"Sideways Continuation" of Dispersive Waves

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In SEP-25, George McMechan and Mat Yedlin describe a method for analyzing the phase velocity of dispersive waves. In this paper I will describe a method of backing out dispersion, in order to see the spatial distribution of sources. Why is this useful? Suppose we have the situation shown in Fig. 1. Fig. 1 shows a map view of a possible source-geophone configuration. The area being surveyed is shallow marine. This type of area is conducive to the formation of Love waves. Now further suppose, as shown in Fig. 1, that there is an island somewhere off to the side that can reflect the Love waves. Now there are two sources: the actual source, and an apparent source which is actually a ghost reflection off the island.

Fig. 2 shows an actual Love wave recorded by Western Geophysical somewhere off the coast of Florida. (This same data was analyzed in McMechan's and Yedlin's article.) Is this a simple Love wave, or is there a ghost from an off-axis reflector? It is not possible to say for sure, just looking at the plot. You might think of applying some sort of deconvolution to the Love wave, in order to remove the effects of dispersion and get a better look at any ghost that may be present. This won't work very well, however. Think about it: The amount of dispersion is a function of distance from source (or apparent source) to receiver. By deconvolving the trace, you have removed the effects of dispersion for a wave that has traveled the distance between the source and receiver. You have not, however, eliminated the effects of dispersion caused by the fact that the apparent source is farther away from the receiver than the actual source is. In other words, when you deconvolve, you succeed in turning the signal from the source back into a spike, but the signal from the apparent source remains a dispersed wave.

As it turns out, there is a way to back out dispersion from multiple sources at varying distances. In order to do this, however, we must make certain assumptions:

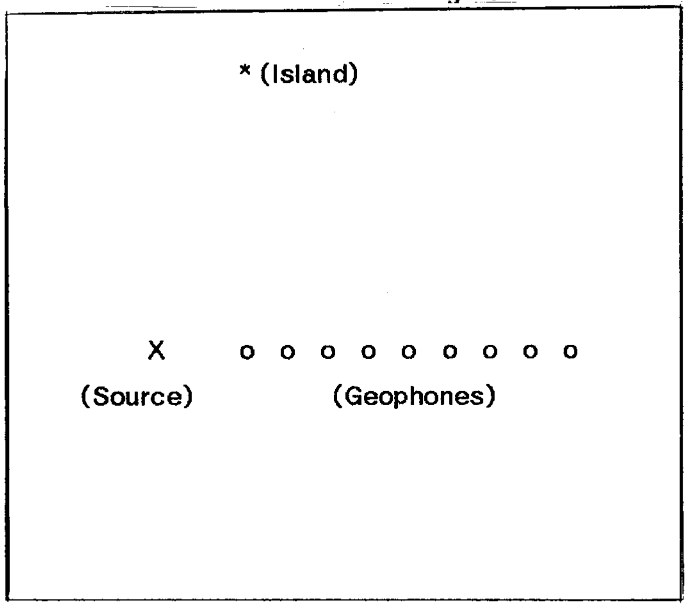

FIG. 1. Map view of a hypothetical marine survey. The source will produce dispersive waves. These waves will bounce off the island, so that the geophones will pick up both the original dispersive wave and its reflection off of the island.

1) We must assume that all the sources went off at the same time, and that they all had the same shape wavelet. This is no problem in the model that I have presented, since of course the source and apparent source are identical.

2) We must assume that velocity is spatially invariant, so that we can apply the McMechan-Yedlin method of finding the phase dispersion curve in the first place, and so that we can back the dispersion out properly.

3) We must assume that one of the sources is much stronger than the rest, so that the McMechan-Yedlin method will work, since this method assumes that there is only a single source.

Suppose these assumptions are valid. Now to begin, we will look at the case of a single source and a single receiver. For this case, the trace seen by the receiver has the form:

$$U(t) = \int_{-\infty}^{\infty} A(\omega) e^{ik(\omega)l} e^{-i\omega t}\, d\omega \tag{1}$$

$U(t)$ is the trace as a function of time, $A(\omega)$ is the source wavelet in the frequency domain, and l is the distance between source and receiver. $k(\omega)$ is the wave number, which is related to phase velocity by the relation $k(\omega)=\frac{\omega}{v(\omega)}$, where $v(\omega)$ is the phase velocity as a function of frequency ω.

