Seafloor-Consistent Pegleg Multiple Attenuation

Larry Morley

Introduction

The notion of surface-consistent source-receiver frequency responses was introduced and applied in a paper by Taner and Coburn at the 1980 SEG. Taner handled surface related amplitude and time shift anomalies by decomposing each seismic data trace, D(ω), into a product of source,geophone, midpoint and offset responses. In particular, he assumed that

$$D(\omega) \approx S(\omega)G(\omega)Y(\omega)H(\omega) \tag{1}$$

Modelling the seismic trace in this way allows for removal of near-surface filtering effects and facilitates conventional surface consistent residual statics corrections.

One might well ask if a similar approach to the multiple suppression problem can be taken. In what sense can one talk about a "seafloor consistent multiple suppression"?

An amplitude-only decomposition of the type described by (1) was performed on a dataset with a water bottom of .3 to .6 second two-way traveltime. The failure of the model to fit the data well is attributed to neglect of phase information in the analysis.

The Decomposition Model

One of the reasons standard predictive decon fails is that it assumes an equal reverberation time at the shot and receiver locations. The near trace and common offset sections of figure 1 are good data examples of such "split-pegleg" phenomenon caused by variation of seafloor topography. A model which takes these different characteristic times into account can allow for a more effective multiple suppression (Morley, '81).

In all model fitting it is desirable to describe the phenomenon of interest with as few free parameters as possible. The separable model of equation (1) gives a very parsimonious description of the seismic dataset. It reduces the number of independent spatial parameters

