to help correlate well logs, VSP's, and regular seismic profiles. This has been the most used application within oil companies and is well represented in the literature (see Balch, *et. al.*, 1982). A SVSP calculated from a sonic log helps identify multiple events on a VSP and along with the formations which produce a given response. A potential research project would explain the differences between a sonic log generated SVSP and an actual VSP.

The third application of SVSP's is to study attenuation in rocks. Spencer, Sonnad, and Butler (1982) have already used SVSP's to investigate attenuation in the earth. Reversing the problem, the measurement of attenuation from VSP's can be developed on SVSP's where the model is known.

The assumptions on which this modeling is based are simple and effective for most purposes. However, they do not allow for the effects of far-offset sources and non-layered geology. Often, a well is drilled into a region of complex geology, such as a fault or an anticline. The next step in modeling is to take into account more complicated models of the earth and different source offsets around the well. A another step in modeling is to include the s wave in the section.

References

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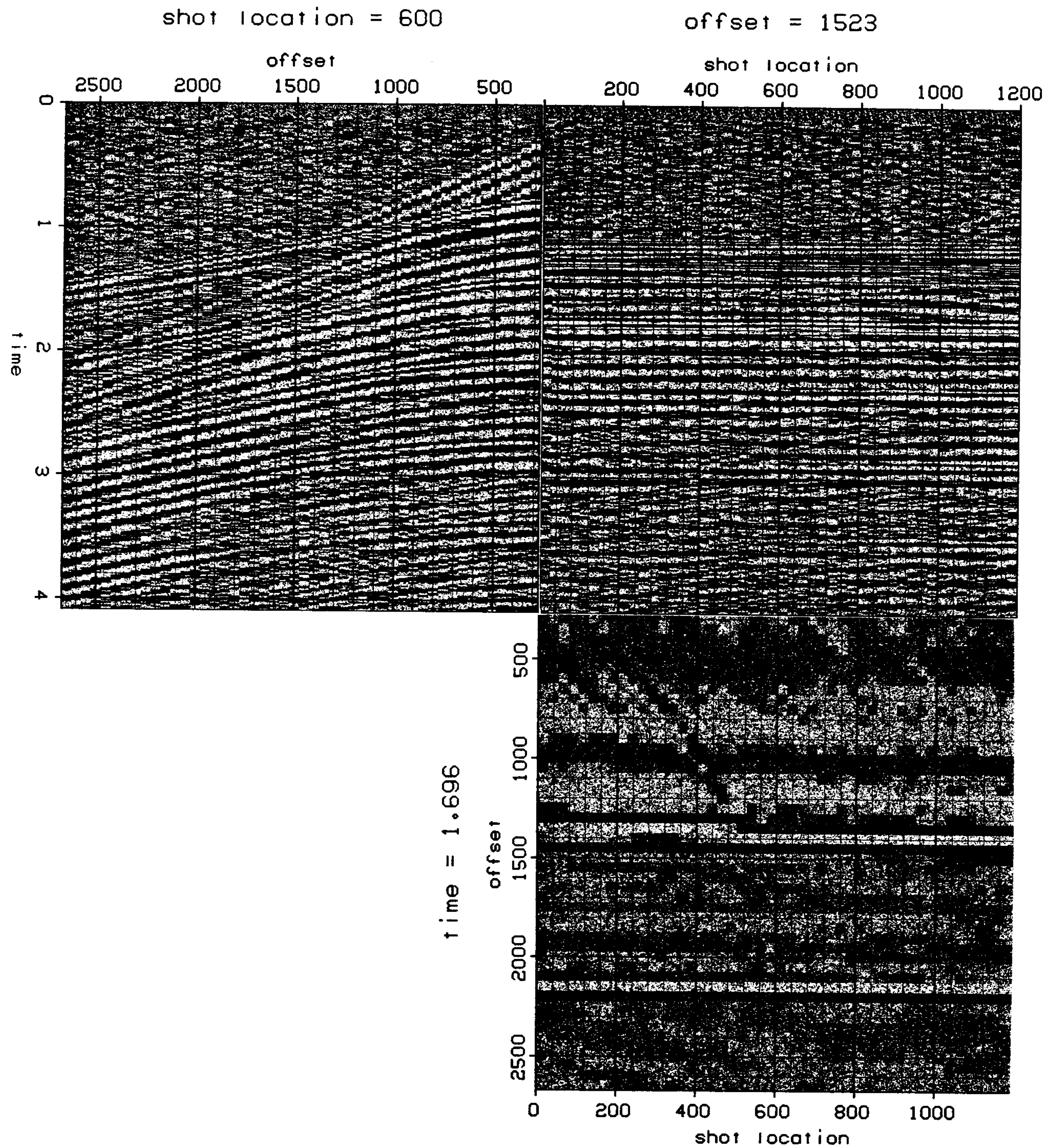
```
# The purpose of this subroutine is to calculate simple synthetic
# vertical seismic profiles at constant time intervals.
# The following model is used:
#
#
#                    D(i)
# layer i    -------------------------------------------------
#                                        U(i+1)
#
#
#                    D(i+1)
# layer i+1 -------------------------------------------------
#                                        U(i+2)
#
#
#       D(i+1) = (1+c(i))*D(i) - c(i)*U(i+1)
#       U(i+1) = U(i+2)*(1 - c(i+1)) + c(i+1)*D(i+1)
#
#       c(i) is the reflection coefficient of layer i.
#
# ngeop - number of geophones     dg - distance between geophones
# nlay - number of layers in model        dx - distance between layers
# ninput - length of input signal
# refco - array containing reflection coefficients
# ntime - time-length of the SVSP
#
subroutine vertpr(signal, refco, dg, dx)
real*8 signal(ninput), refco(nlay), d(1000), u(1000), dpast, dtemp, dg, dx
real*4 output(500)
integer it, il, ig, iig, nll, ngeop, nlay, ninput, ntime

common /number/ ngeop, nlay, ninput, ntime

# Initialize the up- and down-going arrays
do il = 1, nlay {
        d(il) = 0.;        u(il) = 0. }
# Start the loop over time
do it = 1, ntime {
        dpast = d(1)
        if (it <= ninput)          d(1) = signal(it)*(1. + refco(1))
        else                       d(1) = 0.
# Start the loop for each layer
        if (nlay-1 >= it+1)        nll = it + 1
        else                       nll = nlay - 1
        do il = 2, nll {
                dtemp = d(il)
                d(il) = (1. + refco(il - 1))*dpast - refco(il - 1)*u(il)
                u(il) = (1. - refco(il))*u(il + 1) + refco(il)*dtemp
                dpast = dtemp
        }
        d(nlay) = (1. + refco(nlay - 1))*dpast - refco(nlay - 1)*u(nlay)
        u(nlay) = -refco(nlay)*d(nlay)
# Send out the output for each geophone
        do ig = 1, ngeop {
                if (ig == 1)       iig = 1
                else               iig = dg*dble(ig)/dx
                output(ig) = d(iig) + u(iig + 1)
        }
        call outsig(outsig, ngeop)
}
return
end
```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