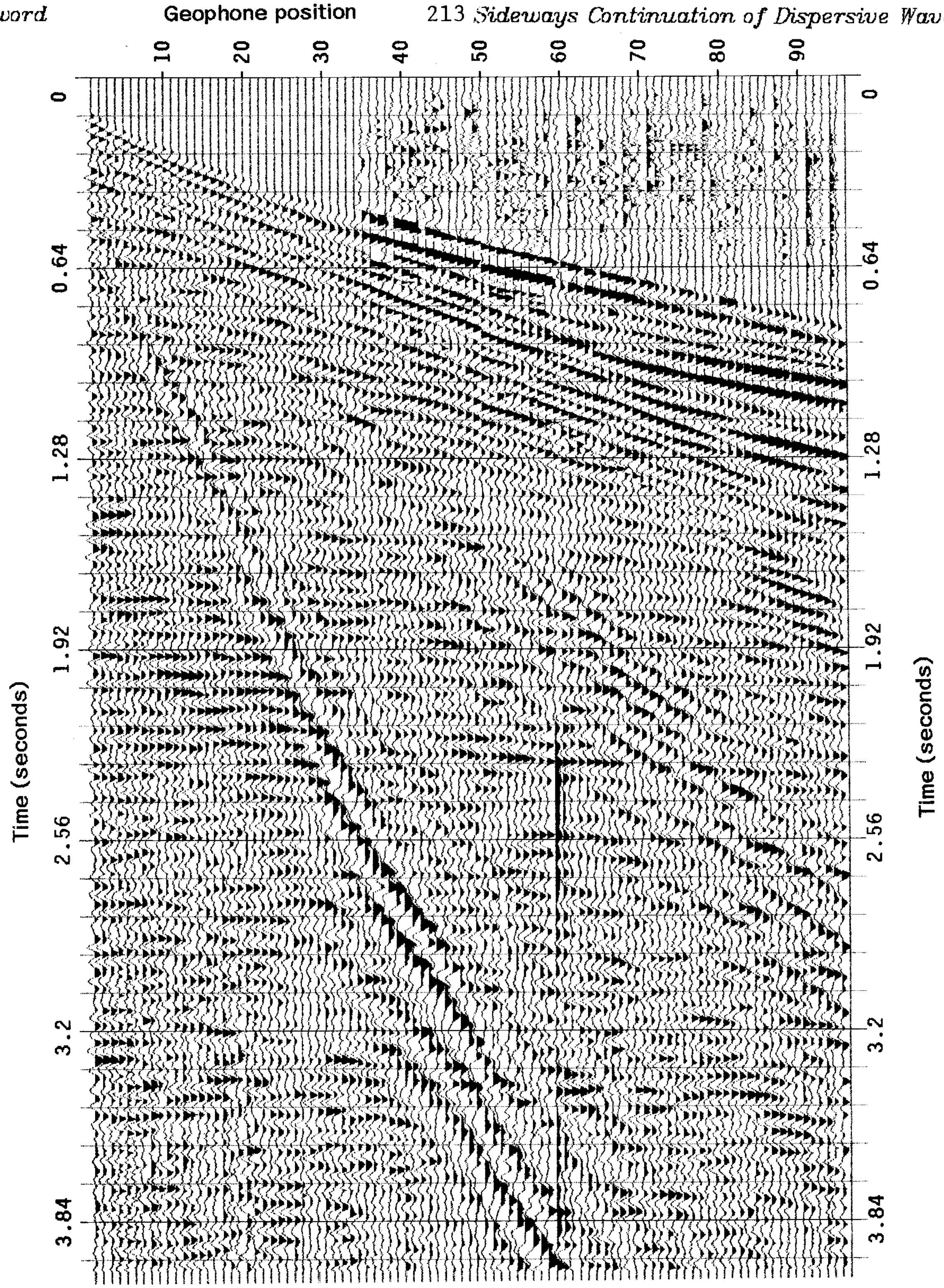

FIG. 2. Love wave recorded off the coast of Florida by Western Geophysical. It is difficult to see any hints of a ghost reflection of the type suggested in Fig. 1. The units on the horizontal axis are arbitrary.

Now suppose that there are many sources, all with the same source wavelet, and all going off at the same time. Then we could write

$$U(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(\omega) V(l) e^{ik(\omega)l} e^{-i\omega t} d\omega dl \qquad (2)$$

This equation is about the same as (1), except for the integration over l and the function $V(l)$. The function $V(l)$ represents the intensity of the multiple sources as a function of l, the distance from a particular source to the receiver. Thus, for the case of a single source with a ghost,

$$V(l) = V_0 \delta(l - l_0) + V_1 \delta(l - l_1)$$

In this case, V_0 and V_1 are the intensity of the shot and the apparent shot respectively, while l_0 and l_1 are their respective distances to the receiver.

So here, then, is what we need to do: we are given $U(t)$, and thanks to the McMechan-Yedlin method, we can find $k(\omega)$ (since $k(\omega) = \omega p(\omega)$). We will assume for the moment that we know $A(\omega)$, although we will see that it doesn't make too much difference if we don't. Given all of this information, we want to solve (2) for $V(l)$. This can be done as follows, using a technique somewhat analogous to the 'Stolt stretch'.

