# Lateral Velocity Anomalies

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## Introduction

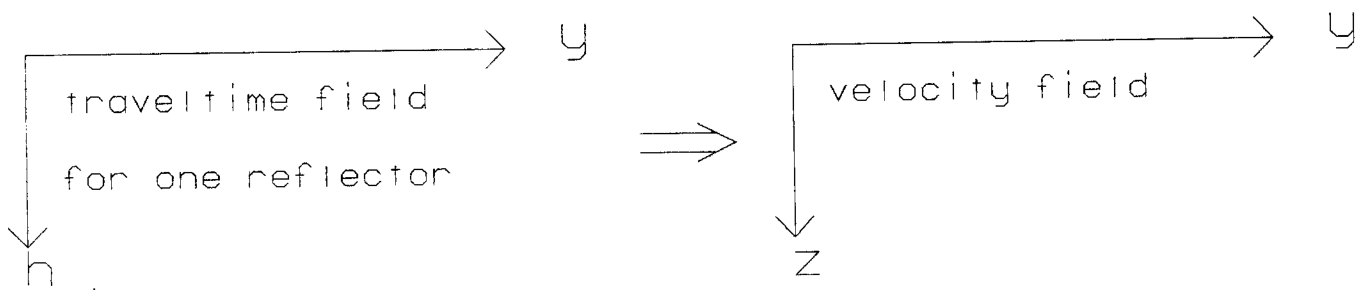
Accurate seismic velocities play a very important role in properly imaging the earth in structurally complex areas. In addition, attempts to detect hydrocarbons directly, i.e. bright spot analysis, often depend on the ability to correctly determine whether a particular region has an anomalously slow or fast velocity. Unfortunately, conventional velocity analysis, which consists of measuring coherency along hyperbolic trajectories in offset-traveltime space(i.e. determining stacking velocities), can lead to meaningless estimates when velocity changes rapidly in the lateral direction. Since the two important applications of accurate seismic velocities mentioned above are precisely cases where the velocity does change significantly in the lateral direction, some improvement to the conventional method of analysis is clearly desirable.

In this paper we will discuss two different approaches to velocity estimation in areas with significant lateral velocity variations. The first approach avoids the problems associated with stacking velocities, by dealing directly with traveltimes. This is the approach taken by Neumann (1981), Kjartansson (1979) and Dines and Lytle (1979), whose methods we review in the first part of this paper. The second approach does deal with stacking velocities, but develops some relations which allow them to be more properly related to the actual interval velocities. Lynn and Claerbout (1979) developed such a method, then Loinger(1981) presented a model which gives a linear relation between interval velocity anomalies and stacking velocity anomalies. In the second part of this paper, we will follow Loinger's approach, with some modifications. In particular, we will discuss the transfer function between interval and stacking velocity anomalies, with its implications on bright spot analysis. Finally, in the third part of the paper, we will compare the traveltime and stacking velocity approaches.

**1. Using Traveltimes Directly**

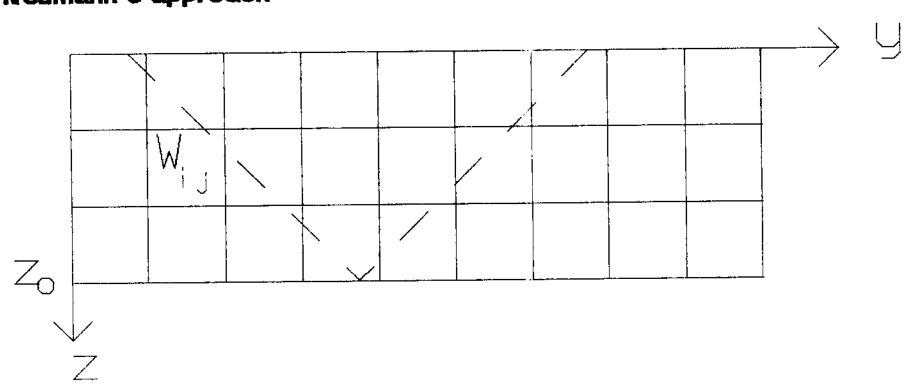
Assume that our data are $T(h,y,z_0)$, i.e. traveltimes as a function of offset and midpoint for a particular reflector at depth $z_0$. Assume also that $z_0$ known, and that rays are rectilinear.

You would like to determine $W(y,z)$ for $z \leq z_0$, where $W = \frac{1}{\textit{interval velocity}} = \textit{slowness}$, for wavelengths of lateral velocity anomalies less than a cablelength.




A problem with this type of approach is that traveltimes are uncertain and expensive to determine.

