Improved Discrimination of Small Events in Stacking by Extrapolation

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Abstract

A method is introduced to improve stacking, not by merely differentially weighting each individual trace in the gather before stack, but by laterally extending the gather and then stacking. The central part of the extended gather is unaltered (apart from muting) and each trace contributes unit weight to the stack. By recognizing that a sharp truncation of the recorded wavefield at the edges of the gather is a prime source of stacking error, the extension method is seen to be an attractive solution to this problem. Yet there are some serious deficiencies with the extrapolation used here: mainly that the extrapolation assumes stationarity of the data along the time axis.

Stacking Problems due to Reflector Truncations on a Gather

Stacking may be viewed as dip-filtering in which only the events with the lowest dips on the gather are extracted. These desired events would be the moveout-corrected primaries, in contrast to multiples or other undesired events that have a substantial residual moveout and are meant to be suppressed by the stacking process. The action of moveout-correcting the trace gather turns a time and space variable dip-filtering operation into a time and space independent operation. Alternately, stacking may be thought of as a dip-spectral estimation process, where (after moveout correction) the zero-dip component is to be obtained. In fact, the same operation could be performed if the gather were 2D Fourier transformed, the k_h=0 energy extracted, and then 2D inverse transformed.

With this analogy in mind, the effect of having only a small number of traces in the common-midpoint gather is the same effect that a finite sample length has on attempts to perform a spectral estimate on it. The finite window contributes an artificial broadening of the spectrum by the convolution of a sinc envelope onto it. In the case of stacking a gather, events that have some residual moveout will yet contribute to the stack solely because of this truncation problem. The finite window here is the few number of traces available for stacking. Upon stacking, the dipping event to be discriminated against destructively interferes with itself in the body of the gather, but at the edges of the gather the wavelet making up the dipping event is "unmatched", so to speak, and appears on the stack. This is especially visible when a multiple event, say, is much stronger on the section than the primary event.

As an example, consider the following case. Let $u(h,t)$ be a group of traces indexed by time t and offset h.

$$u(h,t) \;=\; \varepsilon f(t-t_o) + g(t-t_o-hk_o)$$

where $\varepsilon < 1$, f is a wavelet on a flat event at $t = t_o$, and g is another wavelet superimposed on a dipping event with slope k_o. Waveforms f and g are about the same magnitude, so this models, for example, a weak primary in the presence of a strong multiple. In the frequency (ω) domain,

$$u(h,\omega) \;=\; \varepsilon f(\omega)e^{i\omega t_o} + g(\omega)e^{i\omega(t_o+hk_o)}$$

Now an ideal stack $s(\omega)$ in the frequency domain should produce the flat event $\varepsilon f(\omega)$:

$$\begin{aligned} s(\omega) &= \lim_{h_m\to\infty}\frac{1}{2h_m}\sum_{h=-h_m}^{h_m} u(h,\omega) \\ &= \varepsilon f(\omega)e^{i\omega t_o} + g(\omega)e^{i\omega t_o}\lim_{h_m\to\infty}\frac{1}{2h_m}\sum_{h=-h_m}^{h_m} e^{i\omega hk_o} \\ &= \varepsilon f(\omega)e^{i\omega t_o} + g(\omega)e^{i\omega t_o}\cdot\delta(\omega k_o) \qquad (1) \end{aligned}$$

and if $g(\omega)$ has no zero frequency component, the second term above is zero, since dip $k_o \neq 0$.

Now make the comparison with an actual stack $s_a(\omega)$ where h is summed from $-h_m$ to h_m:

$$\begin{aligned} s_a(\omega) &= \frac{1}{2h_m}\sum_{h=-h_m}^{h_m} u(h,\omega) \\ &= \varepsilon f(\omega)e^{i\omega t_o} + g(\omega)e^{i\omega t_o}\cdot\mathrm{sinc}(\omega k_o h_m) \qquad (2) \end{aligned}$$

That is, a sinc function replaces the delta function as a multiplier in the term containing g.

The implications of this are the following. The term with g now contributes to the stack and, because ε is small, can dominate the stack. It can be minimized by increasing h_m, the number of traces in the stack, or by increasing the dip k_0. But if $g(\omega)$ has considerable low frequency content, the central lobe of the sinc function will contribute by far most of the energy to the stack. Therefore, assume that $g(\omega)$ is restricted in frequency.

The higher lobes of the sinc are the ones that make a contribution to the stack, and depending on the values of k_0, h_m, and the waveform g, will have a wide fluctuation in absolute value between zero and the envelope of the sinc. An estimate of the energy contributed to the stack by the g term can be made by taking the root mean square area under the envelope of sinc($\omega k_0 h_m$) multiplied by $g(\omega)$, since the sinc function is assumed to vary rapidly with respect to $g(\omega)$:

$$\text{Stacking error} \ = \ \left[\sum_{\omega} \left(\frac{g(\omega)}{\omega k_0 h_m} \right)^2 \right]^{1/2}$$

so the stacking error falls off approximately as the inverse of $k_0 h_m$. Increasing either of these parameters improves the stack, but only by an inverse first power.

