# The Size of the Region that Forms a Reflected Wave at a Boundary*

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Thanks to the successes in optical and acoustical holography that have been achieved in recent years, exploration seismologists have turned their attention to the possibility of improving interpretation by using the principles of dynamic wave field reconstruction in accordance with the laws of wave seismics. One of the important questions in the wave theory construction of seismic displays is the question of the dimensions of that region of the medium, in the neighborhood of a seismic ray, which substantially takes part in the transmission of the wave process, and in particular, the dimensions of that area of the boundary which forms a reflected wave. There exist theoretical and experimental data [3-5] that show that the form of the reflected signal depends on the properties of the boundary within an extensive area surrounding the point of specular reflection. Similar results can be obtained using strict wave theory.

The dimensions of that region of the medium which takes part in the transmission of waves can be obtained on the basis of the solution of the scalar wave equation in the form of a Kirchoff integral. It is well-known [1, 6] that in the case of a monochromatic wave process, in the form of a vibration at a given point, theoretically the entire unbounded surrounding medium takes part. In this medium, however, it is possible to distinguish an essential (effective) region. The dimensions of this effective region turn out to be larger when the displayed form of the vibrations is claimed to be more precise, or when the wave is longer. The dependence of the dimensions of the effective area on the wavelength makes difficult the traditional transformation to the impulsive regime by means of the Fourier transform -- within one spectrum, for the various frequency components, the dimension of the effective region varies over a wide range.

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Even as in the case of a monochromatic wave field, the dimensions of the effective region where impulsive vibrations are formed can be obtained in the time domain by calculating the Kirchoff integral.

Let the point source $O$ (Fig. 1) at time $t = 0$ radiate an impulsive spherical wave of length $T$:

$$\varphi = \begin{cases} \frac{1}{\rho} f\ (t - \rho / c) & \text{for } 0 < t - \rho / c < T, \\ 0 & \text{for } 0 \geq t - \rho / c \geq T, \end{cases} \tag{1}$$

where $\rho$ is the distance from the source, and $c$ is the velocity of the elastic wave.

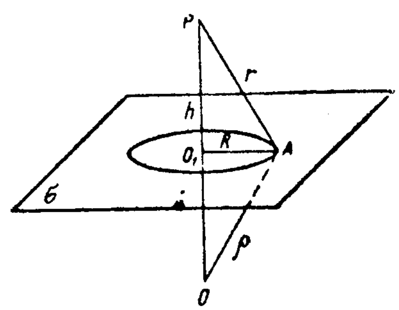


FIG. 1. The effective area of plane of integration $\sigma$ in the case of a transmitted wave.

In accordance with Kirchoff's theory [2], the vibration at an arbitrary point $P$ is given by the integral

$$\varphi_P(t) = \frac{1}{4\pi} \int_\sigma \int \left\{ \frac{1}{r} \left[ \frac{\partial \varphi}{\partial n} \right] + \frac{1}{cr} \left[ \frac{\partial \varphi}{\partial t} \right] \frac{\partial r}{\partial n} - [\varphi] \frac{\partial}{\partial n} \left( \frac{1}{r} \right) \right\} d\sigma; \tag{2}$$

here $\sigma$ is an infinite plane[1] which separates source $O$ and observation point $P$, and on which is evaluated the wave function (1) and its associated derivatives in time and along the normal $n$, and $r$ is the distance from a point on surface $\sigma$ to point $P$.