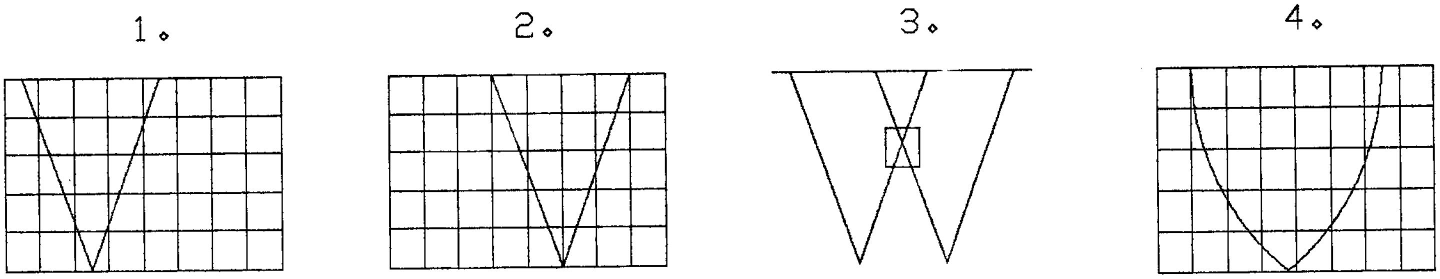
**a) Neumann's approach**




Divide the subsurface into blocks, the unknowns are the slownesses, $W_{ij}$. You find that $T = AW$, i.e. a set of linear equations relating slownesses to traveltimes. This set of equations can then be inverted for slowness $W$. It will be ill conditioned, so more properly some sort of pseudo- inverse will be required.

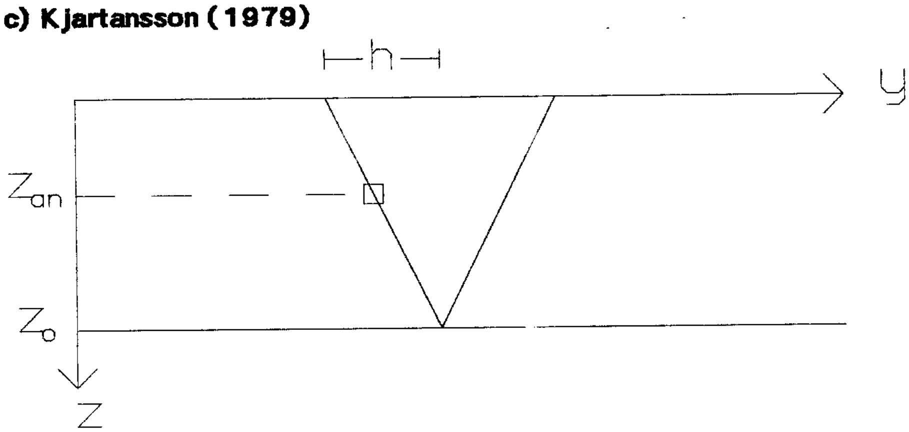
**b) Algebraic Reconstruction Theory (ART)**

This is one way to invert the equation $T = AW$.




1. For a particular ray, divide traveltime equally among all blocks crossed by the ray. (Add some weight if the ray only touches the block, without passing near its center)
2. Repeat for all rays.
3. Average slownesses for all rays passing through one block.
4. Could continue to non-rectilinear ray-tracing in an iterative manner, by refracting through the newly determined slowness field (or a smoothed version of it), then repeating steps 1,2,3 for the new set of rays.

**c) Kjartansson (1979)**




If rectilinear rays are assumed,

$$T(y,h,z_0) = \frac{\sqrt{h^2+z_0^2}}{z_0} \int_0^{z_0} \left[ W\left(y - \frac{h(z_0 - z_{an})}{z_0}, z_{an}\right) + W\left(y + \frac{h(z_0 - z_{an})}{z_0}, z_{an}\right) \right] dz_{an} \quad (1)$$

i.e. collect traveltime along the ray by multiplying distance by slowness. We have chosen to define $z_{an}$ as being positive downward ,to be consistent with the notation used in the latter part of this paper. Note that this is opposite to the direction chosen by Kjartansson. Since $h, z_0$ are known, we can remove the cosine factor by defining:

$$T' = z_0 \frac{T}{\sqrt{h^2 + z_0^2}}$$

Now transform $y$ to $k$ (the equation is stationary in y), resulting in the following set of linear equations, one for each k:

$$\overline{T}_k{}'(h, z_0) = 2 \int_0^{z_0} \overline{W}_k(z_{an}) \cos \frac{kh(z_0 - z_{an})}{z_0} dz_{an} \qquad (2)$$

Where $\overline{W}_k$ and $\overline{T}_k{}'$ are the Fourier transforms of $W$ and $T'$ respectively. Note that the distance from the anomaly to the reflector $z_0 - z_{an}$ acts as the transform variable for h. This has a nice physical interpretation: Shallow velocity anomalies (i.e. $z_0 - z_{an}$ large), correspond to rapid variations in traveltimes as a function of offset. In addition, the value at zero offset, $h = 0$, corresponds to the integral over the variations in depth.

