Five Snell Parameter Imaging Methods

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This article reviews five methods to invert reflection seismic data by transformation into Snell parameter space. Table 1 summarizes the characteristics and comparative advantages of each method.

Table 1: Five Snell Parameter Imaging Methods

Name	Snell Transform	Imaging Method	Comments
Phinney Beams [1]	common shot gather (G) slant stacks	ray tracing: (+) few aliasing and edge effects (+) lateral velocity variations easier	(-) poor actual results
Controlled Directional Receptivity [2]	both common shot gather (G) and reciprocal gather (S) slant stacks		(+) oldest and most widespread use
Simplan [3]	common shot gather (G) slant stacks	wave equation: (+) automatic, no picking Snell parameters or events (+) cleaner results	(+) uses zero offset migration operator
Slant-Midpoint [4][5]	common midpoint gather (H) slant stacks		midpoint advantages: (+) dip moveout always positive (+) earth coordinates interpretationally meaningful
Snell Traces [6]	common midpoint gather (H) Snell trace remapping		" (+) no slant stack artifacts (-) not exact theory

Overview

The motivation for Snell parameter imaging methods is that they are more accurate than conventional stacking-migration or migration of constant offset sections (Claerbout SEP-15). Steep dips, wide offset angles, and multiple dips at the same location are better handled. These methods are also more efficient than brute force migration before stack (equation 1), because they operate on 2-D sections.

Snell parameter imaging is divided into two stages. First, gathers are transformed into traces of constant Snell parameter. Second, sections of constant Snell parameter traces are migrated.

Each stage may use a ray tracing or wave equation algorithm. The wavefield Snell transformation of gathers is called slant stacking, while the ray tracing method is called Snell trace mapping. The ray tracing method seems to work better during this stage because it avoids aliasing and edge artifacts introduced during slant stacking. On the other hand, during migration wave equation methods seem to give cleaner results. Ray tracing migration also requires the arbitrary picking of reflections and their Snell parameters. However, the treatment of lateral velocity variations is better understood in a ray tracing context.

There are two choices of coordinate systems in which to work (figure 1). Three of the Snell parameter imaging methods work in shot-geophone coordinates by stacking common shot gathers. The other two operate in midpoint-offset coordinates. In midpoint-offset coordinates dip moveout on gathers is always positive and sections can be interpreted in more meaningful earth coordinates.

Summary of Each Method

The five methods will be described in the context of the double square root equation (1). This frequency domain equation is used to migrate unstacked data.

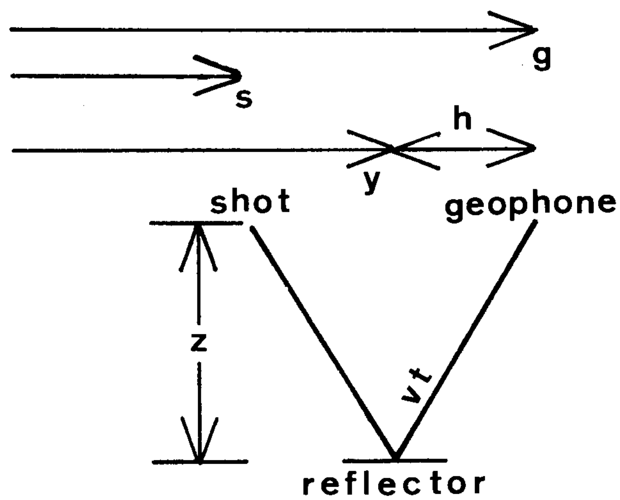
$$\frac{dP}{dz} = -i\frac{\omega}{v}\left[\sqrt{1 - G^2} + \sqrt{1 - S^2}\right] P \tag{1a}$$

$$\frac{dP}{dz} = -i\frac{\omega}{v}\left[\sqrt{1 - (Y + H)^2} + \sqrt{1 - (Y - H)^2}\right] P \tag{1b}$$

The notation is explained in figure 1.