Let us take (2) into the frequency domain and divide both sides by $A(\omega)$, so that

$$\frac{U(\omega)}{A(\omega)} = \int_{-\infty}^{\infty} V(l) e^{ik(\omega)l} dl \qquad (3)$$

Now, instead of using $K(\omega)$, with k as a function of ω, let us invert to get ω as a function of k: $\omega = \omega(k)$. Then $k(\omega) = k(\omega(k))) = k$, so we can write (3) as

$$\frac{U(\omega(k))}{A(\omega(k))} = \int_{-\infty}^{\infty} V(l) e^{ikl} dl \qquad (4)$$

Taking the inverse Fourier transform of both sides, we get

$$V(l) = \int_{-\infty}^{\infty} e^{-ikl} \frac{U(\omega(k))}{A(\omega(k))} dk \qquad (5)$$

That was pretty simple. This means that we simply take U as a function of ω, which we know, find values for ω as a function of k, which we can do, and interpolate U using those values. Doing this will give us $V(l)$ convolved with the stretched and transformed version of $A(\omega)$. A, then, merely represents a term that we can convolve out at our convenience.

We have now seen how we can get $v(l)$ for one trace. But remember that in Fig. 1, we had a number of receivers. What kind of result should we get? Fig. 3 shows one possibility, if we assume that the situation shown in Fig. 1 holds.

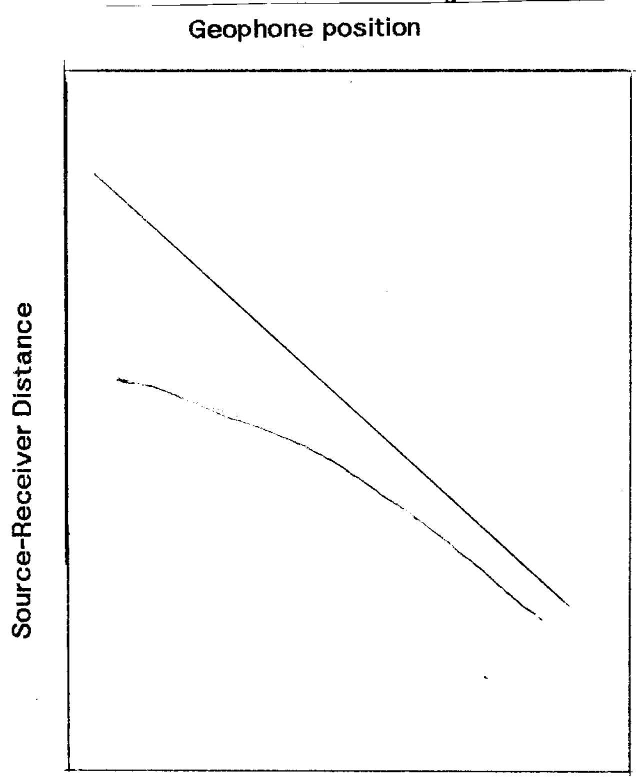

FIG. 3. Expected result of processing if ghosting is present. The straight line represents the direct arrival, while the hyperbolic line represents the ghost.

Especially note the axes, since they are not necessarily something you have seen before: distance from source to receiver as a function of receiver position. It is not too difficult to convince yourself that the straight diagonal line is what you would expect from a single in-line source. The curved line is then the trace caused by the off-axis reflector. Remember that this curved line represents the total distance from the apparent source to the receiver, so this line actually represents (distance from source to reflector) + (distance from reflector to receiver).

Now that we have seen the theory, what kind of results do we get in practice? Fig. 4 shows the answer. I used the McMechan-Yedlin method to analyze the dispersive wave in Fig. 2, and then applied the appropriate "sideways continuation" to the data.

There indeed appears to be some sort of ghost. Where it comes from, we of course don't really know. In fact, we don't really know if it is indeed a ghost, or if it is simply some sort of artifact. The problem is that we don't seem to have any other data sets that contain dispersive waves, so we have nothing with which to compare the current results. While we now see that the method I have described is at least not fatally flawed in some way, there is not much more we can say until we get more data sets that we can analyze and compare.