Function (1) must satisfy the homogeneous wave equation -- that is, it and its first derivatives must be continuous [2]. We will denote by $h$ the perpendicular dropped from point $P$ to $\sigma$ (see Fig. 1), and we will describe plane $\sigma$ by a circle with radius $R = \sqrt{r^2 - h^2}$ which can go to infinity, and with its center at point $O_1$. Then with the substitutions

The fact that the infinite plane in this case can replace a closed area surrounding both the source and the observation point is proven in many works ( *cf*, for example, [1, 2]).

$$\sigma = \pi R^2; \quad d\sigma = 2\pi R\, dR = 2\pi r\, dr$$

equation (2) can be given in any of these three forms:

$$\varphi_P(t) = \frac{1}{4\pi}\int_\sigma\!\!\int \{M\}\, d\sigma = \frac{1}{2}\int_0^\infty \{M\} R\, dR = \frac{1}{2}\int_h^\infty \{M\} r\, dr, \tag{3}$$

the last of which is preferable from the point of view of clarifying the physics involved; here $M$ represents the expression in braces in (2).

Doing the differentiating under the integral in equation (2), we obtain:

$$\varphi_P(t) = \frac{1}{2}\int_h^\infty \left\{ \frac{1}{\rho r}\left[\frac{\partial f(t-\rho/c)}{\partial \rho}\right]\frac{\partial \rho}{\partial n} - \frac{1}{\rho^2 r}\Big[f(t-\rho/c)\Big]\frac{\partial \rho}{\partial n} + \right.$$

$$\left. + \frac{1}{c\rho r}\left[\frac{\partial f(t-\rho/c)}{\partial t}\right]\frac{\partial r}{\partial n} + \frac{1}{\rho r^2}\Big[f(t-\rho/c)\Big]\frac{\partial r}{\partial n}\right\} r\, dr. \tag{4}$$

Let us explain the idea of the square brackets in (2) and (4). For each point on the surface of integration $\sigma$, the values of the wave function and its derivatives are set for the time interval $0 < (t-\rho/c) < T$. These values are determined by the delay of the wave from the source to the point. The integration is carried out for later times $0 < (t-\rho/c-r/c) < T$ when the elementary wave, which is radiated by the given point, reaches the point of observation $P$. The Kirchoff integrals (2) and (4) seemingly connect within themselves two physical processes that are distinct in time: the vibrations at the point of observation $P$, and the preceding vibrations at the points on the surface over which the integral is taken. For example, non-zero values of the function and its derivatives at point $A$ (see Fig. 1) are given for $t > \rho/c$, but non-zero values at this point in the vibration $\varphi_P(t)$ can occur only for $t > (\rho/c + r/c)$. In particular, the differential expressions under the integral (4) are calculated for an argument $(t-\rho/c)$, but upon integration the argument increases to $(t-\rho/c-r/c)$.

The variable of integration $r$ in (4) describes the delays at $P$ in the summed elementary waves from the imaginary sources distributed on $\sigma$. Since the integration proceeds to $r \to \infty$, even elementary waves that are infinitely delayed relative to the beginning of vibration are formally summed. But the length of vibration at the point of observation, and at any other point in the space, is limited, according to (1), and therefore, following [2], it is possible to suppose that in the case of an impulsive wave, integration over an infinite plane takes on a formal character. Apparently, there exists a bounded region, within $\sigma$, over which an integral gives the full vibration $\varphi_P(t)$, and the integral over the remaining part of $\sigma$ is zero. Let us examine a particular case: the point of observation $P$ is symmetrical to source $O$ relative to plane $\sigma$ (see Fig. 1). Then $\rho = r$; $\partial\rho/\partial n = -\partial r/\partial n$; $\partial r/\partial n = \cos(n, r) = h/r$. Since for function (1) it is true that

$$\frac{\partial f(t-\rho/c)}{\partial\rho} = -\frac{1}{c}\frac{\partial f(t-\rho/c)}{\partial t}, \tag{5}$$

integral (4) takes on the form

$$\varphi_P(t) = \int_h^\infty \left\{ \frac{h}{r^3} f\left(t - \frac{2r}{c}\right) + \frac{h}{cr^2} \frac{\partial f\left(t - \frac{2r}{c}\right)}{\partial t} \right\} dr. \tag{6}$$