Let us now see an illustration of the effect of gather truncation on the stacking of some real data. Figure 1 displays two common shot gathers acquired by GECO in the Barents Sea. The gathers are not adjacent to each other but are from shots spaced 5 kilometers apart. Obviously this data has a serious water bottom multiple problem. Yet on close examination there can be seen faint primaries in the gathers. Figure 2 shows the same two gathers moveout-corrected with the velocity given in the table on the figure. Flat events can be seen at 1.1 sec, 1.8 sec, 2.5 sec, 3.0 sec, 3.8 sec, and 5.0 sec. Some of these events are undoubtably peglegs off the sea floor, but for the remainder of this paper the following assumptions shall be made. First, the events flattened by the velocity function of figure 2 are assumed "real" and it is desired that they be stacked in. The objective here is to enhance weak events in the presence of stronger ones, rather than a complete multiple removal process. Second, because of the flat sea floor and relatively gentle dips present in the acquisition area, these shot gathers will be assumed to be common midpoint gathers. With respect to the goals of this paper, this assumption does no harm.

Figure 3 shows a stack of 50 adjacent corrected gathers, including those shown in figure 2. To the right of the stack is plotted the rightmost gather used in the stack. The primary events mentioned above have stacked rather poorly compared to the multiples. On this section, primary events all have a shallow dip component up to the left, whereas the sea floor is very flat. The contribution that the edge traces have to the stack is large, as can be seen by comparing the stack to the adjacent gather. Waveforms seen only on the far trace seem to dominate the stack. Needless to say, the near-offset traces of the gather contribute just as much to the stack. Therefore a raw, unweighted stack on this particular data set is a poor process to use if the weak primaries are meant to be enhanced. The next section shall propose some improvements to stacking to remedy this.

Extrapolating Gathers

There are two possible remedies to the truncation problem described in the last section. The first is to weight the traces independently: large weights for the middle traces of the gather, and small weights for the outside traces. Again make the analogy with spectral estimation: just as applying a Gaussian envelope to a truncated time series improves the resolution in the frequency domain, the same means will improve the discrimination of the primaries from the dipping multiples in our case. Some stacking methods, among them Western's Optimum Weight CDP Stacking method, use such an approach. A set of optimum weights are found to apply to the traces before stack. The weights obtained usually greatly minimize the contribution of the outermost traces on each side of the gather.

A second remedy is to create a reasonable extrapolation laterally off the ends of the gather, allowing it to taper to zero at large offsets. This is analogous, in the spectral estimation problem, to performing an extrapolation of the truncated time series by means of a prediction error filter, and using the expanded time series to find its spectrum. This paper busies itself with the extrapolation approach.

In the method which we shall use, the relative errors in stacking with or without extrapolation should be kept in mind. The class of gathers that are of interest to us shall be the same class described in the first section: common midpoint gathers on which there are weak primaries among strong multiples. Let us make some heuristic definitions of relative event strengths. First, let e_p be a measure of the strength of the primary reflection, and e_m the strength of the multiple. Let now e_{tp} be a measure of the error introduced in the stack by truncation of the primary, and e_{tm} the error corresponding to the truncation of the multiple. Then $\varepsilon_1 \equiv e_p / e_m < 1$ means the primary is weaker than the multiple. Another assumption will be that the truncation error energy of an event is smaller than the total energy of that event: $\varepsilon_2 \equiv e_{tm} / e_m = e_{tp} / e_p < 1$. The problem introduced by truncation is that even though $\varepsilon_2 < 1$, e_{tm} may be greater than e_p. By stacking at the primary velocity, e_m is eliminated, while e_{tm} remains; e_{tp} also remains, but it is *second order* with respect to e_p or e_{tm}. In the proposed extrapolation, e_{tm} is to be minimized by extrapolating multiple events, but it is not as important to extrapolate the primary, because e_{tp} is a second-order correction.

