Stacking smiles

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Abstract

Simple geometrical considerations show that the zero offset section obtained from a Common Mid-Point (CMP) gather in presence of a single point diffractor is a combination of:

(i) Part of the diffraction hyperbola of the scatterer.

(ii) Other patterns depending on the relative locations of the diffractor and the CMP and on the midpoint spacing (aliases).

Superimposing many CMP's, pattern (i) is strengthened while pattern (ii) is not; thus patterns (ii) become of negligible amplitude if the CMP sampling rate is high enough.

The geometrical approach can explain the Dip Moveout (DMO) action on aliasing noise, clarifying why the latter cannot be removed at wavenumbers that are integer multiple of the Nyquist of the survey.

The "V" pattern

Let us consider a seismic section due to a single diffractor located at (x_0, z_0) (Figure 1). A gather with common midpoint in y will record reflection at times given by the double square root equation:

$$t_h = \frac{1}{v}\left[\sqrt{(y-h-x_0)^2 + z_0^2} + \sqrt{(y+h-x_0)^2 + z_0^2}\right] \tag{1}$$

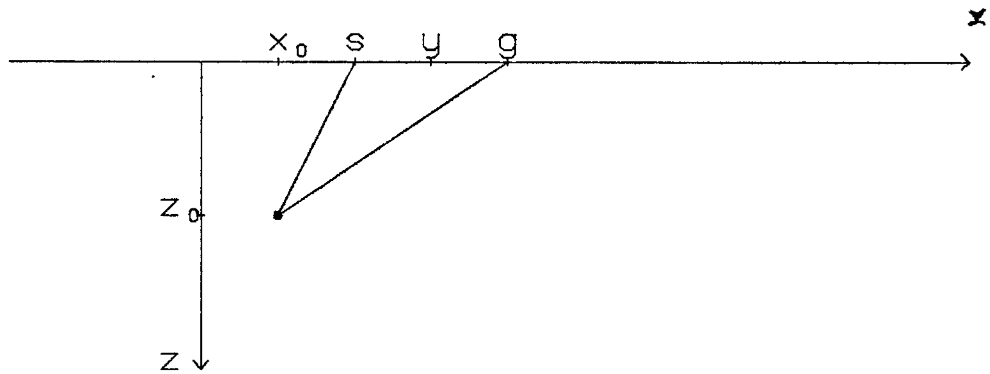

FIG. 1. Point diffractor model.

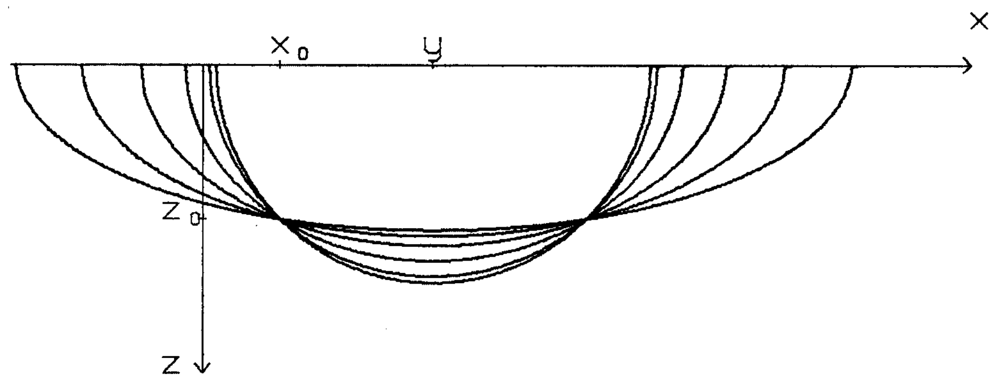

FIG. 2. Pre-stack full migration of a CMP gather.

The result of full migration before stack of a CMP is shown in Figure 2. On every CMP gather, each offset is migrated to an ellipse with foci at its shot and geophone location; all ellipses pass through the diffractor; they intersect in two symmetrical positions at $y \pm \left| y - x_0 \right|$. One location coincides with the point diffractor, the other corresponds to the alias.