Since differential equation (5) is satisfied for any argument of the wave function, it is possible to change the differentiation with respect to time in (6) to differentiation with respect to $r$:

$$\frac{\partial f\left(t - \frac{2r}{c}\right)}{\partial r} = -\frac{2}{c} \frac{\partial f\left(t - \frac{2r}{c}\right)}{\partial t};$$

with the result that

$$\varphi_P(t) = \int_h^\infty \left\{ \frac{h}{r^3} f\left(t - \frac{2r}{c}\right) - \frac{h}{2r^2} \frac{\partial f\left(t - \frac{2r}{c}\right)}{\partial r} \right\} dr. \tag{7}$$

Integrating the second component of (7) by parts,

$$\frac{h}{2}\int_h^\infty \frac{1}{r^2} \frac{\partial f\left(t - \frac{2r}{c}\right)}{\partial r} dr =$$

$$= \frac{h}{2r^2} f\left(t - \frac{2r}{c}\right)\Bigg|_h^\infty + \int_h^\infty \frac{h}{r^3} f\left(t - \frac{2r}{c}\right) dr \tag{8}$$

and substituting (8) in (7), we obtain the correct result

$$\varphi_P(t) = -\frac{h}{2r^2} f\left(t - \frac{2r}{c}\right)\Bigg|_h^\infty = \frac{1}{2h} f\left(t - \frac{2h}{c}\right), \tag{9}$$

which, however, does not clarify the physical essence of the phenomena, since it was obtained using an infinite upper limit of integration. The impression is created that all of the infinite plane $\sigma$ participated in the formation of vibration $\varphi_P(t)$. In order to limit the area of the plane, let us look at instantaneous values of $\varphi_P(t_i)$ in integral (7) for several typical times $t_i$.

From what was previously stated about the physical meaning of the Kirchoff integral, it follows that each momentary state of the vibration at the point of observation is determined by the distribution of wave motion on the surface of integration $\sigma$ at a fixed preceding moment of time. In particular, for the geometry given in Fig. 1, when $\rho = r$, a certain distribution of functions on surface $\sigma$. corresponds to time of integration $t_i$. These functions entered into the expression under the integral at time $t_i/2$. This is illustrated in Fig. 2, in which is shown the full vibration (9) which was determined at point $P$ by integral (7). Also shown schematically is the wave situation at plane $\sigma$ which corresponds to several instantaneous values of $\varphi_P(t)$ in this integral at fixed moments of time $t_0$, $t_1$, $t_2$, $t_n$.

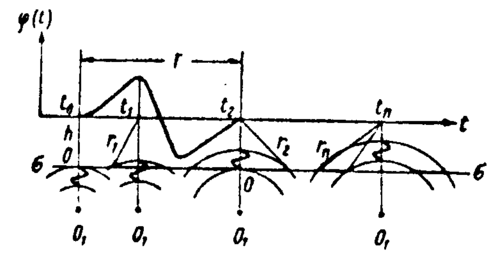

FIG. 2. An illustration of particular moments in the calculation of instantaneous values of integral (7).

Before time $t_0 = 2h/c$, the vibration at point $P$ disappears, since for all time $t < h/c$ the function under the integral, which is distributed on $\sigma$, is identically equal to zero, according to (1).

For $t_0 = 2h/c$, which is characterized by the beginning of vibrations at point $P$, the integral is still identically zero, because at that time the agitation on surface $\sigma$, except at one point $O$, is still non-existent (see Fig. 2). The integral over a zero-valued surface is zero-valued.

The instantaneous value of $\varphi_P(t_1)$ is evaluated over the vibrations of the points on surface $\sigma$, points which lie on the circle of a cone which is formed by $r_1 = ct_1/2$. We will split integral (7) into two parts: that within the described circle, and that outside the circle on plane $\sigma$:

$$\varphi_P(t) = \int_h^{r_1} \{N\}\, dr + \int_{r_1}^{\infty} \{N\}\, dr, \tag{7a}$$

where $N$ is the expression within the braces in (7).