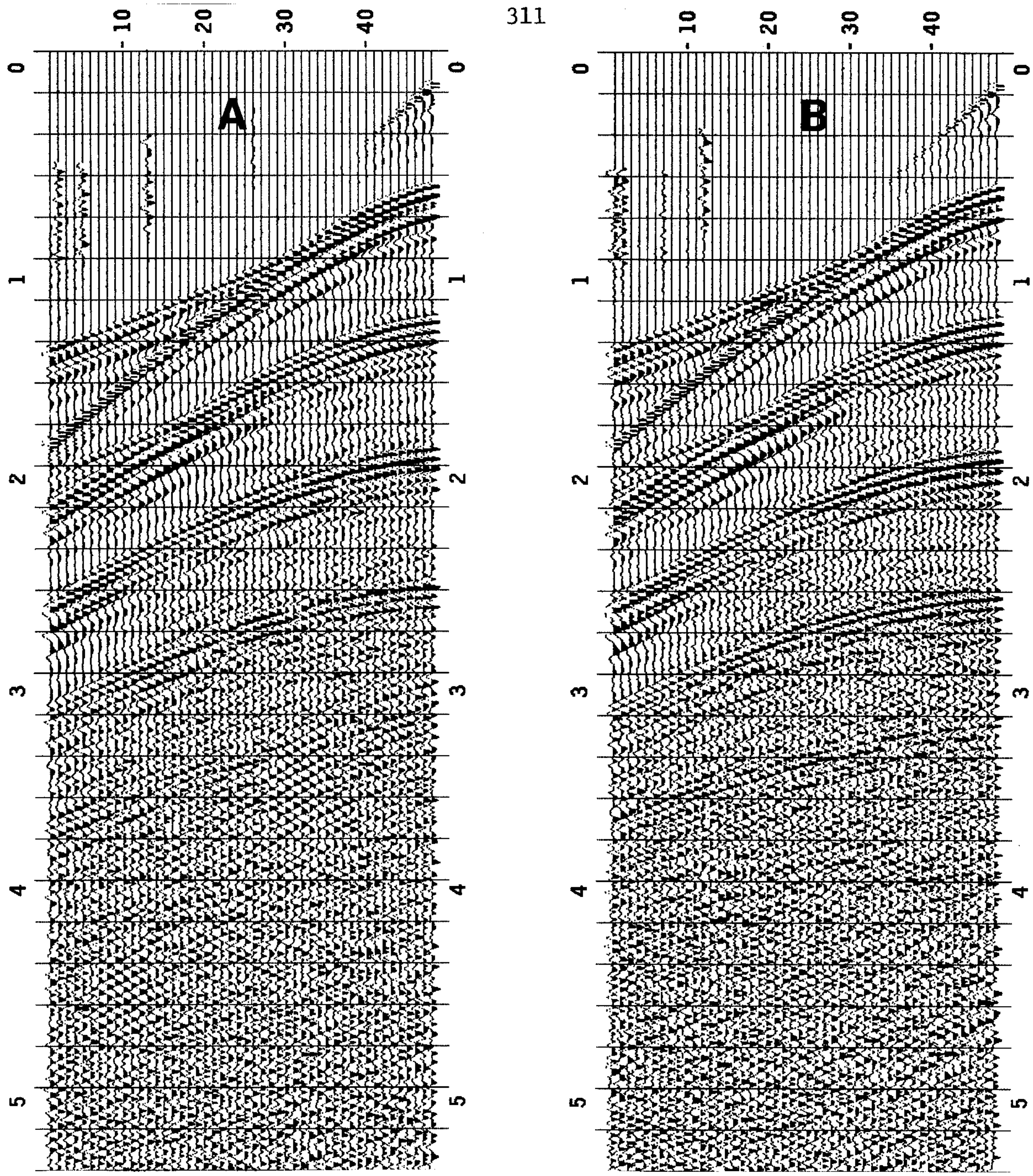

FIG. 1. Two common shot gathers from a GECO survey shot in the Barents Sea. 1500 points per trace, sample rate 4 msec, hydrophone spacing 50 meters, distance from shot to near trace 256 meters. For the purposes of this paper these will be treated as common midpoint gathers (see discussion). Gather (a) is at the beginning of the stacked piece of line (figures 3 and 7) and gather (b) is 10 shotpoints to the right, in the direction of boat motion. These gathers have been gain corrected with a window of 200 points. Strong multiples dominate the gathers down to 3.5 seconds (to the fourth multiple). To see better the underlying primaries and peglegs, examine figure 2.

