Modified Radial Coordinates for RMS Velocity Estimation

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Abstract

In Snell midpoint coordinates, velocity is a function of the (h, τ) coordinates of a reference Snell wave. Redefining the expression of radial space as a function of Snell wave traveltime gives a transformation to (v_{RMS}, t_0) space. This transformation is useful when energy has been stacked to the reference Snell wave arrival coordinates. In this space, several (h, τ) images with different p_0 can be superposed.

1. Snell Coordinates Velocity Estimation.

In Snell midpoint coordinates (Claerbout, SEP 15, p. 81-87) we can measure velocity directly in CMP gathers. Velocity is a function of the arrival coordinates (h, τ) of a reference Snell wave. h is half–offset and τ the two–way traveltime of the vertical component of a reference wavefront with fixed Snell's parameter p_0. An imaging step stacks the energy to these reference Snell wave coordinates, then the RMS velocity is obtained measuring slopes about the origin of the focused energy positions. This constitutes the *LMO* method. The relationship between velocity and slopes $2h/\tau$ is given by:

$$v_{RMS}^{-2} = p_0 \left[p_0 + \frac{\tau}{2h} \right] \tag{1}$$

From this equation it follows there is a one–to–one correspondence between RMS velocities and slopes $2h/\tau$. Exploiting this feature we modify radial coordinates so a mapping to v_{RMS} velocity space is possible.

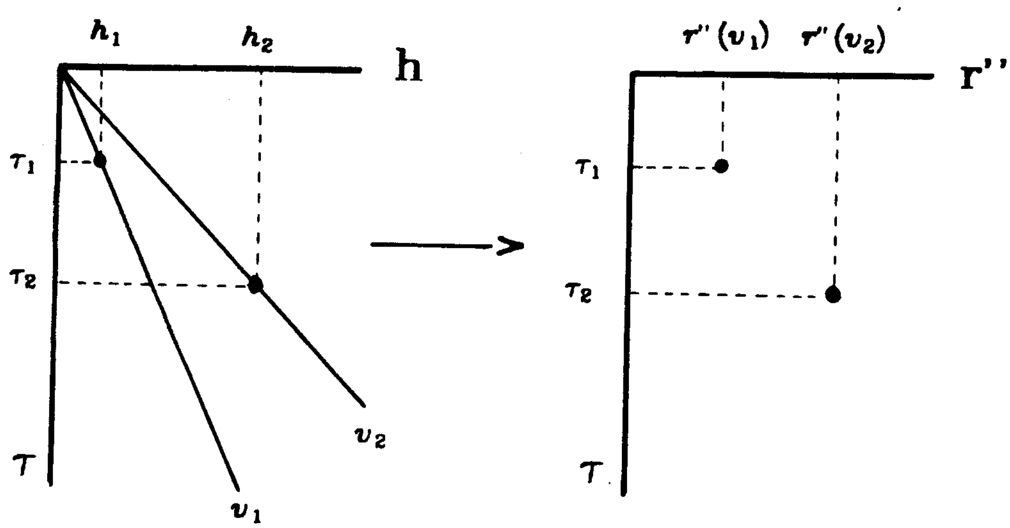

FIG. 1. Transformation to RMS velocity space. (a) Imaged Snell (h,τ) coordinates for a fixed reference Snell's parameter p_0. In these coordinates we can estimate RMS velocity by measuring slopes about the origin of imaged focused positions. (b) Transformation to (v_{RMS},τ) space using equations (5) and (6).

2. Modified Radial Coordinates.

The radial coordinate r is defined at the earth surface by

$$r \equiv \frac{2h}{v\,t} \tag{2}$$

For our purposes a useful definition of the radial coordinate, keeping in mind Snell waves, is given by

$$r'' \equiv \frac{2h}{t'} \tag{3}$$

Where t' is the slant retarded time

$$t' = t - 2(g - s) + 2\int_0^z \frac{\cos\vartheta}{v(\xi)}\,d\xi \tag{4}$$

Note that this definition of radial coordinate has units of velocity.

In Snell coordinates at the image point we have $t' = \tau$; equations (1) and (4) give

$$\tau'' = \frac{2h}{\tau} = \frac{p_0 v^2}{\sqrt{1 - p_0^2 v^2}} \tag{5}$$

Using this equation we can sample values of τ'' so that the radial axis increases linearly with velocity; from equation (1)

$$v_{RMS} = \frac{1}{\sqrt{p_0(p_0 + 1/\tau'')}} \tag{6}$$

With equations (5) and (6) we can transform from (h,τ) to (τ'',τ) to (v_{RMS},τ) coordinates. This transformation is useful only when data has been stacked about some reference Snell wave arrival coordinates. Only after stack can we measure velocity by identifying focused energy. (Figure 1).

3. RMS Velocity Estimation using several reference Snell waves.

Equation (5) defined a transformation into modified radial space. Suppose we want to image data in Snell midpoint coordinates for several reference Snell waves. Assume there are no multiple reflections. After imaging all events with a given velocity will have the same radial coordinate τ'', but the τ coordinate will depend on the p_0 used during the imaging step. (Figure 2).