The second of these integrals is identically equal to zero, since on the part of the plane over which it is evaluated, the function under the integral is identically zero until $t_1/2 = r_1/c$. To evaluate the first integral (see formulas (7-9)), in place of infinity, the upper limit of (9) is now $r_1 = ct_1/2$, and $t$ must be replaced by $t_1$. The substitution of the upper limit into (9) transforms the argument of the function to zero; that is, its value is taken at the first moment of vibration. But, by the strength of (1), $f(0) = 0$. The precise instantaneous value is provided by the substitution of the lower limit of integration:

$$\varphi_P(t_1) = \frac{1}{2h}\left(t_1 - \frac{2h}{c}\right). \tag{9a}$$

An analogous situation exists for all instantaneous values of $\varphi_P(t_i)$ up until the time when the relation $2h/c \le t \le 2h/c + T$ is satisfied, where $T$ is the length of the impulse (1).

At time $t_2 = 2r_2/c = 2h/c + T$, the vibration at point $P$, according to (1), ceases. Indeed, up until time $t_2/2$ on the plane of integration, points are involved in the vibration which lie on the circle (see Fig. 2) at the base of the cone formed by $r_2 = ct_2/2 = h + cT/2$. The instantaneous value of function $\varphi_P(t_2)$ is obtained from expression (7a) with the condition that the limit of integration $r_1$ be replaced by $r_2$. Then the second integral in (7a), as before, is identically zero, since the expression under the integral is identically zero. In the evaluation of the first integral, the substitution of the upper limit $r_2$ into (9) results in the argument of the function becoming zero, while with the substitution of the lower limit the function becomes equal to $(t_2 - 2h/c)$; but since $t_2 = 2h/c + T$, then according to (1) $f(T) = 0$, and consequently, $\varphi_P(t_2) = 0$.

At all times $t_i > 2h/c + T$, the vibration at the point of observation, according to (1), ceases, and the value of integral (7) must be zero. At the corresponding time $t_i/2$ the values of the functions under the integrals on plane $\sigma$ are agitated over a washer-like disk, closer to the center of which the vibration has already ceased, but to the outside of which the vibration has not yet begun. If we define $r_n$ to be that which forms the outer cone (see Fig. 2), then the length of that which forms the inner cone is $r_n - cT/2$. Integral (7) can conveniently be broken into three parts: the hole of the washer-shaped disk, the disk itself, and the rest of plane $\sigma$:

$$\varphi_P(t_n) = \int_h^{r_n - cT/2} \{N\}\, dr + \int_{r_n - cT/2}^{r_n} \{N\}\, dr + \int_{r_n}^{\infty} \{N\}\, dr. \tag{7b}$$

The first and third integrals of the right part of (7b) are identically zero, since the expressions under those integrals are identically zero. Thanks to its limits, the second integral in (7b) always turns out to be over the values of function (1) from the beginning time of vibration $f(0)$ to the end time of vibration $f(T)$. Since these values, according to (1), are always zero, the result of integration is zero as well. Formally, the calculation of the second integral in (7b) results in the substitution of the limits in (9) with account being taken of the fact, that $t_n = 2r_n / c$:

$$-\frac{h}{2r^2} f\left(t_n - \frac{2r}{c}\right)\Bigg|_{r_n - cT/2}^{r_n} = -\frac{h}{2_n^2} f(0) +$$

$$+ \frac{h}{2(r_n - cT/2)^2} f(T) = 0.$$

Consequently, at any time

$$t_n > 2h/c + T, \quad \varphi_P(t_n) = 0.$$

The preceding analysis indicates that in order to obtain the exact value of the function which describes the impulsive vibration at point $P$, it is sufficient to limit the integration to only a part of plane $\sigma$, the dimensions of which are determined only by the length $cT$ of the impulse and the distance $2h$ of the observation point from the source[1]. In the example that has been examined, the essential area on plane $\sigma$ is a circle at the base of the cone formed by $r_2 = h + ct/2$, the radius of which (see Fig. 2) is