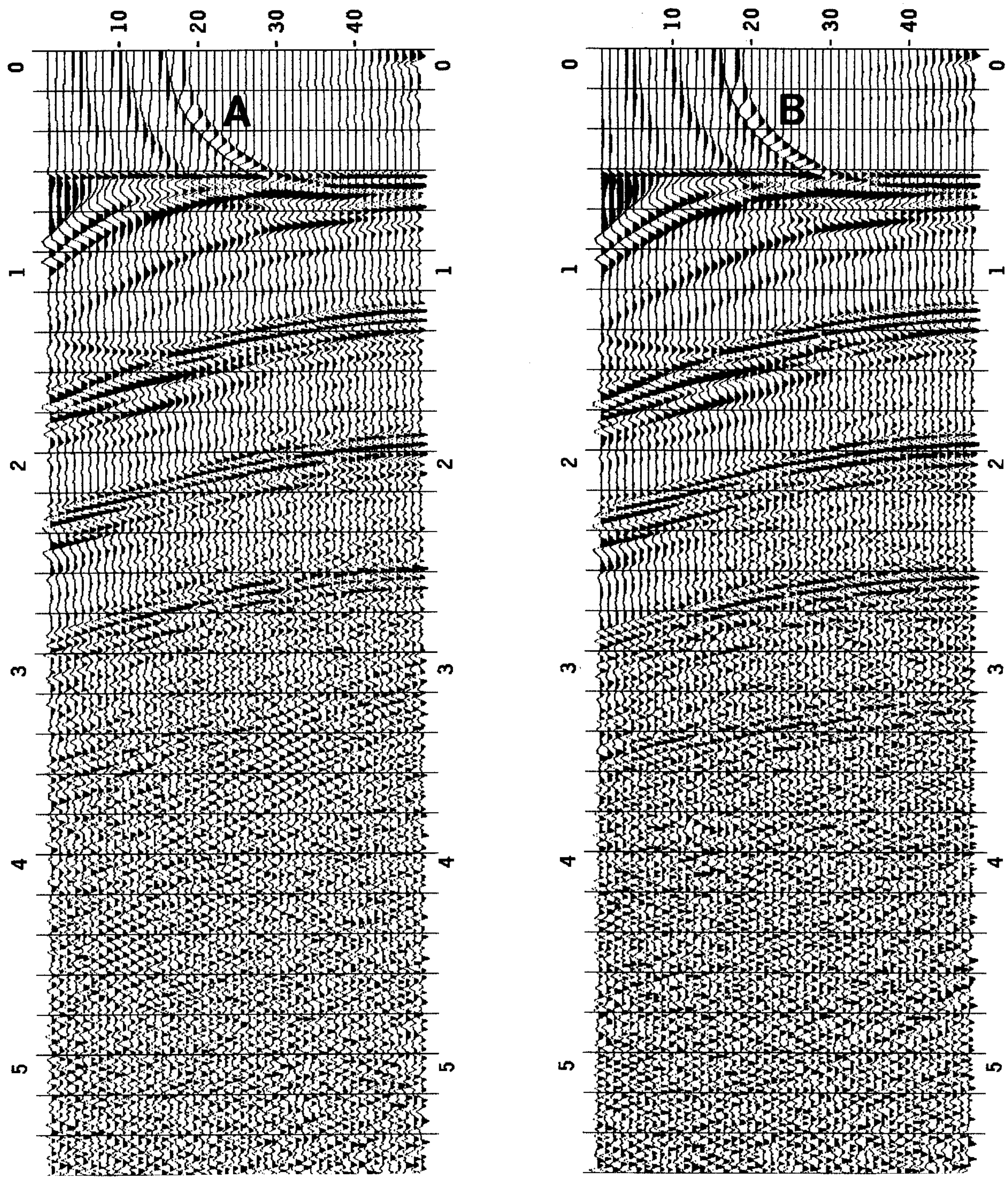

FIG. 2(a,b). The gathers of figure 1 moveout-corrected with the velocity function listed in figure 2(c) (next page). By looking at these long enough one can be convinced that there are flat events present down to 5 seconds. The velocity function of figure 2(c) seems to indicate that some of these are water layer peglegs. They shall be imagined as "primaries".

Time	RMS Velocity
0.640	1480
1.800	1950
2.500	2000
3.900	2500
4.600	2500
6.000	3000

FIG. 2(c). The moveout velocity function used on the data in this paper.

Figure 4 illustrates the relative errors of truncation in the case of spectral estimation. Figure 4(a) is a truncated time series consisting of two sinusoids: a large low-frequency one and a small high-frequency one. The objective is to resolve these two sinusoids in the frequency domain, which one can see in 4(a) are not resolved because of the truncation of the sinusoids. Figure 4(b) shows the large sinusoid extended (analytically). Even though the small high frequency event is not as resolved as it could be by extrapolation, the two events have been separated in the frequency domain by extending only the large sinusoid. In 4(a), the small event is swamped by the side lobes of the large one, while in 4(b) it is adequately resolved. In other words, we shall not care about the error in our extrapolation of the primary but shall attempt to extrapolate the dominant multiple accurately to drive down the error e_{tm} with respect to e_p.

Stacking Extrapolated Gathers: Examples

Let us now extrapolate and stack the field data of figure 1 in an attempt to resolve the primaries better. The procedure is illustrated in figure 5. Since the water layer multiples are the dominant events on the data, the goal is to extrapolate them accurately. All other events are to be taken as primaries. Each temporal frequency component of the gather was extrapolated separately by first estimating a prediction error filter from it, then successively applying the filter to the unknown points. Since this method extrapolates events linearly, NMO at water velocity was applied to the gather before this step. After the extrapolation, the gather was recorrected to the velocity function of figure 2, and muted. An example of an extended gather, just before stacking, is shown in figure 6. These happen to be the extensions of the gathers displayed in figure 2. Since the extrapolation was performed after a moveout correction at water velocity, The extensions to the water layer multiples display the proper moveout. There is also the tendency to tack on the same extrapolation to all other coherent events on the gather, with the apparent moveout of water