Now transform over h, with $\eta$ being the transform variable. To do this, one must assume h unbounded (i.e. unlimited offsets )

$$\hat{T}_k(\eta, z_0) = \int_0^{z_0} \overline{W}_k(z_{an}) \int_{i=-\infty}^{i=\infty} \exp j \left[ \eta h \pm \frac{kh(z_0 - z_{an})}{z_0} \right] dh \; dz_{an}$$

$$= \int_0^{z_0} \overline{W}_k(z_{an}) \left[ \delta \left( \eta - \frac{k(z_0 - z_{an})}{z_0} \right) + \delta \left( \eta + \frac{k(z_0 - z_{an})}{z_0} \right) \right] dz_{an}$$

$$= \frac{z_0}{|k|} \overline{W}_k \left[ z_0 \left( 1 - \left| \frac{\eta}{k} \right| \right) \right] \qquad (3)$$

Where $|k| > |\eta|$. Finally, identifying

$$z_{an} = z_0 \left( 1 - \left| \frac{k}{\eta} \right| \right)$$

we find

$$W_k(z_{an}) = \frac{|k| - |\eta|}{z_{an}} \hat{T}_k \left( \eta , \frac{z_{an}}{1 - \left| \frac{\eta}{k} \right|} \right) \qquad (4)$$

Clearly there will be some smoothing due to the limited offset range. This equation could be used as an inversion formula (as written), or, perhaps better, it could be turned around and used as a direct formula for T given W.

**2. Using stacking velocity**

Let us plot in the usual $t^2 - x^2$ domain the traveltimes correspondent to a reflector in a constant velocity medium, for a CMP gather, and let one of the traveltimes increase. The interpolating straight line will change its slope, thus changing the apparent r.m.s. velocity.

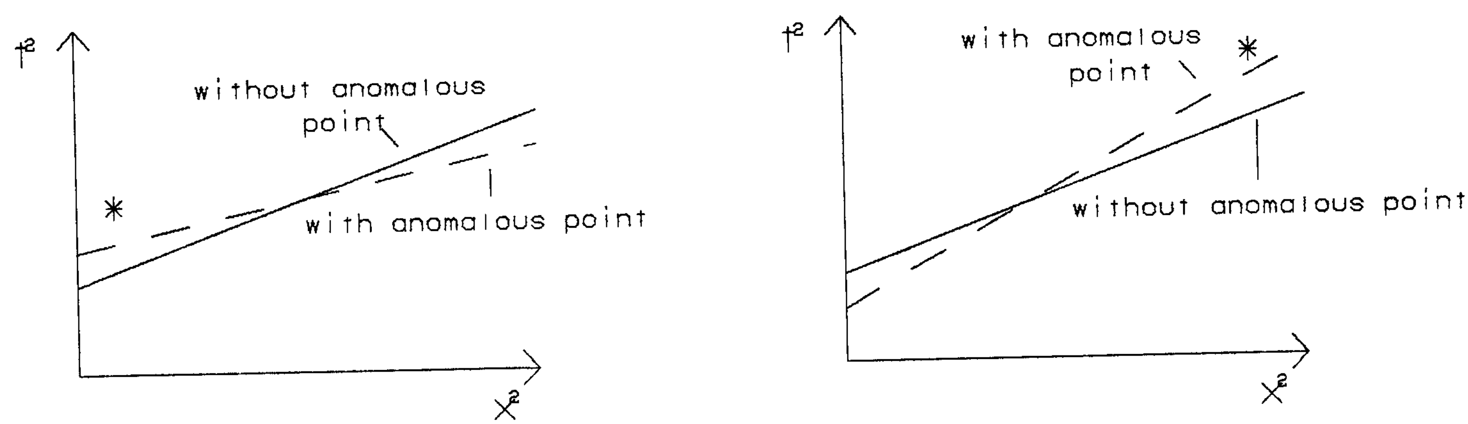



FIG. 1. a) $t^2 - x^2$ plot with increased traveltime for one trace at small offset. b) $t^2 - x^2$ plot with increased traveltime for one trace at large offset.

As seen in figure 1, increasing the traveltime of one trace of the CMP gather for small offsets will increase the rms velocity, whereas at large offsets, the same increase in traveltime will decrease the calculated rms velocity. More specifically, given a set of n pairs of $r,s$ which you want to fit with a straight line, $s_i = mr_i + q$, and the origin of the $r_i$ chosen such that $\sum_{i=1}^{n} r_i = 0$, the choice of $m$ that minimizes the sum of the squared deviations from the line is

$$m = \frac{\sum_{i=1}^{n} r_i s_i}{\sum_{i=1}^{n} [r_i]^2} \tag{5}$$

A variation $\Delta s$ at $\bar{r}$ will produce a variation in $m$

$$\Delta m = \frac{\bar{r}\Delta s}{\sum_{i=1}^{n} r_i^2} \tag{6}$$

Looking at the standard hyperbolic traveltime equation,

$$t_i^2 = t_0^2 + \frac{x^2}{v_{rms}^2} \tag{7}$$

we can identify $m = \frac{1}{v_{rms}^2}$, $s_i = t_i^2$, and $r_i = x^2 - \frac{1}{n}\sum_{i=1}^{n} x^2$, resulting in