To combine several images in (h,τ) space, each for a different reference Snell wave, into a single radial space, the time–depth coordinate should be redefined. We need to refer all Snell traveltimes τ to a single reference τ. Doing this for a vertically propagating Snell wave $(p_0 = 0)$ we have

$$t'' = \int_0^{\tau} \frac{1}{\sqrt{1 - p_0^2 v(\xi)^2}} d\xi \tag{7}$$

or, using v_{RMS}

$$t'' = \frac{\tau}{\sqrt{1 - p_0^2 v_{RMS}^2}} \tag{8}$$

The usefulness of equations (5) and (8) is limited to small reference propagation angles, where Dix's equation is valid.

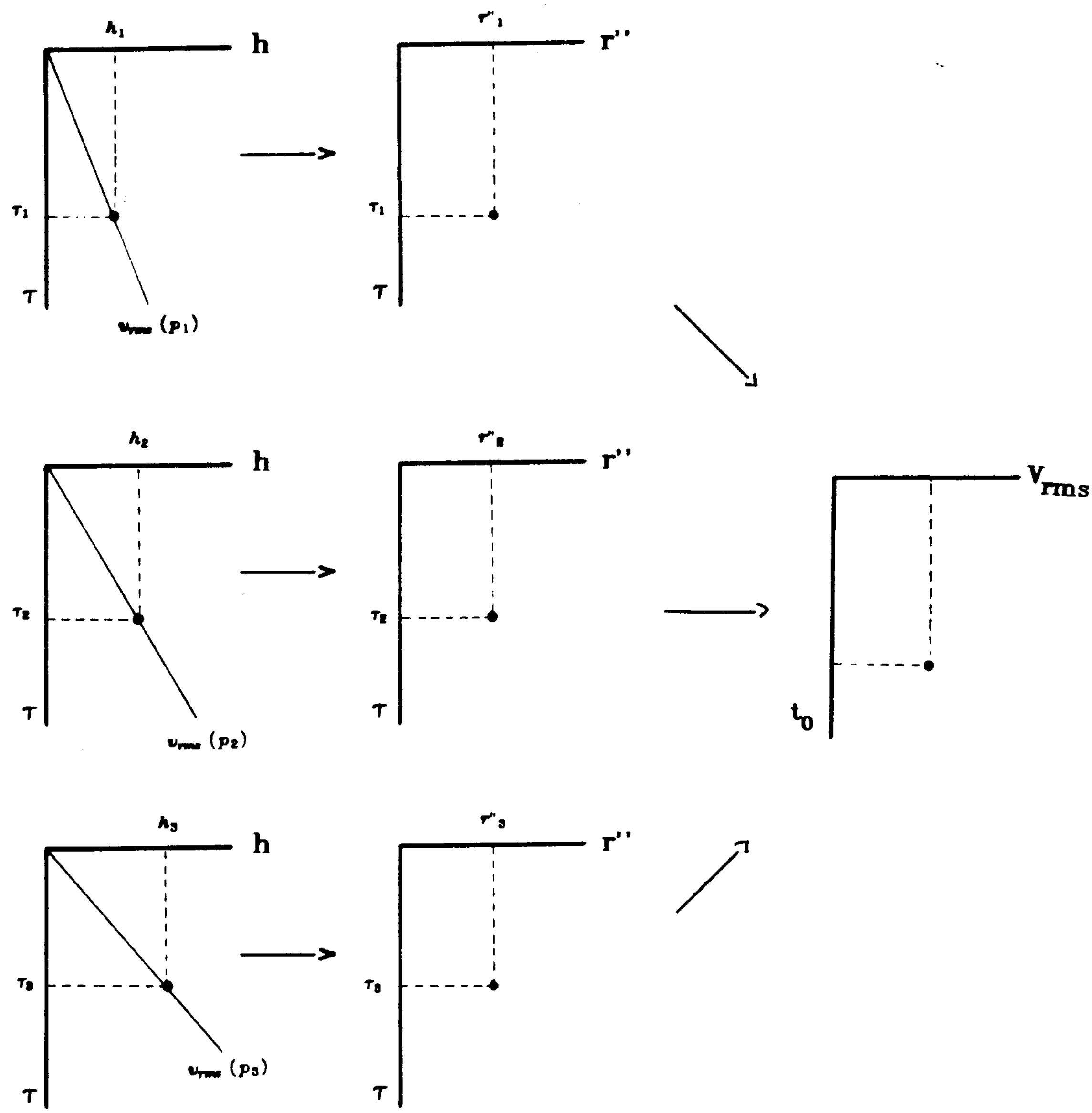

FIG. 2. Transformation to RMS velocity space. This figure shows how we can combine several images in (h, τ) space into a single (v_{RMS}, t_0) RMS velocity – vertical traveltime space.