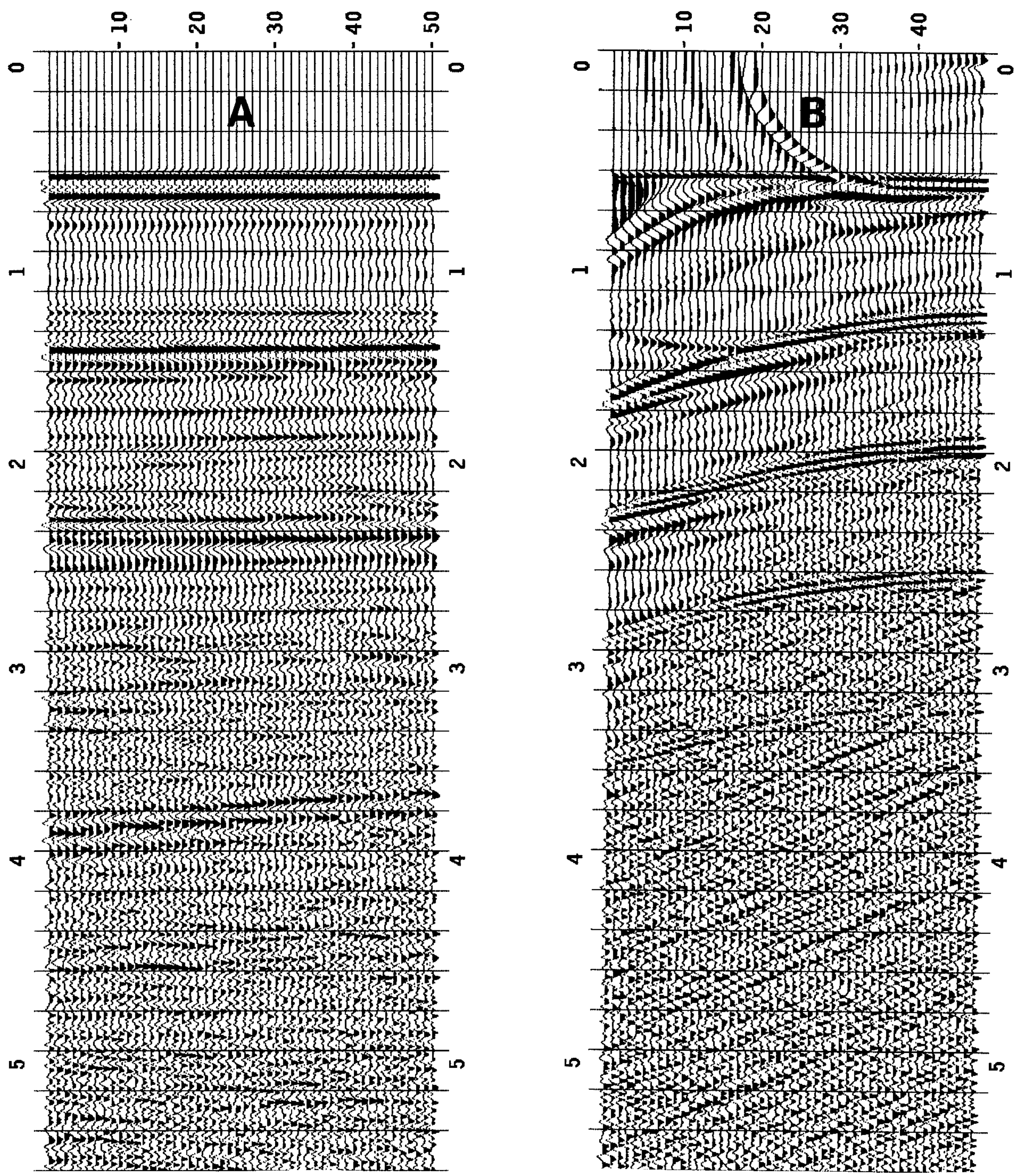

FIG. 3. (a) Stack of 50 gathers. The leftmost trace is stacked from the gather of figure 2(a), and the rightmost trace is stacked from gather (b) immediately to the right in this figure. By comparing gather (b) to the rightmost edge of the stacked section one can see the large contribution the truncated multiples make to the stack, especially those at the far-offset edge.