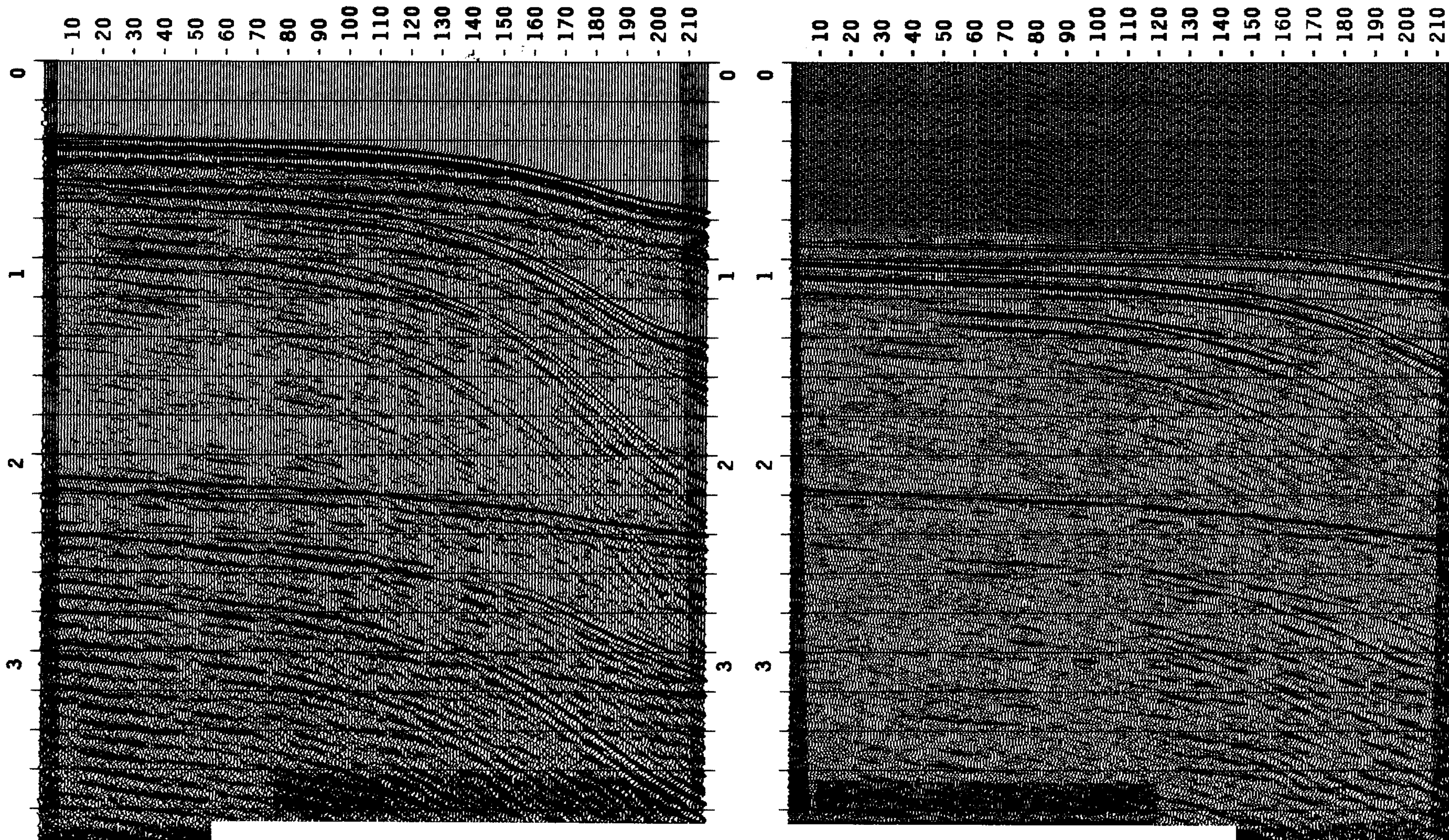

FIG. 1. 1a (left): Near trace section showing hard bottom multiples. Two peglegs from the primary at 2.2-2.4 seconds can be seen cutting across the section from 2.5-3.1 seconds and from 2.9-3.8 seconds.

1b (right): Constant offset section from an offset half-way down the cable. Note that the first order pegleg (from 2.5-3.1 seconds on the near trace section) is split while it was unsplit on the near trace section.

from a total of $N_s N_h$ (as in standard trace-by-trace predictive decon) to $N_s+N_g+N_y+N_h$. In most cases this amounts to a significant reduction in the number of free spatial parameters. If fewer free parameters are required to describe the data then we are less likely to attenuate primaries while trying to suppress multiples. This should result in an improvement in the post-stack primary to multiple ratio (to the extent that the model fits the data).

We will adopt a convolutional model very similar to (1). The only refinement is the addition of an "average response" factor, A(ω). This last term depends mainly on the average shot waveform - something we will *not* attempt to deconvolve here. Our goal is to attenuate the water column reverberations (leaving as much primary as possible intact!). The model, therefore, is:

$$D(s,g,\omega) \approx S(s,\omega) G(g,\omega) Y(y,\omega) H(h,\omega) A(\omega) \qquad (2)$$

Intuitively we expect S to contain shot ghost responses, water reverberation effects characteristic of shot location and residual shot waveforms. Receiver ghosts and water reverberations in the receiver vicinity should be embedded in G.

Taking log magnitudes on both sides of (2) gives:

$$D_{ij} \approx S_i + G_j + Y_{\frac{i+j}{2}} + H_{\frac{i-j}{2}} + A \qquad (3)$$

where i denotes shot index and j, receiver index. All quantities in (3) are now the log magnitudes of their values in (2). If the analysis is done with equation (3) the question of phase information immediately arises. In this paper we shall concern ourselves only with amplitude variations. We know that the shot and receiver reverberation effects we are after are causal positive real. We shall attempt to suppress them with standard Wiener-Levinson spectral whitening techniques.

The problem of estimating least square solutions for S,G,Y, and H in equation (3) is completely analogous to the classic residual statics estimation problem (Taner,'74 or Wiggins,'76). The problem is overdetermined (since we have $N_s N_h$ equations for $N_s+N_g+N_y+N_h$ unknowns) but is known to be underconstrained in the long wavelength solution components. This second aspect of the problem will become apparent when we examine real data solutions.

The complexity of this problem is increased by a factor of **nf** over conventional residual statics, where **nf** is the number of frequency planes on which we are doing the decomposition. This forces us to resort to a more approximate iterative method of solution to keep the problem computationally viable. Our method of solution follows.