$$\Delta m = \frac{-2}{v_{rms}^3}\Delta v_{rms} = 2 W_a \Delta W_a \tag{8}$$

where $W_a$ is the apparent rms slowness $\left[\frac{1}{v_{rms}}\right]$ and $\Delta W_a$ is the change in apparent rms slowness due to the time variation $t_i$ at $x_i$. To cast the right side of equation (8) into a useful form, we use

$$\frac{1}{n}\sum_{i=1}^{n} x^2 = \frac{1}{n\Delta x}\sum_{i=1}^{n} x^2 \Delta x \approx \frac{L^2}{3}$$

and

$$\sum_{i=1}^{n} x^4 = \frac{n}{n\Delta x}\sum_{i=1}^{n} x^4 \Delta x \approx n\frac{L^4}{5}$$

where L= cable length, giving

$$r_i = x^2 - \frac{L^2}{3}$$

and

$$\sum_{i=1}^{n} r_i^2 = \sum_{i=1}^{n}\left[x^2 - \frac{L^2}{3}\right]^2 = \sum_{i=1}^{n} x^4 - 2\frac{L^2}{3}\sum_{i=1}^{n} x^2 + \frac{nL^4}{9}$$

$$= \frac{4}{45} nL^4$$

Finally,

$$\Delta m = 2 W_a \Delta W_a = 2 t_i \Delta t_i \frac{\left[x_i^2 - \frac{L^2}{3}\right]}{\left[\frac{4}{45} nL^4\right]} \tag{9}$$

To get $t_i \Delta t_i$ in terms of interval slowness $W_{in}$, we begin by rewriting the hyperbolic travel-time equation (7) in terms of apparent rms slowness $W_a$

$$t_i^2 = W_a^2 \left[ 4z^2 + x^2 \right]$$

then differentiating,

$$2t_i \Delta t_i = 2W_a \left[ 4z^2 + x^2 \right] \Delta W_a$$

and using the fact that for a single anomaly in interval slowness $\Delta W_{in}$ at depth $z$ and of thickness $s$, $\Delta W_a = \Delta W_{in} \frac{s}{2z}$, we find

$$t_i \Delta t_i = 2W_a sz \left[ 1 + \frac{x^2}{4z^2} \right] \Delta W_{in} \tag{10}$$

which can be substituted into equation (9) to yield

$$W_a \Delta W_a = \frac{2W_a sz \left[ 1 + \frac{x^2}{4z^2} \right] \left[ x^2 - \frac{L^2}{3} \right] \Delta W_{in}}{\frac{4}{45} nL^4}$$

or

$$\Delta W_a = \frac{15}{2} \frac{sz}{nL^2} \left[ \frac{3x^2}{L^2} - 1 \right] \left[ 1 + \frac{x^2}{4z^2} \right] \Delta W_{in} \tag{11}$$

So far we have found the change in apparent rms slowness corresponding to a change in interval slowness at a particular offset $x$. A more useful form would be to have the anomalous interval slowness specified at a particular midpoint coordinate $y_{an}$, and then look at the change in apparent rms slowness as a function of midpoint.

The following relations can be derived from figure 2,

$$L' = \frac{(z - z_{an})}{z} L$$

$$x = \frac{2z}{z - z_{an}} (y - y_{an}) = \frac{2L}{L'} (y - y_{an})$$

It is worth noting that in a layered medium, the relation between $L$ and $L'$ would be more complex; we could, however, relax the straight ray hypothesis. In a constant velocity medium we have,

$$\Delta W_a(y) = \frac{15}{2} \frac{sz}{nL^2} \left[ 3 \left[ \frac{2(y - y_{an})}{L'} \right]^2 - 1 \right] \left[ 1 + \frac{L^2}{4z^2} \left[ \frac{2(y - y_{an})}{L'} \right]^2 \right] \Delta W_{in}(y_{an}) \tag{12}$$

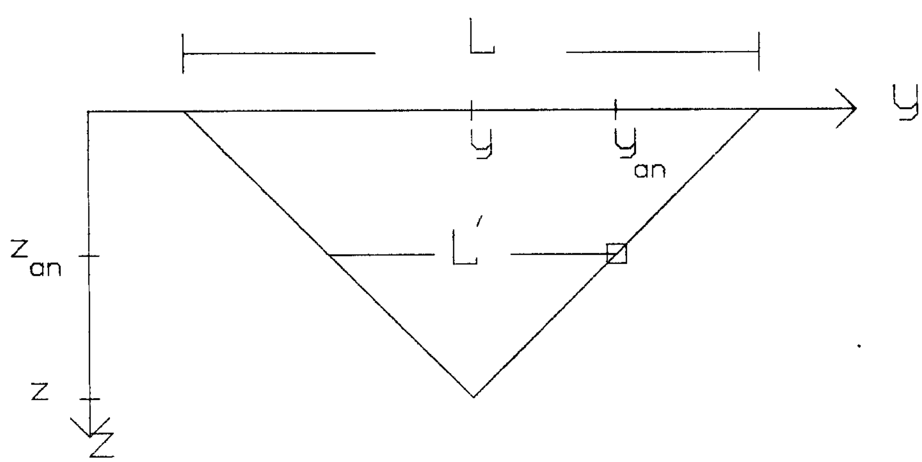



FIG. 2.