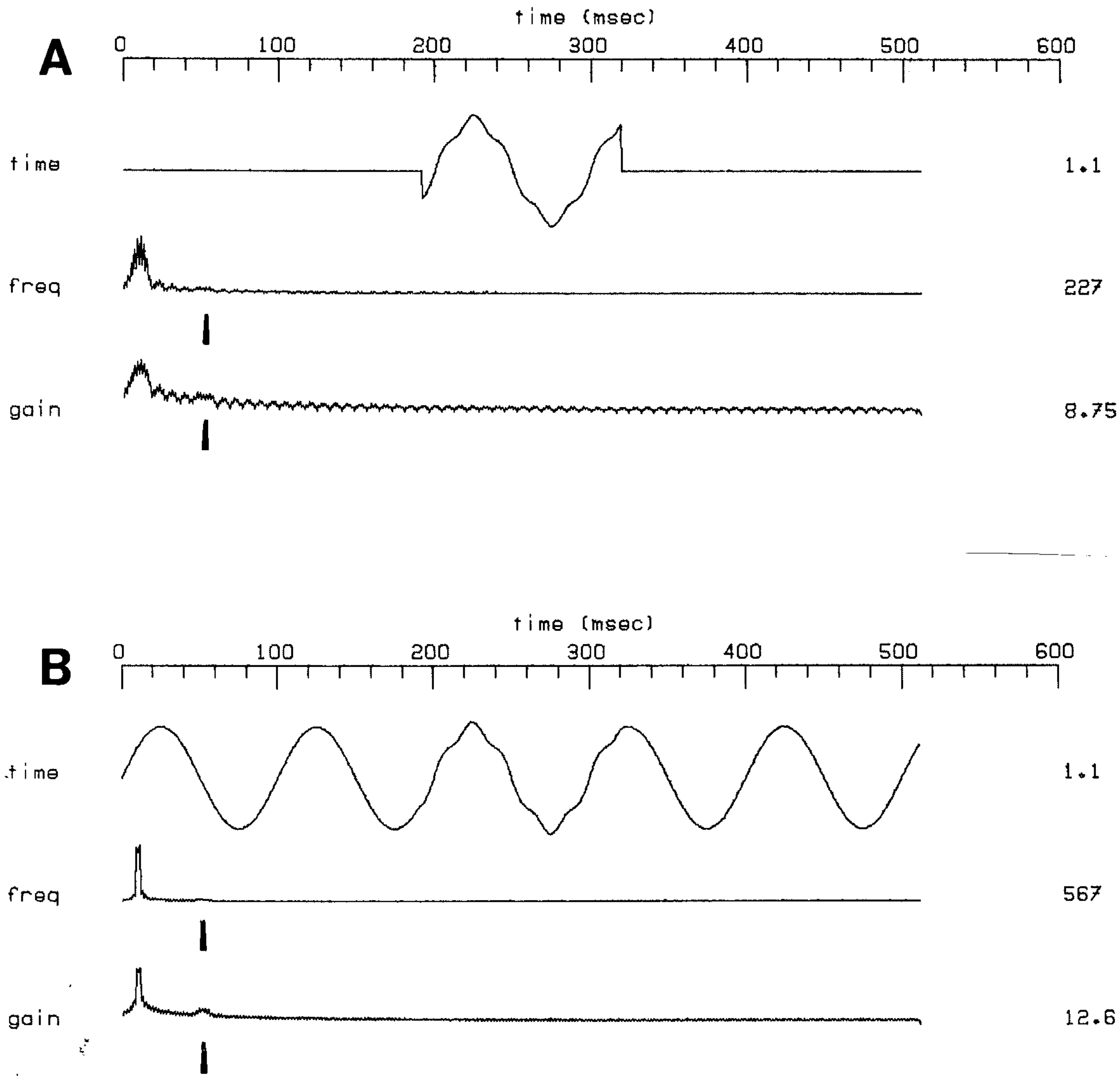

FIG. 4. (a) displays two truncated sinusoids on a 512-point trace. The larger sinusoid has a wavelength of 100 points, the smaller has an amplitude of one-tenth the larger and a wavelength of 20 points. Both are nonzero over a 128 point window. "Freq" is the amplitude spectrum of the time trace, and "gain" is the spectrum raised (lowered?) to the 0.4 power. The smaller sinusoid can hardly be recognized on the transformed traces because of the wide side lobes from the large truncated sinusoid. In (b), the large sinusoid has been extended over the entire trace, while the smaller sinusoid remains unchanged. Even though energy has been added to the large sinusoid, the smaller one is now resolved in the frequency domain because of the much narrower side lobes of the former.

velocity. For reasons discussed in the last section, this should not be a critical error to make as long as the event remains second order with respect to the multiple truncation, or to the primary itself. Another thing to notice is that the zero-offset trace is now included in the extrapolated gather. Flat events now lie on the gather, meaning that the events seen at the zero-offset trace will contribute much to the stack. Our procedure as described can do nothing to suppress the multiples at their minimum times, but then it is not designed to remove the multiple totally from the stack, but rather to eliminate edge artifacts associated with the multiple. With this in mind, the extrapolation will be of greatest benefit on the far traces of the gather. Figure 7 is a stack of 50 adjacent extended gathers. The stack has improved over that of figure 3, by removing the truncation error on the far-offset traces. However it seems inferior in its lower half, for reasons which shall be discussed in a later section.

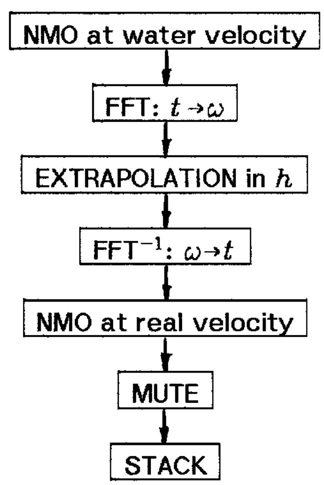

FIG. 5. Extrapolation and stacking algorithm. The extrapolation as well as the mute is subject to a cosine taper. Water velocity moveout is performed with a moveout correction corresponding to v = 1480 m/sec, while the real moveout correction uses the velocity function of figure 2(c).

Iterative Refinement of the Extrapolation

Iterative extrapolation methods described in previous reports (Claerbout, SEP-26) can be used on common-midpoint gathers. What is needed to start the iteration is a reasonable estimate of an extrapolation, such as the extrapolated gathers of the previous section. Therefore such a method can be used to refine the initial extrapolation produced in the previous section.