Least Square Estimators:

After extracting the average magnitude, A(ω), from the s-g plane, our problem is to find the $\min\limits_{S,G,Y,H} E$, where:

$$E = \sum_i \sum_j (D_{ij} - S_i - G_j - Y_{\frac{i+j}{2}} - H_{\frac{i-j}{2}})^2 \tag{4}$$

The i,j subscripts range over the appropriate regions of the s,g plane. The minimum coincides with the vanishing of the partial derivatives of E with respect to S_k, G_l, Y_p, and H_q. Setting, for example,

$$\frac{\partial E}{\partial S_k} = 0$$

yields

$$\sum_j (D_{kj} - S_k - G_j - Y_{\frac{k+j}{2}} - H_{\frac{k-j}{2}}) = 0 \tag{5}$$

or

$$S_k = \frac{1}{N_g} \sum_j (D_{kj} - G_j - Y_{\frac{k+j}{2}} - H_{\frac{k-j}{2}}) \tag{6a}$$

Similarly,

$$G_l = \frac{1}{N_s} \sum_i (D_{il} - S_i - Y_{\frac{i+l}{2}} - H_{\frac{i-l}{2}}) \tag{6b}$$

Making the index transformation

$$m = \frac{i+j}{2} \quad ; \quad n = \frac{i-j}{2}$$

we can rewrite (4) as:

$$E = \sum_m \sum_n (D_{mn} - S_{m+n} - G_{m-n} - Y_m - H_n)^2 \tag{7}$$

Zeroing $\partial E / \partial Y_p$ and $\partial E / \partial H_q$ in (7) gives:

$$Y_p = \frac{1}{N_h} \sum_n (D_{pn} - S_{p+n} - G_{p-n} - H_n) \tag{6c}$$

$$H_q = \frac{1}{N_y} \sum_m (D_{mq} - S_{m+q} - G_{m-q} - Y_m) \tag{6d}$$

Equations (6a-d) tell us what we may have intuitively expected; the L_2 norm solutions to the decomposition problem can be obtained by averaging model residuals over the direction orthogonal to the response component of interest. It is worth noting in passing that L_1 solutions can be obtained by replacing the averaging operators in (6) with median operators.

The solution strategy involves cycling through equations (6) for all desired values of k,l,p and q until convergence at the spatial wavelengths of interest is obtained. The outermost loop is over temporal frequency.

Data Results

Despite the analogies we have already drawn between the surface-consistent statics problem and the marine multiple suppression problem, there is an important difference in the way the decomposition information is used in the two cases. For statics it suffices to back out the effects of S and G from the data; this results in a signal that is more coherent from trace to trace but may still contain a constant or long wavelength residual static. In the multiple suppression problem we have to remove the Y dependence as well. For those experienced with the statics problem this runs counter to intuition since it is customary to think of Y as the "geology" component of the model. The "A" term in (3) plays that role here. Removing just the S and G responses makes the multiples appear to be independent of shot and geophone location but does not necessarily suppress them.

A program using the above algorithm was run on the first 24 (of 48 possible) offsets of the dataset partially displayed in figure 1. Before starting the analysis the seafloor multiples were suppressed by dip filtering along the midpoint axis at constant offset. This was necessary because the seafloor multiples are wide angle reflections - not fitting our vertical incidence multiple model. The data was then windowed from 2-4 seconds to concentrate on the deep peglegs. The logarithms of the S,G, and Y amplitude responses after four iterations are displayed in figure (2).

The main feature on all these plots is a trend towards higher "quefrencies" from left to right. This is due to the increased water depth or reverberation time at the right side of the section (see figure 1). The scalloping effect on the right half of the G and Y plots is believed to be due to residual seafloor multiples not completely removed by dip filtering. The vertical noise streaks show that the useful bandwidth of the data runs from 5% to 40% of the Nyquist frequency.

The plots in figure (2) were created by four iterations of equations (6) in the order "d-a-b-c". In order to get a feel for the uniqueness and accuracy of this solution we obtained another solution by iterating four times in the order "b-a-c-d". The difference between the