We would now like to consider the effect of an anomalous layer, by summing over $y_{an}$, but we first must cast EQ 12 into a form where $\Delta y$ corresponds to one trace width.

$$\Delta' = \frac{L'}{2n}$$

so we can identify $\frac{1}{2n}$ in EQ 12 as $\frac{\Delta'}{L'}$.

Summing EQ (12) over $y_{an}$ , and writing the sum as an integral yields,

$$\Delta W_a(y) = \int \frac{15}{L^2 L'} sz \left[ 3\left(\frac{2(y - y_{an})}{L'}\right)^2 - 1 \right]\left[ 1 + \frac{L^2}{4z^2}\left(\frac{2(y - y_{an})}{L'}\right)^2 \right] \Delta W_{in}(y_{an})\Delta' \quad (13)$$

$$= \frac{15}{L^2 L'} sz \left[ 3\left(\frac{2y}{L'}\right)^2 - 1 \right]\left[ 1 + \frac{L^2}{4z^2}\left(\frac{2y}{L'}\right)^2 \right] * \Delta W_{in}(y) \quad (14)$$

That is, $\Delta W_a$ is the anomalous interval slowness convolved with an impulse response, where the "impulse" is just a unit magnitude anomaly in interval slowness, of thickness $s$, at depth $z_{an}$ and width equal to one apparent trace spacing (apparent cable length for that depth over number of channels). Some insight into the nature of this impulse response can be gained by looking at its Fourier transform. Thus, transforming the impulse response $F(z,z_{an},y)$ over $y$ to $\bar{F}(z,z_{an},K)$, then using a dimensionless wavenumber $k = \frac{KL}{2}\frac{(z - z_{an})}{z} = K\frac{L'}{2}$, we find

$$\overline{F}(z,z_{an},k) = \frac{15sz}{L^2k^5}\left[\left((2+2c)k^4 + (-34c-6)k^2 + 72c\right)\sin k + \left((6+10c)k^3 - 72ck\right)\cos k\right] \quad (15)$$

where $c = \frac{L^2}{4z^2}$. This transfer function is plotted in figure 3 for $c = 0$ and $c = .4$.

For $c = 0$, and $k$ small, the transfer function is $\approx k^2$, that is, a second derivative operator. A similar result was derived and discussed by Lynn and Claerbout (1982). To see what a more typical value of $c$ would be, we need to first discuss the effect of muting the data. Muting makes the cable length $L$ a function of depth. For a linear mute that reaches full offset at 2 seconds, with a velocity of 2000 meters/sec and a true cable length of 2500 meters,

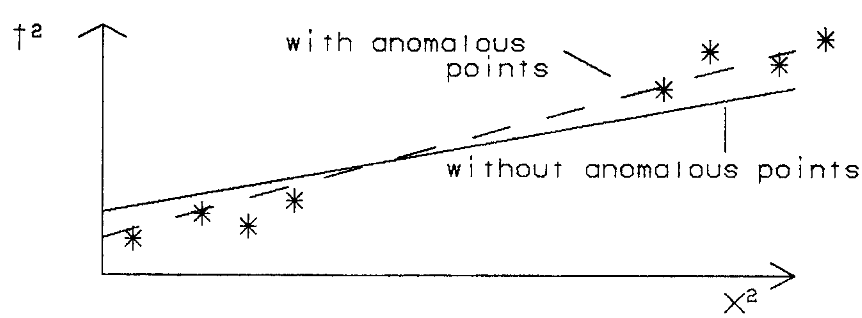
$$L(z) = \frac{2.5}{2}z = 1.25z$$

so a more typical value of $c$ is

$$c = \frac{L(z)^2}{4z^2} \approx .4$$

The most striking difference between the $c = 0$ and $c = .4$ cases is at the low wavenumber end. Note that for $c = .4$ the transfer function is not equal to zero at $k = 0$; in particular it is positive. This is an intuitively satisfying result, since it says that an increase in the DC component of interval slowness will produce an increase in the DC component of rms slowness.

Another interesting feature that can be seen in figure 3, is that the magnitude of the transfer function achieves a maximum at $k \approx 3.5$. This feature is shared by both the $c = 0$ and $c = .4$ cases. This feature can be thought of as a resonance phenomenon, since for given depth of reflector, depth of anomaly and cablelength, there is a particular wavelength of interval slowness anomaly that will give a maximum response in anomalous apparent slowness. More quantitatively, the resonance occurs at $k = \frac{KL'}{2} \approx 3.5$ or $\lambda \approx L'$. That rms slowness should be particularly sensitive to velocity anomalies of wavelength $L'$ makes sense, if we recall the simple picture that introduced this section on stacking velocity.



Clearly there is a maximum change in rms velocity when all of the large offsets sense a positive anomaly and the small offsets sense a negative anomaly. Such a situation will occur at $\lambda \approx L'$ and again at $\lambda \approx \frac{L'}{2}$, thus explaining both the minimum at $k \approx 3.5$ and the maximum at $k \approx 7$.