Let us now apply pre-stack partial migration. This can be carried out either by diffracting the ellipses into the zero offset section, and getting the prestack migration smiles, or expanding the time spikes of the CMP gather directly to the smiles. In both cases, as well known, (Deregowski and Rocca, 1981) the resulting pattern from a spike at time t_h, half offset h and CMP y, will be the truncated ellipse:

$$\frac{(x-y)^2}{h^2} + \frac{t^2}{t_0^2} = 1 \qquad (2)$$

The ellipse bottoms under the midpoint y at the normal moved out time:

$$t_0^2 = t_h^2 - \frac{4h^2}{v^2}$$

The truncation is at:

$$\left| x - y \right| < \frac{2h^2}{vt_h}$$

The envelope of the smiles from several offsets traces belonging to a CMP gather is shown in Figure 3 and in the appendix to have a symmetric "V" pattern. It is the superposition of parts of two hyperbolas: one due to the actual diffractor, the other due to its alias. The finite extension of the part of the hyperbola shows that not all wavenumbers of the diffractor spectrum can be recovered using a single CMP. The width of the "V" (see the Appendix for the calculations) is approximately equal to:

$$\frac{1}{2} W \approx \frac{h_{max}^2}{t} \frac{dt}{dx} \tag{3}$$

If we apply NMO only, instead of NMO and DMO, we would have got the apex of the "V" only.

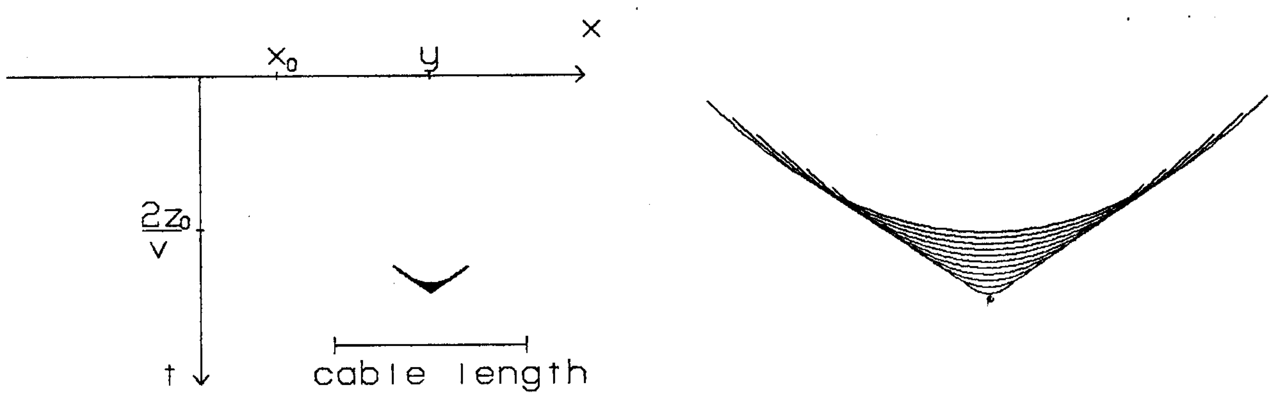

FIG. 3. The "V". (A blow-up to the right).

Superposition of "V" patterns.

The results of the combination of more than one CMP gather is shown in Figure 4; in Figure 4a we show the results obtained using both DMO and NMO. Figure 4b shows the results of conventional processing, i.e. applying NMO using the cosine corrected velocity ($v / \cos\theta$).

The alias pattern changes a lot in the two cases. In the depth domain the sampled hyperbola of Figure 4b migrates into a set of circles that intersect in the diffractor location only. On the other hand, after NMO and DMO, the hyperbola with "bristles"of Figure 4a will migrate into the same diffractor and into the aliases at depth z_0 and abscissae $x_0 + 2n\Delta y$ (Δy is the midpoint spacing). These aliases are unavoidable as can be seen from Figure 2. Figure 4a shows clearly why DMO reduces the aliasing noise. A frequency domain explanation to the same phenomenon is given in the companion paper "Why Dip Moveout ?" in this report.