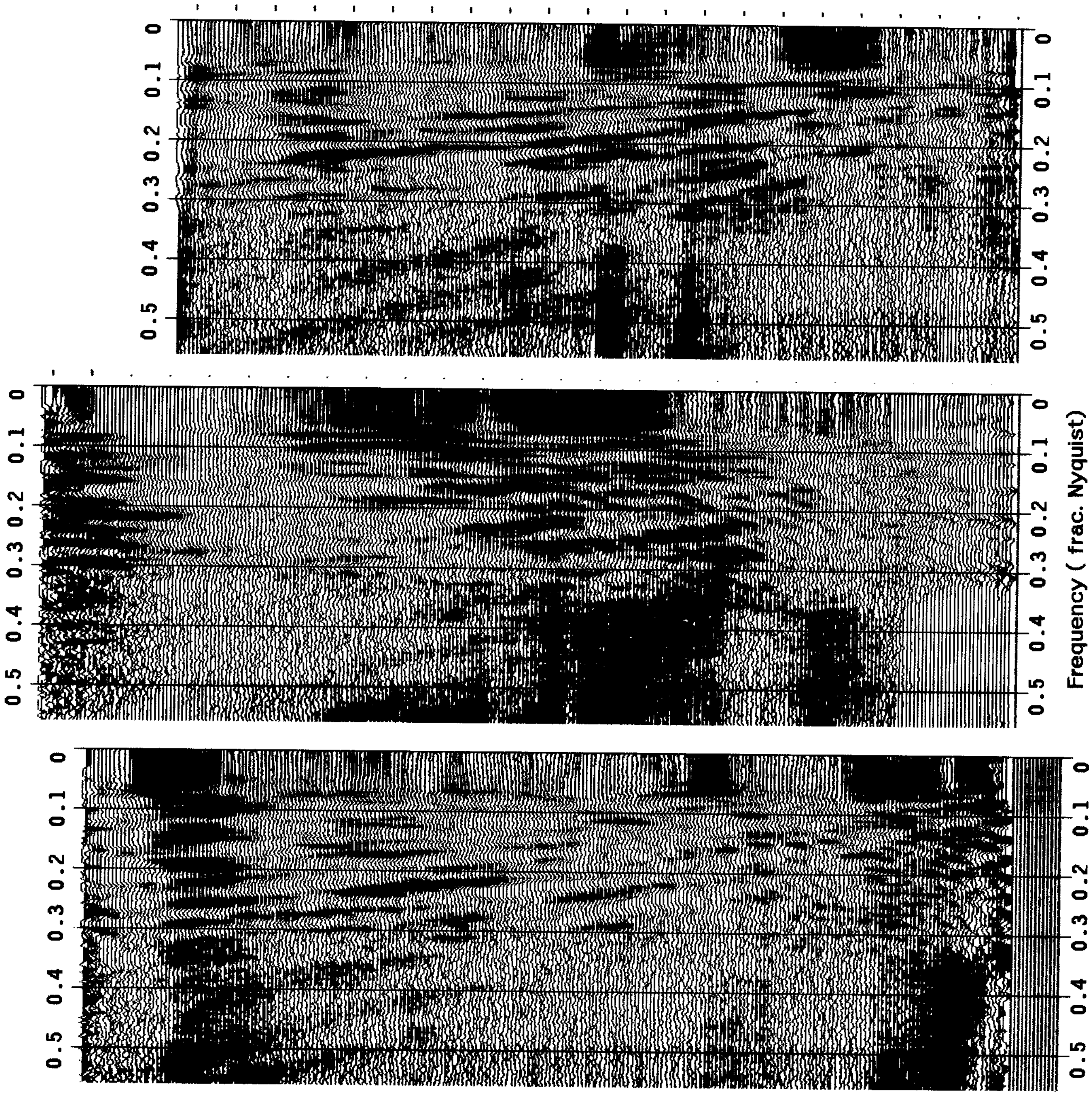

FIG. (2). Log amplitude plots for shot, geophone and midpoint residual responses (top to bottom). Clip level is 0.5 on all plots. Plots are displayed so that common ground locations are vertically aligned. Trend towards higher quefrencies from left to right is due to increased water depth at right side of section.

two "S" amplitude responses is plotted in figure (3) at the same clip level as the plots of figure (2). This plot shows that the main errors are confined to long wavelengths. The difference plots for the two "Y" and "H" solutions (not displayed) also support this observation. We can therefore be confident that the decomposition is unique at wavelengths less than a cable length or fifty traces in this case.