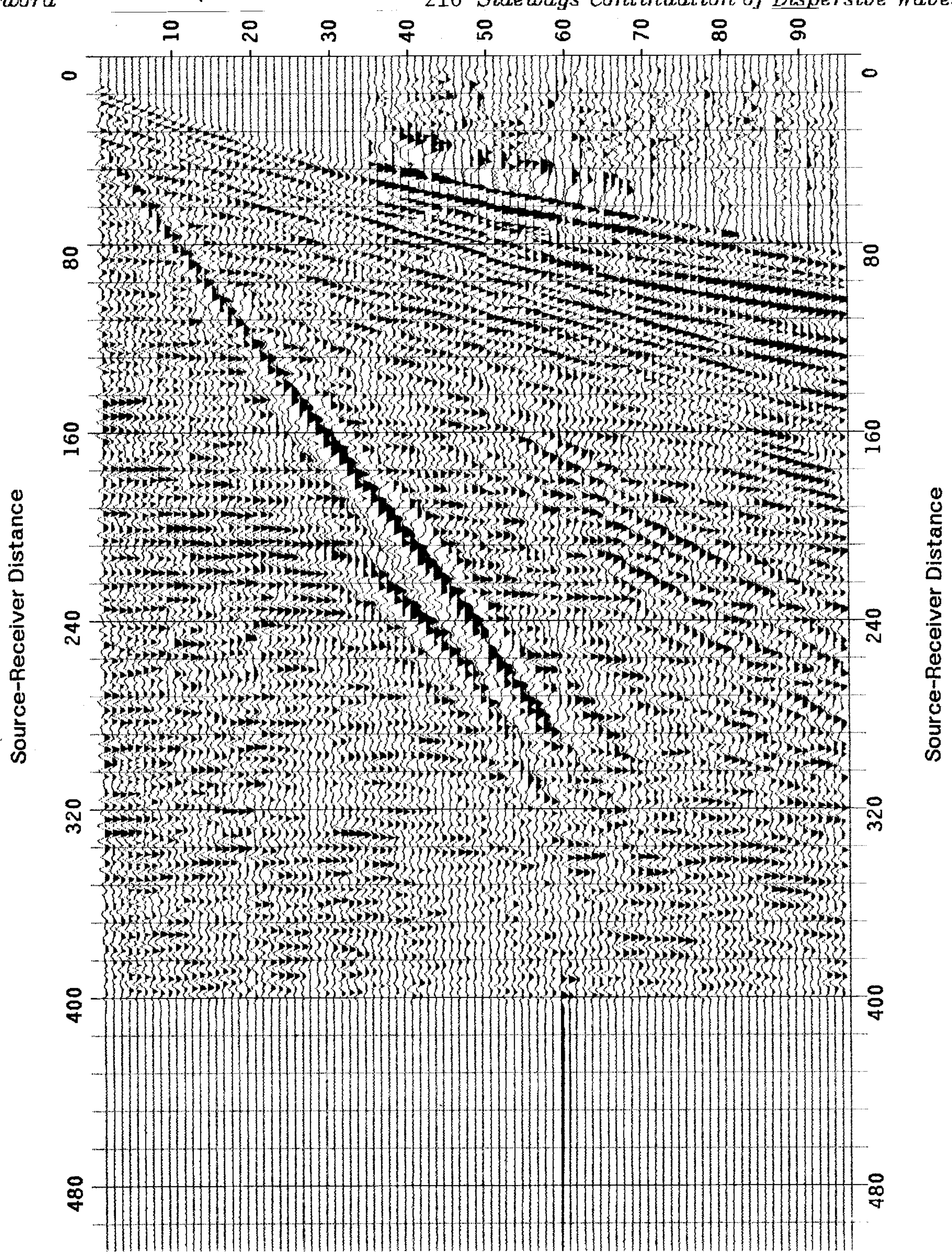

FIG. 4. The results of processing the data in Fig. 2. The apparent ghost reflection is clearly visible. The units on the horizontal and vertical axes are arbitrary.

ACKNOWLEDGMENTS

The idea of looking for distributed sources in the first place, as well as the suggestion that some sort of stretch and integration might be the best way to go about it, are both due to Professor Claerbout.

REFERENCES

McMechan, G.A., and M. Yedlin, 1980, Analysis of Dispersive Waves by Wave-Field Transformation: SEP-25, pp. 101-114.

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55. Given

1) An apple and a banana will balance a orange.
2) An apple will balance a banana and a pear.
3) Two oranges will balance three pears.

What will balance a single apple?

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56. Ten students enroled in a course in number theory. In order to remember their names more easily, the professor seated them in the order shown below. Can you determine the system that he used?

Don Edwards	Jessi Xander
Robert Worden	Rose Ventnor
Edith Reed	Leigh Thompson
Rolf Oursler	Toni Nesbit
Jeff Ives	Pete Norris

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57. A secretary types four letters to four people and addresses the four envelopes. If she inserts the letters at random, each in a different envelope, what is the probability that exactly three letters will go in the right envelopes?

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58. A mathematician decided to make sporting proposition with his son. "I have," said the father, "ten crisp new ten-dollar bills and ten crisp new one-dollar bills. You may divide them any way you please into sets. We'll put one set into hat A, the other set in hat B. Then I'll mix the contents of each hat and put one hat on the right and one on the left side of the mantel. You pick either hat at random, then reach into that hat and take out one bill. If it's a ten, you may keep it. Otherwise, you will have to mow the lawn for a month."

The boy agreed. How should he divide the 20 bills in order to maximize the probability of drawing a ten-dollar bill, and what will that probability be?