$$R = \sqrt{r_2^2 - h^2} = \sqrt{cT(h + cT/4)}; \tag{10}$$

when the distance $2h$ is much larger than half the length of the impulse ($2h \gg \sqrt{cTh}$),

$$R \approx \sqrt{cTh}. \tag{10a}$$

A transfer from the case of the free propagation of wave impulses to the case of reflection from a plane layer can be accomplished by introducing the concept of an imaginary source [2]. Let us suppose that at point $P$ (Fig. 3) we combine the source of the impulsive wave (1) and the point of observation. Here $\sigma$ is a reflecting plane whose reflection coefficient does not vary with angle of incidence, and $O_1$ is the imaginary source, which is symmetrical to $P$ with respect to plane $\sigma$.

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Limiting the area of integration is being considered here, but not limiting plane $\sigma$ itself; if $\sigma$ were limited, diffraction events would be observed that are not discussed here.

The field of the wave reflected from $\sigma$ coincides with the field of the imaginary source $O_1$ and is described in equation (1). The result of integrating the field of the imaginary source over plane $\sigma$ (taking into account the reflection coefficient) will be the same as in the previously analyzed case of a freely propagating wave. On this basis it is possible to say that in the formation of a reflected wave that is detected at $P$, only a part of the boundary $\sigma$, lying within a radius $R$ in (10), takes part.

The generalization of this result to the case where the source and the point of observation $P'$ do not coincide, but lie on one plane (see Fig. 3), leads to the result that the area of the boundary that forms the wave is an ellipse whose major axis is oriented along the projection of the line connecting the source and the receiver point:

$$R_1 \approx \frac{1}{\sin\psi}\left[cT\frac{h}{\sin\psi}\right]^{\frac{1}{2}}. \tag{11}$$

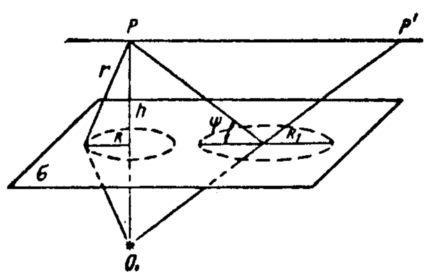


FIG. 3. The effective areas of reflecting boundary $\sigma$ in the cases of normal and diagonal incidence of a wave formed at $P$.

The estimates obtained have a clear physical explanation. The leading edge of the reflected impulse is formed in the neighborhood of the point of specular reflection. For the formation of the following phases of the vibration at later times, the longer it has been since the beginning of vibration, the larger an area of the boundary takes part. The shorter the signal falling on the boundary, the smaller the effective zone which is concentrated on the point of specular reflection as $T \to 0$, and therefore, the closer the form of the wave is to that predicted by geometrical seismics. The longer the signal, the larger an area of the boundary forms vibrations behind the front of the reflected wave.

Field seismic reflection data agree with the results obtained. It is well known that even in favorable seismo-geological conditions the beginning phases of vibration of reflected waves are more stable than the end phases, and this causes their predominant use for phase correlation. The changeability of the phase relations and amplitudes of later phases on real seismograms can be caused by changes in the boundary which are important for interpretation, and the idea that reflections do not originate from one point can prove very useful. As evidence of this we can cite the success of interpretation of data by the method of controlled directional receptivity (CDR) [5], which is based on the idea that in complex areas, a reflected wave is formed over a wide area of the boundary.

The central result of this work is that a reflected impulse, detected at one point of the profile, contains within itself information about a wide area of the plane of the reflecting boundary. Improved methods of seismic data processing should be directed towards extracting this information.

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# 地球物理数据处理基础

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