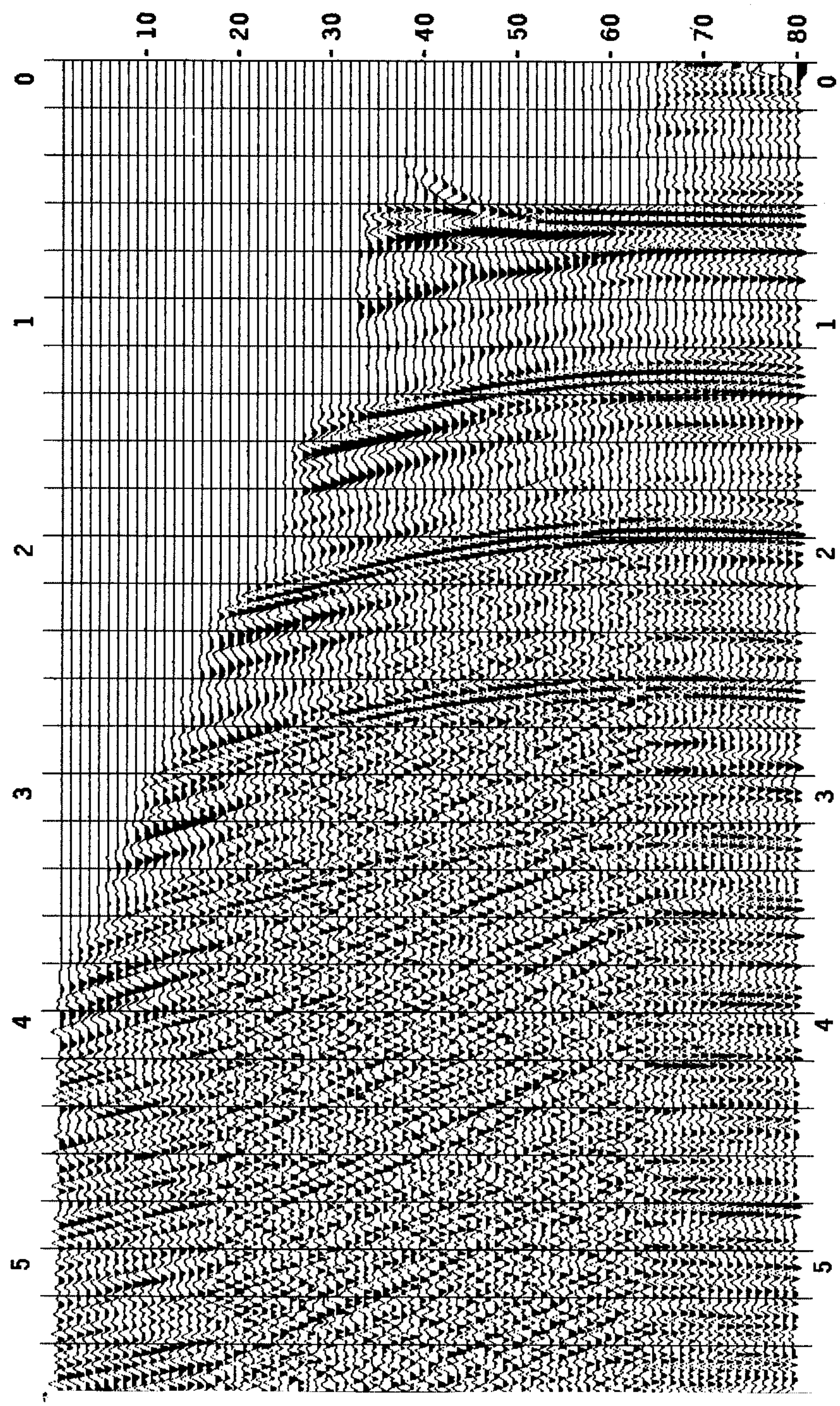

FIG. 6. The gather of figure 3(b) extended with the process of figure 5. This gather corresponds to the step after muting and immediately before stacking in figure 5. The gather has been gain-corrected with a 200-point window. Many problems with the extrapolation procedure are exemplified in this gather; see the section "Problems" for a discuss of them.