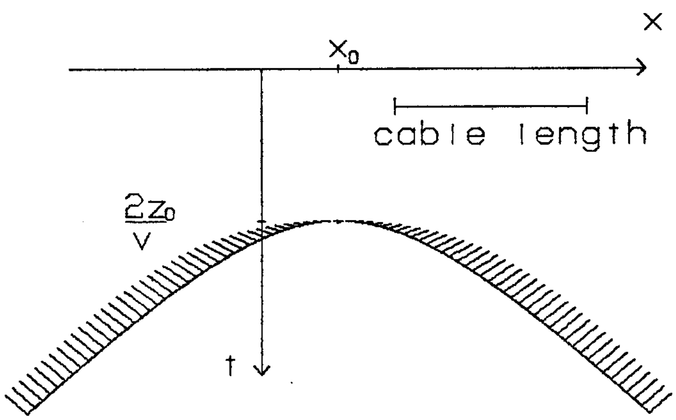

FIG. 4a. Zero offset section after DMO and NMO.

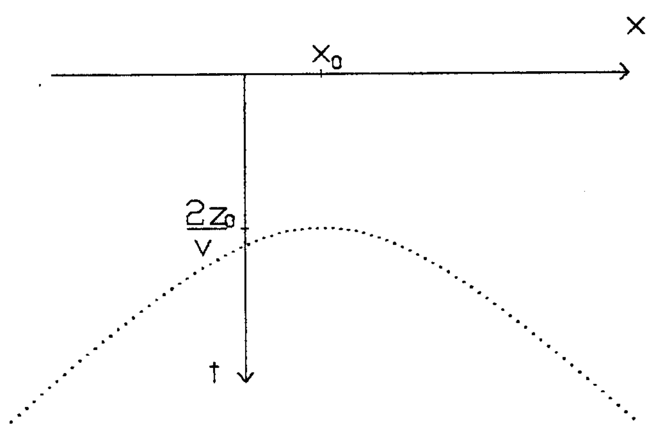

FIG. 4b. Zero offset section after NMO only.

Figures 2 and 4a also show that a complete cancellation of the aliasing noise is impossible, even with perfect DMO correction. The residual aliasing noise spectrum after DMO will peak at wavenumbers multiple of $nk_0/2$, since the aliasing noise consists of spikes evenly spaced of $2\Delta y$. ($k_0 = 2\pi/\Delta y$). Again this agrees with what found in the companion paper.

Stacking with incorrect velocity

If the NMO is applied with incorrect velocity, the apex of the "V" will remain in the same position, but the "V" itself will open or close if the applied velocity is too high or too low respectively. In this case the limb of the "V" will not lie on the hyperbola and there will be some defocusing, as well known. This mechanism might explain why the velocity analyses are much crisper after DMO.

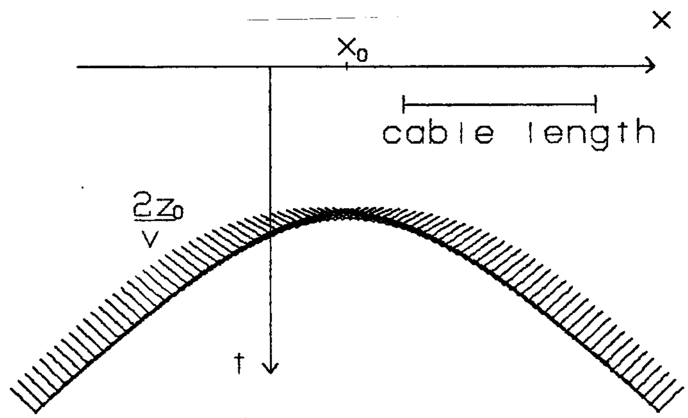

FIG. 5. Stacking with DMO and NMO with incorrect (10% low) velocity.

The zero offset section corresponding to a common shot gather.

As we did for the CMP's ,it is possible to find the zero offset section corresponding to a CSG, again in the case when a single diffractor is present. Each trace then will carry a single spike that will be transformed to a smile by PSPM. The smiles will have other locations than in the case previously seen. Their new envelope is shown in Figure 6.

Again we see part of the diffraction hyperbola due to the diffractor, but the picture is clearly different from the one retrieved from a CMP. The aliasing noise now appears to cluster along two parabolas with horizontal axes, moving in opposite directions along the y axis from the shot location and tangent to the diffractor hyperbola. Changing the shot location, the two parabolas would move as it would the reproduced part of the hyperbola.