One remaining question about the resonance is why the transfer function should be negative at $k \approx 3.5$. The essence of the resonance phenomenon is that the effect of the small offsets on rms velocity is reinforced by the large offsets, so that in evaluating the sign of the effect, we can concentrate on the small offsets. As discussed earlier, a decrease in interval velocity at small offsets will produce an increase in apparent rms velocity. When the cable is centered over a negative lobe of the anomaly (recall that we are looking at the response to a particular wavelength of anomaly), the low offsets will be sensing the negative part, and the high offsets the positive part, with the net effect being a positive value for the anomalous rms velocity. The anomalous interval and rms velocities are thus 180 degrees out of phase, i.e. the transfer function is negative.

**3. Comparison between traveltime and stacking velocity approaches**

So far we have only looked at the response to an anomaly at a particular depth. In order to compare the responses of traveltime and stacking velocity to anomalous interval velocity we will need to integrate the transfer functions over depth. Thus for a reflector at depth $z$, we find

$$\Delta W_a(z,k) = \frac{15z}{L^2}\int_0^z G(z,z_{an},k)\Delta W_i(z_{an},k)dz_{an} \tag{16}$$

where, from EQ (15),

$$G(z,z_{an},k) = \frac{1}{k^5}\left[\left((2+2c)k^4 + (-34c-6)k^2 + 72c\right)\sin k + \left((6+10c)k^3 - 72ck\right)\cos k\right] \tag{17}$$

and as in EQ (15), $c = \frac{L^2}{4z^2}$. Equation (16) is the one to invert if we want to retrieve interval velocities from anomalous rms velocities. If the number of anomalous levels is equal to the number of horizons chosen for the rms velocity analysis, the system of equations is lower triangular. Obviously we can always choose to consider more reflectors than anomalous levels.

Now suppose that the interval slowness, $W_i$, is constant , except for at depth $z_{an}$, where there is an anomaly $\Delta W_i(k, z_{an})$, of thickness $\Delta z_{an}$. Equation (16) then becomes,

$$\Delta W_a(z,k) = \frac{15z^2}{L^2} G(z, z_{an}, k) \frac{\Delta z_{an}}{z} \Delta W_i$$

and the relative response,

$$\frac{\Delta W_a(z,k)}{W_a(z,k)} = \frac{15z^2}{L^2} G(z, z_{an}, k) \frac{\Delta z_{an}}{z} \frac{\Delta W_i}{W_i} \tag{18}$$

We can go through the same procedure with the traveltimes. Under the same model of anomalous interval velocity at one depth, EQ (2) becomes,

$$\Delta T(h,z,K) = 2\cos\left[\frac{hK(z-z_{an})}{z}\right] \Delta W_i \Delta z_{an}$$

where $K$ is the conventional wavenumber. Then the relative response,

$$\frac{\Delta T(h,z,k)}{T(h,z,k)} = \frac{2\cos\left[\frac{hK(z-z_{an})}{z}\right] \Delta W_i \Delta z_{an}}{2zW_i} \tag{19}$$

$$= \cos\left[\frac{2hk}{L}\right] \frac{\Delta z_{an}}{z} \frac{\Delta W_i}{W_i} \tag{20}$$

in terms of the dimensionless wavenumber $k$. As in the earlier part of the paper, where we saw that $h$ and $z - z_{an}$ served as transform variables, we see from EQ (19) that for small $h$ our relative response is insensitive to the depth of the anomaly. If we had only one anomaly, and knew it's depth, then clearly any offset would work fine. If, on the other hand we had more than one anomaly, and wished to also resolve their depth, then clearly the maximum sensitivity to depth would occur when $h = h_{max} = \frac{L}{2}$, giving

$$\frac{\Delta T}{T} = \cos k \frac{\Delta z_{an}}{z} \frac{\Delta W_i}{W_i} \tag{21}$$

$\frac{\Delta T}{T}$ for the $h = \frac{L}{2}$ case (EQ 21), is plotted in figure 4 along with $\frac{\Delta W_a}{W_a}$. Note that the transfer functions are equal for $k = 0$, a result which can also be derived by comparing EQs (18) and (19) in the limit as $k \to 0$. Note also that at resonance the $\frac{\Delta W_a}{W_a}$ response is greater than the $\frac{\Delta T}{T}$ response by a factor of about 7. The effect on the $\frac{\Delta T}{T}$ response of using a different offset, would just be to change the period of the cosine, leaving the amplitude unchanged. Thus, regardless of the particular offset used, the rms slowness will be more responsive to a particular spatial bandwidth of anomaly than will be the traveltime. Of course, with traveltimes one has the possibility of statistically combining the results derived from several offsets, and thereby improving the response.