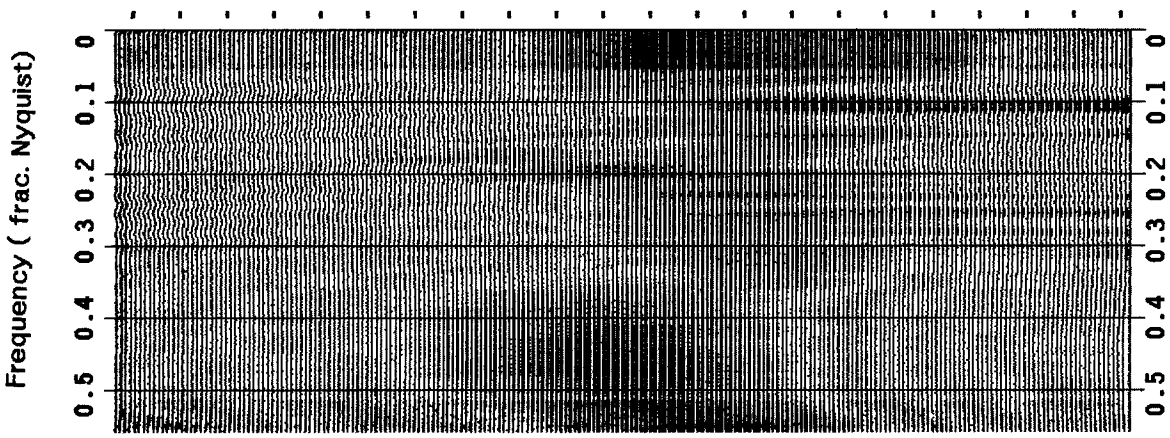

FIG. (3). Error in shot response estimate. Most of the error energy is in the long spatial wavelength components. i.e. - wavelengths greater than a cable length.

Despite the uniqueness of the decomposition, there was not enough residual in the S,G, and Y responses to generate a non-white (i.e.- non-delta) inverse operator. Plots of the data prior to the stacking operations of equations (6) show why. Figure 4a consists of four plots of common shot gather 100 after the first four iterations of the decomposition algorithm. The solution has almost converged after only one iteration. The trace to the right of each iteration is the shot response or average response over all geophones for that shot. There is a dip with offset on most of the events which causes the amplitudes to destructively interfere in the stack; there is no noticeable pattern that one could call a common shot response. This problem is partially corrected by starting the analysis with moved-out