Conclusion

In this short paper we have given a simple explanation of the action of DMO on aliasing noise removal and on velocity analysis. We have seen why DMO improves spatial resolution and why there will always be some noise at the Nyquist wavenumber. CSG processing is just a reordering and does not solve the residual aliasing noise problem.

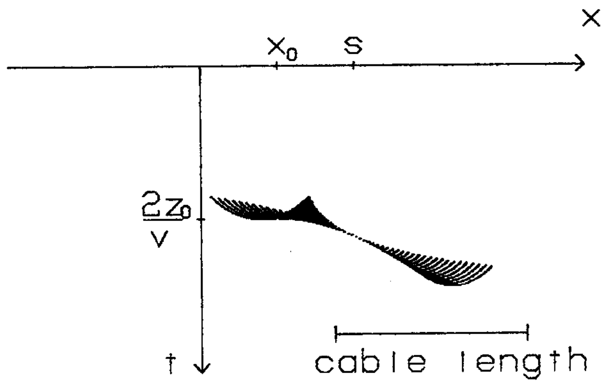

FIG. 6. Zero offset section corresponding to a CSG. (Split spread, one cable length on each side of the shot).

Appendix

We determine here the shape of the envelope of the smiles, for CMP gather $y = 0$, generated by a point diffractor located at (x_0, z_0); the ellipses that correspond to the migration of the CMP $y = 0$ have equation:

$$\frac{x^2}{b^2 + h^2} + \frac{z_0^2}{b^2} = 1 \tag{A-1}$$

They are centered at 0 and have focal distance $2h$. The length of the minor semiaxis b is determined by:

$$b^4 - b^2\left[x_0^2 + z_0^2 - h^2\right] - z_0^2 h^2 = 0 \tag{A-1a}$$

that ensures that the ellipses cross the diffractor.

Define $\tau = vt/2$ then the "smile" is:

$$\frac{x^2}{h^2} + \frac{\tau^2}{b^2} = 1 \tag{A-2}$$

Substituting h^2 as a function of b^2, from (A-1a) into (A-2):

$$x^2\left[b^2 - z_0^2\right] - \tau^2\left[b^2 - x^2 - z_0^2\right] - b^2\left[x_0^2 + z_0^2 - b^2\right] = 0 \tag{A-3}$$

To find the envelope, the derivative with respect to b^2 should vanish, therefore:

$$b^2 = \frac{1}{2}\left[\tau^2 + x_0^2 + z_0^2 - x^2\right] \tag{A-4}$$

Substituting (A-4) into (A-3):

$$\tau^4 - 2\tau^2\left(x_0^2 + z_0^2 + x^2\right) + x^4 - 2x^2\left(x_0^2 - z_0^2\right) + \left(x_0^2 + z_0^2\right)^2 = 0$$

Solving for τ we get two hyperbolas; one with apex at (x_0, z_0):

$$\tau^2 = \left(x - x_0\right)^2 + z_0^2 \tag{A-4a}$$

and the other at $(-x_0, z_0)$:

$$\tau^2 = \left(x + x_0\right)^2 + z_0^2 \tag{A-4b}$$

To find the width of the "V" pattern let us find the abscissa of the point of tangency between the smile (A-2) and the hyperbola (A-5a):

$$x = \frac{x_0^2 + z_0^2 - b^2}{x_0} \tag{A-6}$$

Solving (A-1a) for b^2 we get:

$$b^2 = \frac{x_0^2 + z_0^2}{2}\left[1 + \sqrt{1 + h^2\,\frac{h^2 + 2z_0^2 - 2x_0^2}{(x_0^2 + z_0^2)^2}}\right] - \frac{h^2}{2} \tag{A-1a'}$$

and inserting into (A-6) we have:

$$\frac{1}{2}W = \frac{x_0^2 + z_0^2 - b_{\min}^2}{x_0} \tag{A-7}$$

Where $b_{\min}$ is found by substituting the largest offset into (A-1a').

This result is consistent with the ones obtained by Levin (1971) and Deregowski (1982).

REFERENCES

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Levin, F.K., 1971, Apparent velocity from dipping interface reflections, GEOPHYSICS v. 36, p. 510 - 516.