## Conclusions

In this paper we have discussed two very different methods for determining interval velocities, one based on traveltimes and one based on stacking velocities. While the use of stacking velocities instead of traveltimes entails a loss of the offset dimension, and hence a loss in redundancy, we showed that for a single offset the methods are quite comparable. Indeed for a certain spatial bandwidth of anomaly, the stacking velocities are much more responsive than are the traveltimes. One big advantage to using the stacking velocity method is that stacking velocities are a standard product of a conventional processing sequence, whereas traveltimes are not. One could, then, imagine that the stacking velocity method described here could easily be added on to a standard processing sequence at very little cost. By contrast, any method using traveltimes must, of course, begin with the determination of traveltimes, a major effort in itself.

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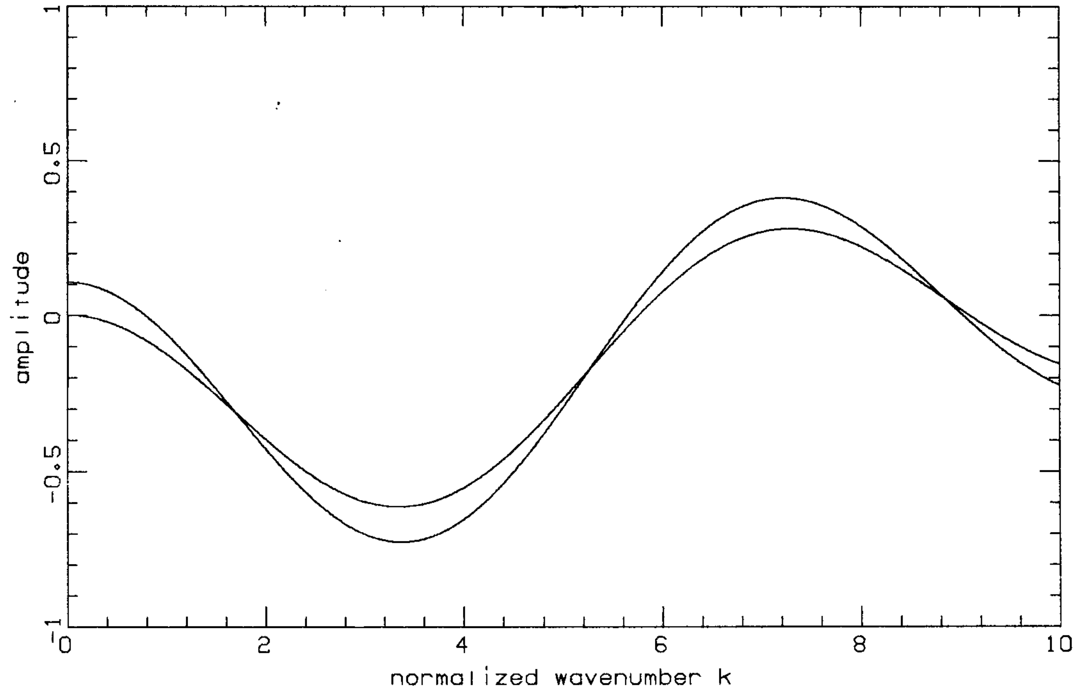



**FIG. 3. Plot of transfer function ( EQ 15 ). Note non-zero value at $k = 0$, and large negative value at $k \approx 3.5$.**