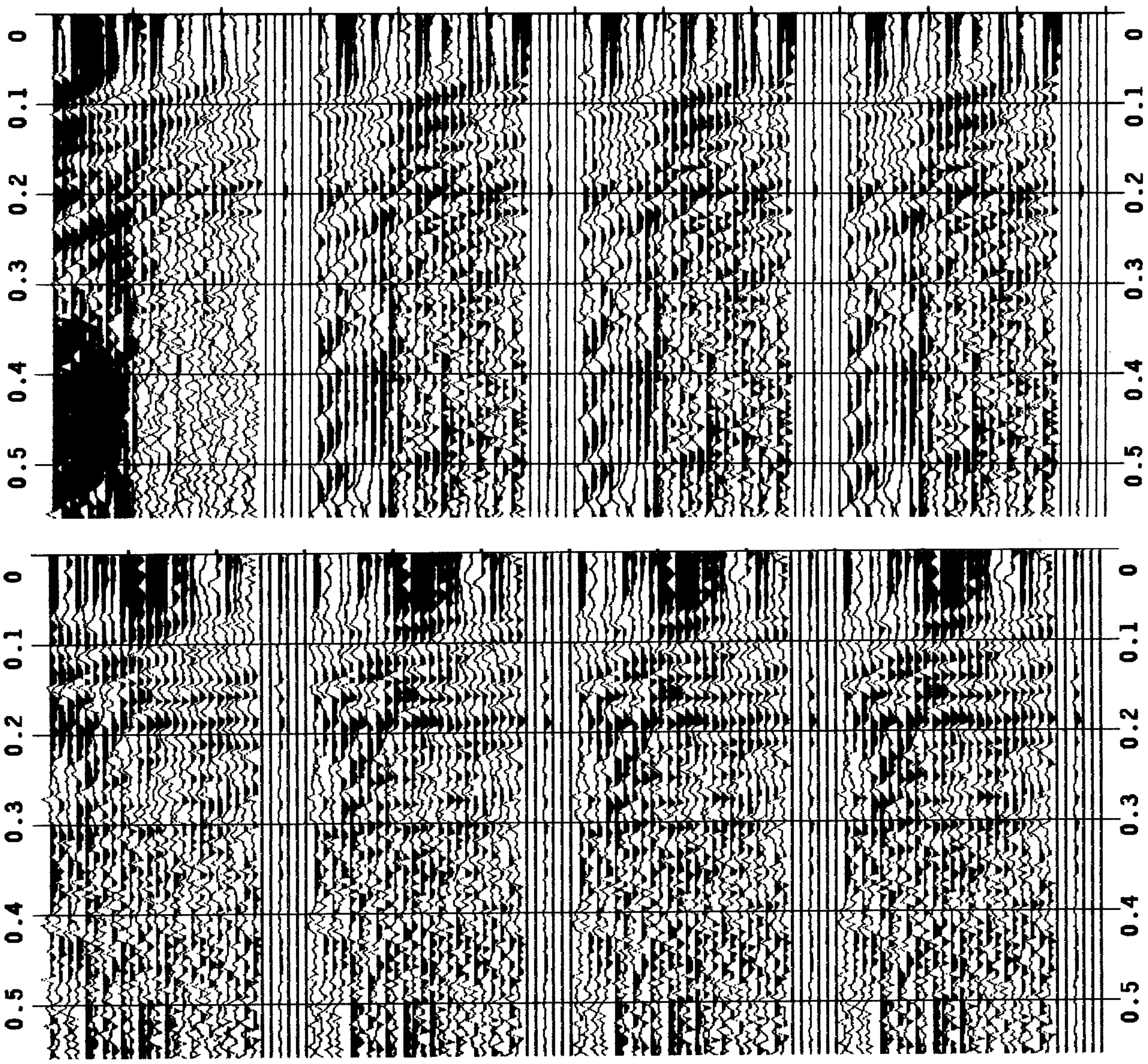

FIG. (4). Plots of log amplitude response of CSG #100 *without* NMO (a-top) and *with* NMO (b-bottom). Clip level is 1.5 in both cases. Each panel from left to right is plotted after successive iterations of the decomposition algorithm. The traces to the right of each panel are the (normalized) sum traces or estimated shot point response. There is much more destructive interference in the stack without NMO than with NMO.

data (figure 4b). In this case there is a pattern which stacks constructively but again the amplitude of the stack is too low to imply a non-trivial predictive decon filter.

Figure (5) is the sum of the S,G, and Y amplitude responses for the constant offset section displayed in figure (1b). Figure (6) shows the gapped decon operators corresponding to figure (5). It contains two branches - one for the shot pegleg and another for the geophone pegleg. The amplitude of these branches is so small, however, that the operators do not cause any visible changes in the data.

Questions for future study

(1) It seems desirable to time limit the interesting features in the dataset before doing the decomposition. This would decrease the frequency resolution and reduce the tendency for destructive interference while stacking over shots or geophones in the frequency domain. It would also reduce the computational load by cutting down on the number of frequency planes. In the residual statics problem this is done by time windowing the marker horizon before starting.

For the multiple suppression problem this might be achieved by starting with a database of the decon operators themselves. The decomposition could then be used to reduce the effective number of free parameters in this database.

(2) Should NMO be done before the decomposition? Figure (4) shows that it can make a big difference. This effect is not presently understood.

(3) A full complex analysis (as opposed to one on magnitudes only) might help in the multiple suppression problem. The dereverberation operators are causal but are not necessarily minimum phase.

REFERENCES

Morley,L.C. (1981), Split Backus Deconvolution Operators, SEP 26 - May, 1981, pp. 95-107.

Taner, M.T, and Coburn, K.W. (1980), Surface Consistent Estimation of Source and Receiver Response Functions, paper G-43, 1980 SEG meeting, Houston.

Taner, M.T., Koehler, F., and Alhilali, K.A. (1974), Estimation and Correction of Near - Surface Time Anomalies, Geophysics, v.39, pp 441-463.

Wiggins, R.A., Larner, K.L., and Wisecup, R.D. (1976), Residual Statics Analysis as a General Linear Inverse Problem, Geophysics, v.41, no.5, pp 922-938.