In general, slant stacking along offset coordinate x is defined by time domain and frequency domain equations of the form

$$t' = t - px \tag{2a}$$



Coordinate Name	Notation	Transform	Frequency Domain	Transform	Normalized Wavenumber	Snell Parameter
shot	s	$y+h$	k_s	$\frac{k_y-k_h}{2}$	$S=\frac{k_y}{\omega}$	$p_s=\frac{k_s}{\omega}$
geophone	g	$y-h$	k_g	$\frac{k_y+k_h}{2}$	$S=\frac{k_y}{\omega}$	$p_g=\frac{k_g}{\omega}$
midpoint	y	$\frac{g-s}{2}$	k_y	k_g+k_s	$Y=\frac{vk_y}{2\omega}$	$p_y=\frac{k_y}{\omega}$
half-offset	h	$\frac{g+s}{2}$	k_h	k_g-k_s	$Y=\frac{vk_h}{2\omega}$	$p_h=\frac{k_h}{\omega}$
time	t		ω			
depth	z		k_z			

FIG. 1. Separated source-receiver coordinate systems in shot-geophone (s,g,t,z) and midpoint-offset (y,h,t,z) coordinates.

$$p_x = \frac{k_x}{\omega} \tag{2b}$$

The offset x may be g, s, or h. The derivation of equation 2b is given in Claerbout (SEP-15). Equation 2b is the dispersion relation of a plane wave with a Snell parameter p_x. Slant stacking simulates that plane wave by summing together trace records given a time delay which linearly increases with offset (equation 2a). (There is no physically realizable plane wave in stacks over half-offset h.)

Slant-Midpoint Method

This method is most familiar to readers of SEP reports. Slant stacks are done over common midpoint gathers using equation 2b in terms of h.

$$H = p_h v \tag{3}$$

Inserting equation 3 into equation 1b gives a formula to migrate this kind of Snell parameter section.

$$\frac{dP}{dz} = -i\frac{\omega}{v}\left[\sqrt{1-(Y+p_h v)^2} + \sqrt{1-(Y-p_h v)^2}\right] P \tag{4}$$

The main advantages of this method are the use of wave equation migration and working in midpoint coordinates. The main disadvantage is the difficulty of producing good slant stacks.

Simplan

The Simplan method slant stacks over common shot gathers rather than common midpoint gathers. The slant stack equation is now

$$G = p_g v \tag{5}$$

Equation 5 is inserted into equation 1a to obtain the appropriate migration equation.

$$\frac{dP}{dz} = -i\frac{\omega}{v}\left[\sqrt{1-p_g^2 v^2} + \sqrt{1-S^2}\right] P \tag{6}$$

Equation 6 can be broken down into two operations. First, the Snell parameter section is migrated with a single square root operator $\sqrt{1-S^2}$. Then a moveout correction $\sqrt{1-p^2v^2}$ is applied. The advantage of using this method is is being able to use conventional wave equation migration to do the single square root part. The disadvantages are having to work in shot-geophone coordinates.

A variation of this method is to slant stack common *geophone* gathers with $G=pv$. The migration equation and procedure is about the same.

Phinney Beams

Phinney's first method of slant stack imaging is mathematically similar to Simplan. However, ray tracing techniques were used to implement the migration of equation 6. There seemed to be some problems with propagating amplitudes and the results were not very clean.

Controlled Directional Receptivity

This is the oldest method of Snell parameter imaging. It combines features of both shot-geophone and midpoint-offset coordinate systems. The slant stacks are done in the former, while ray tracing migration works in the latter. Both common shot and reciprocal gathers are used. The midpoint-offset ray parameters of a given reflector are determined from the Snell parameters measured off the slant stacks. From figure 1 these relations are

$$p_y = p_g + p_s \qquad p_h = p_g - p_s \tag{7}$$

The ray trace migration uses these new ray parameters. The motivation for transforming ray parameters between coordinate systems is that before the advent of powerful electronic digital computers common shot and common geophone slant stacks could be performed using analog electronics directly off the field records. Russian geophysicists seem to have overcome most of the practical difficulties of migration ray tracing and produce rather clean looking sections.

Snell Traces

This last method is intended to overcome the major drawback of the previous four methods, that is of producing a clean slant stack. Slant stacks are simulated by predicting what information they extract from a common midpoint gather, then extracting this information directly. Slant stacks pull information out of a gather along a diagonal line if their are no dipping events and constant velocity. The extraction trajectories for depth variable velocities are computed by Ottolini elsewhere in this SEP report.

The wave equation migration operator is found by using geometrical optics. It was discovered by Ottolini that the point scatterer response of a Snell trace section is a hyperbola compressed in the z direction by the cosine of the offset angle. This leads to the migration operator

$$\frac{dP}{dz} = -i2\frac{\omega}{v}\left[\frac{1 - Y^2}{1 - p_h^2 v^2}\right]^{1/2} P \tag{8}$$

REFERENCES

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