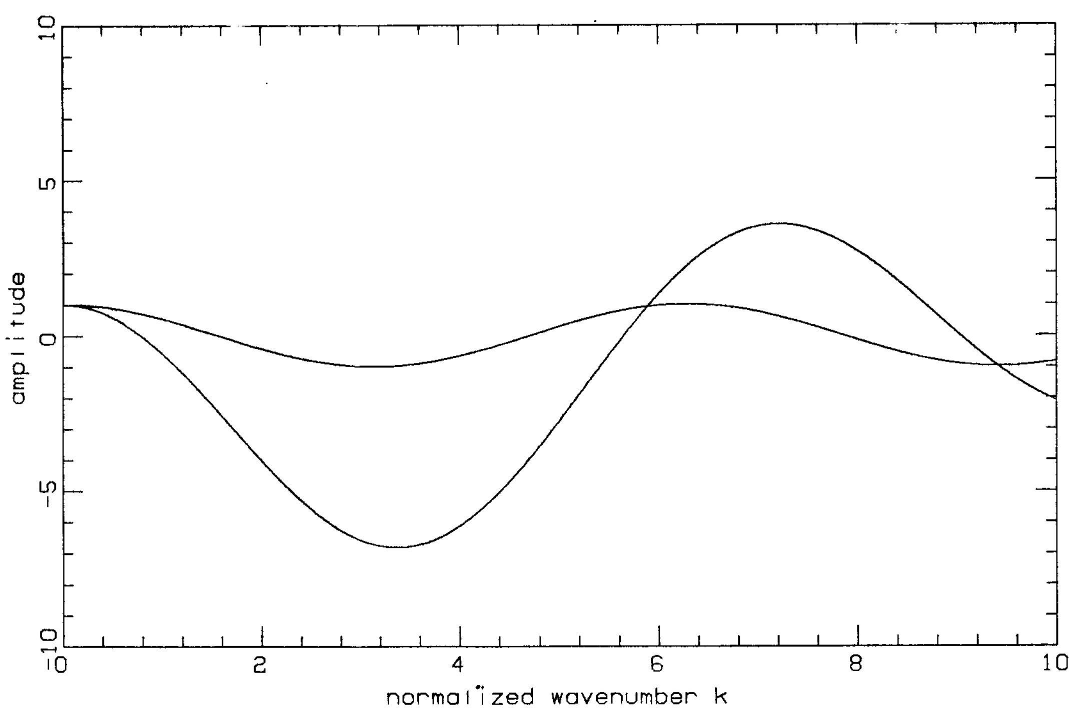
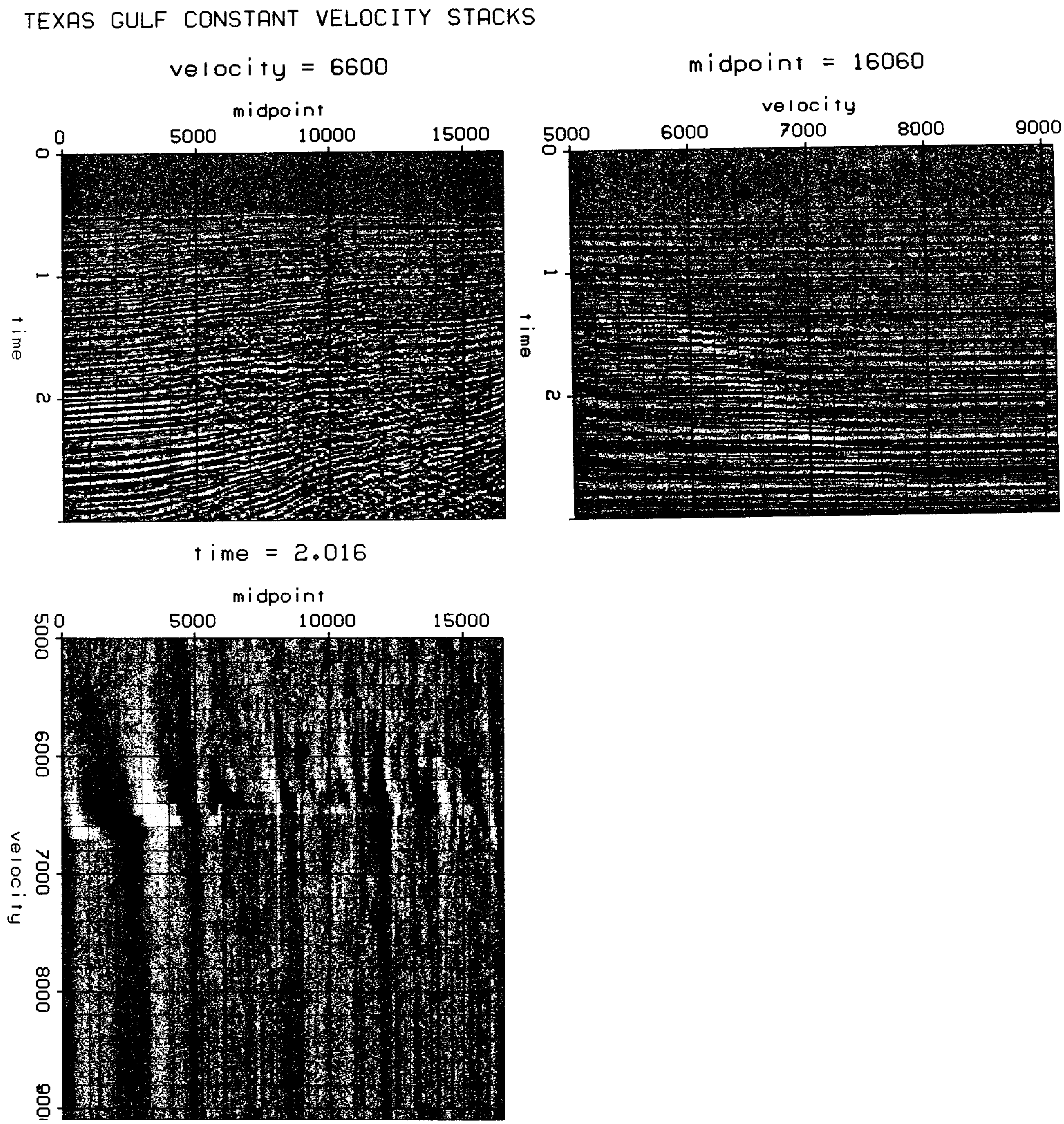



**FIG. 4. Comparison of $\frac{\Delta W}{W}$ and $\frac{\Delta T}{T}$ (EQs 18 and 21 resp).**


TEXAS GULF CONSTANT VELOCITY STACKS
velocity = 6600
midpoint
0
5000
10000
15000
time
0
1
2
midpoint = 16060
velocity
5000
6000
7000
8000
9000
time
0
1
2
time = 2.016
midpoint
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velocity
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