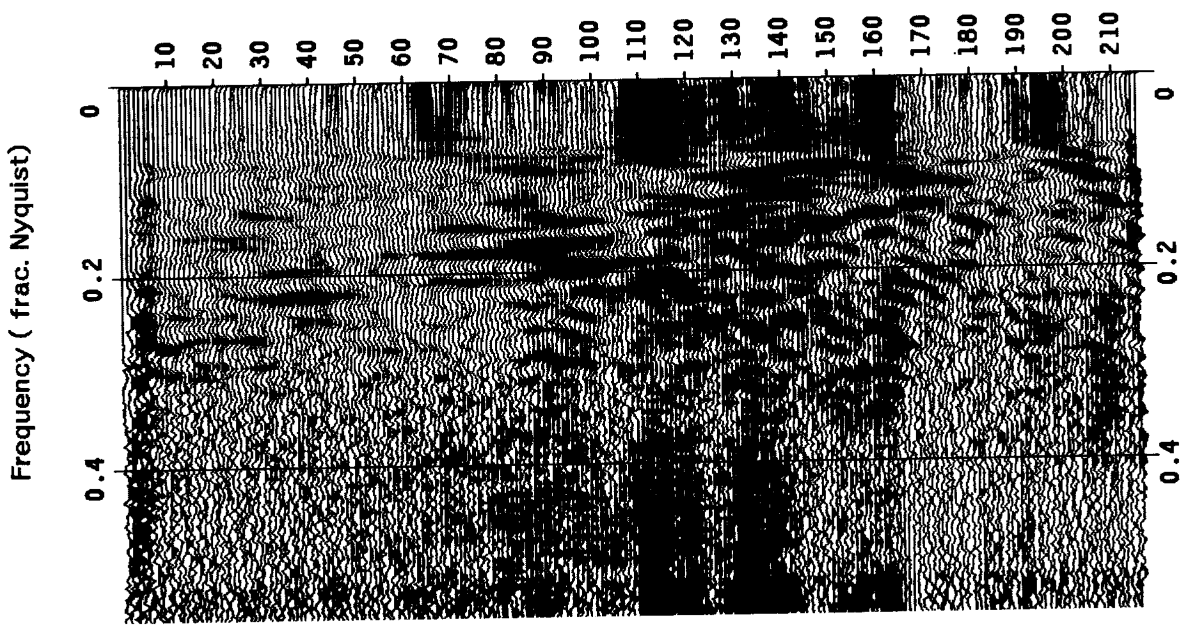

FIG. 5. Sum of S,G, and Y amplitude responses for same constant offset section as figure 1b. Clip level is 0.5.

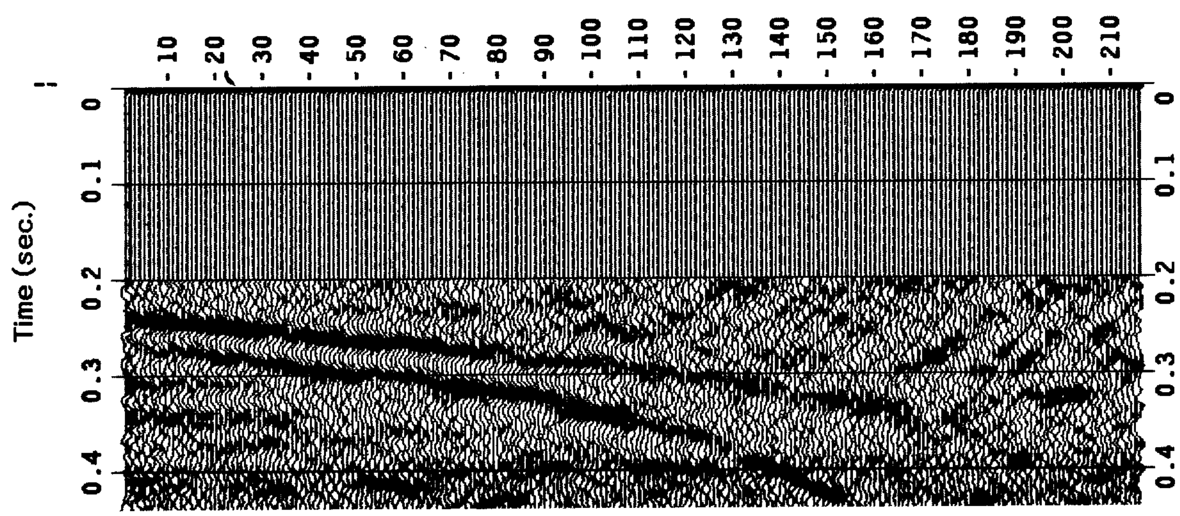

FIG. 6. Gapped decon filter corresponding to amplitude response of figure 5. The amplitude of the two branches is unfortunately too small (clip level .02) to change the data with this filter.