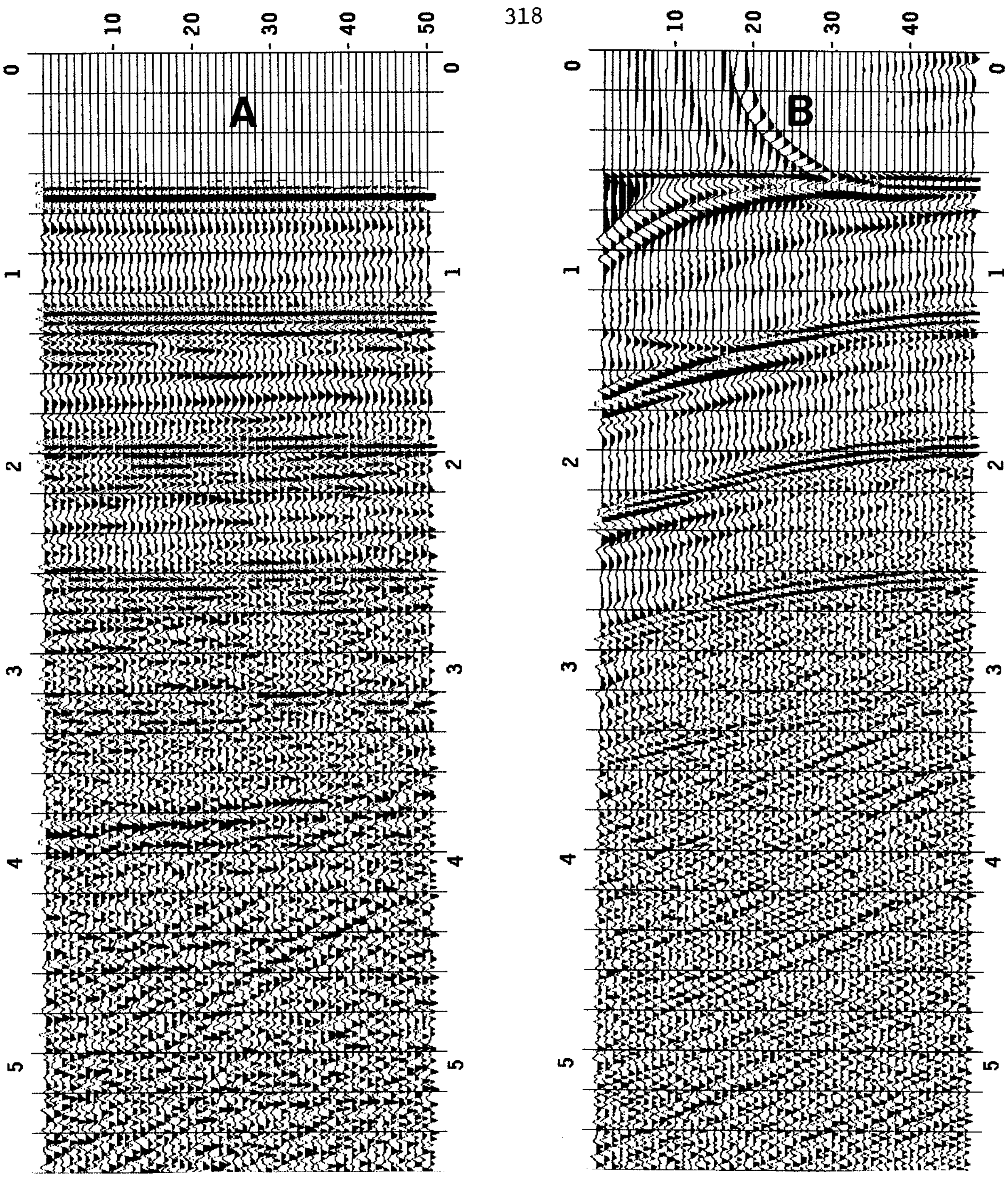

FIG. 7. (a) is the stack of 50 extended gathers. This stack can be compared directly with figure 3(a) which is a stack of the same gathers, unextended. The rightmost gather of figure 3(b) is here repeated in (b), to illustrate how the contribution of the multiples at far offsets has been removed. In the lower section, the stack is unnecessarily worse than that of figure 3; this stems from the time-stationarity assumption in the extrapolation. See the section "Problems".

Iterative extrapolation seeks to minimize some chosen functional while constraining the original data to remain the same; the free variables are the extrapolation points. The functional to be minimized may be the energy at non-zero offset in the migrated domain. For an accurate migration, the gather must image at zero offset. The iterative extrapolation would proceed as follows:

1) Migrate the gather (given the velocity),

2) Favorably weight the energy near zero offset (imaging),

3) Diffract (inverse migrate),

4) Restore the traces of the original gather.

Since for migration a single velocity function must be chosen, all extensions of events will be consistent with the velocity chosen; if extrapolated at water velocity, the extrapolated events will have a moveout corresponding to that of water velocity. It does not seem worthwhile to iteratively refine the extrapolation here for two reasons. First, it is costly, and second, it does not seem necessary to increase the accuracy of the extrapolation. In either case, the primary events are extrapolated at the wrong moveout, so the second-order errors in the method are still present. The goal of removing the truncation artifacts from the stack seems to be reasonably met by a one-shot extrapolation like that of the previous section.

Problems

Figure 6, an example of an extrapolated gather, summarizes all that is wrong with the extrapolation as it now stands. First, extrapolating a section that is moveout-corrected at water velocity guarantees that the new near-offset traces will be virtually flat. This will contribute many events to the stack, and what is worse, many of these events are totally artificial, as the bottom half of the gather of figure 6 shows. This leads to the second problem, namely that the extrapolation algorithm as it now stands depends on the traces being stationary in the time dimension. For our data, this is not true: multiples are dominant for the first 3 seconds of data, while diffraction energy (seen on figure 6) is dominant on the next 3 seconds. The extrapolation filter calculated here will "see" only the water multiple events, because they are by far the strongest on the gather, and as a consequence events on the lower half of the gather are artificially extrapolated at a moveout consistent with water velocity. These events are mostly diffractions on figure 6, and this explains why diffractions from 3 to 6 seconds are stacking in so well in figure 7. Finally, the method works to the best advantage on far-offset truncated energy. Unfortunately in our examples the tapered mute (figure 6) must be credited with most of the work of eliminating the far-offset multiples, since it extends to 4 seconds.

Here are some steps that may be taken to solve the problems listed above. The extrapolation filter may be made to accommodate nonstationarity in the time traces. This can be done by designing a two-dimensional extrapolation filter to be used in the (t,h) domain (Thorson, SEP-28). Second, it need not be necessary to perform the NMO at water velocity in figure 5. There may be enough moveout present on the multiples on the near-offset traces to have them extrapolate at a dip and subsequently destructively interfere in stacking.

ACKNOWLEDGMENTS

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REFERENCES

Claerbout, J.F., Missing Data: Wanted, Good Convergence in a Few Steps: SEP-26, p.23.

Thorson, J., Extrapolating Nonstationary Traces by Autoregressive Filters: SEP-28 (this report).

Western Geophysical, Attenuation of Long Period Multiples: 1978 (a brochure published by Western Geophysical).