Migration Before Stack by Transformation Into Snell Trace Coordinates

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Migrating Snell trace midpoint sections is a method which handles wide offset angles and steep dips better than migrating constant offset sections. It avoids the dip selectivity of CDP stacking. It permits the more accurate velocity analysis *after* migration. A partial prestack migration version of this theory improves conventional processing.

A *radial trace* (introduced by Taner 1975) is the seismic dataset at a constant ratio of offset to travel time. It is extracted along a diagonal across a CDP gather. A *Snell trace* is a radial trace generalized to depth variable velocity. A Snell trace has a Snell parameter p corresponding to the angle of the trace on the CDP gather. For a given Snell parameter, a *Snell trace midpoint section* is created by extracting a Snell trace of that parameter from each CDP gather. Figure 1 illustrates these concepts.

There is a close relationship between Snell traces and *slant stacks*. A Snell trace is the portion of a CDP gather selected by a slant stack under certain conditions. Snell traces do not as rigorously satisfy the general migration-before-stack equations as do slant stacks. However, Snell traces do not exhibit the false events common in slant stacks which degrade the migration of such sections (Ottolini et. al. 1978; Schultz and Claerbout 1978).

Snell Trace Midpoint Section Point Scatterer Response

The point scatterer response on a constant velocity Snell trace midpoint section is better behaved than on a constant offset section. It is always a hyperbola. This allows the easy design of an accurate migration operator in Snell trace trace coordinates in contrast to conventional migration-before-stack.

Consider the separated shot-geophone recording geometry shown in figure 2. The travel time t for a midpoint y, half-offset h, depth z, and constant velocity v is given by

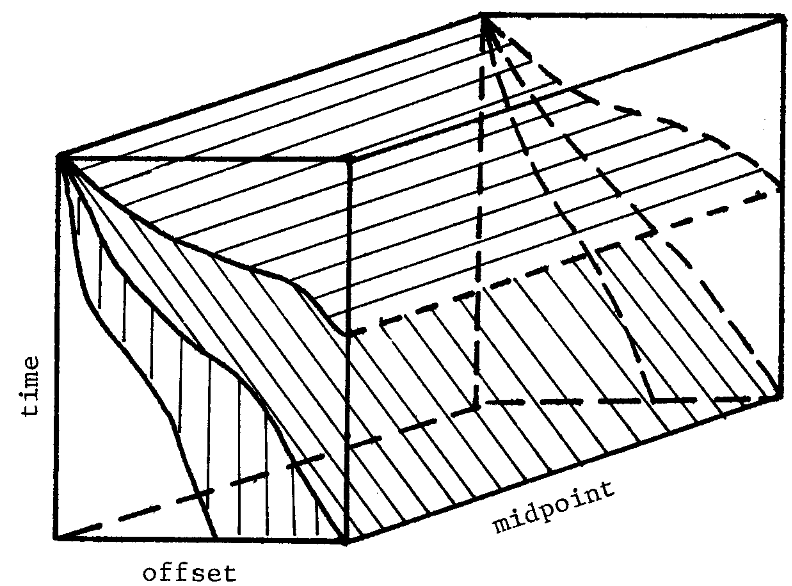

FIG. 1. This is the proposed processing geometry. Shown above is the unprocessed data cube. CDP gathers are vertical planar sections seen edge on. Constant offset midpoint sections are vertical planes along the elongated side of the cube. We propose resampling the data into Snell trace midpoint sections (shaded planes.) Snell traces are the edge on view of such sections seen on CDP gathers. They lie on diagonal trajectories which are a ratio of offset to time. When velocity varies with depth, Snell trace midpoint sections are somewhat wavy.

$$vt = \sqrt{z^2 + (y+h)^2} + \sqrt{z^2 + (y-h)^2} \tag{1}$$

For small offset angles ($h<<z$) h approaches zero. Equation 1 then reduces to a hyperbolic curve as a function of y and t for fixed z and h. However, when $h>z$, equation 1 gives the "table top" curve shown in figure 3. Such a point scatterer response causes difficulties in designing a stable and accurate migration operator (Deregowski and Rocca 1981; Yilmaz and Claerbout 1980).

Let us determine the point scatterer response for a constant ratio r of offset to time.

$$h = rt \tag{2}$$

Simple algebra obtains

$$vt = 2\left[\frac{z^2}{1 - \frac{4r^2}{v^2}} + y^2\right]^{1/2} \tag{3}$$

Equation 3 is that of a hyperbola with the z axis scaled by

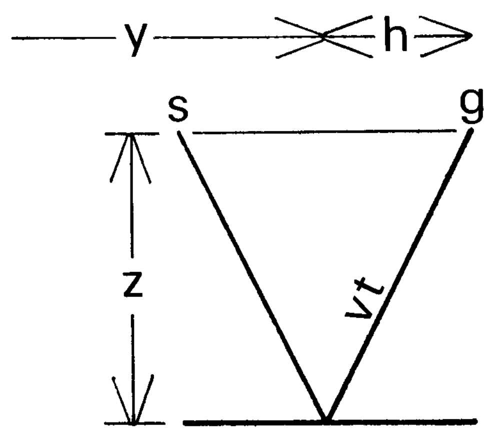

FIG. 2. This is the separated shot-geophone recording geometry. The midpoint and half offset of the shot and geophone are y and h. Reflector depth, travel time, and constant velocity are z, t, and v.

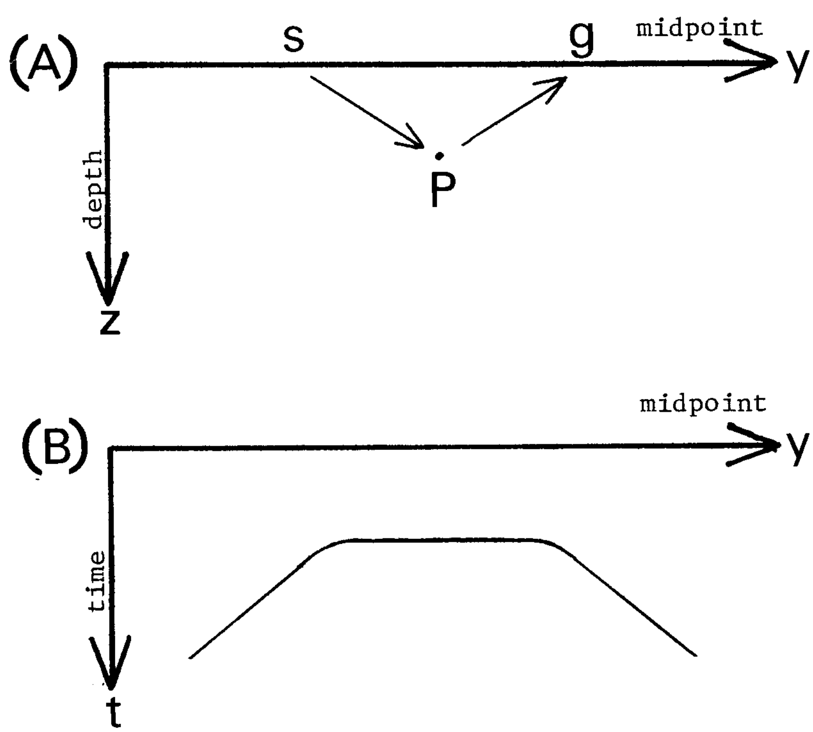

FIG. 3. Part (b) is the point scatterer response of the separated shot-geophone geometry in part (a). The offset shot to geophone is comparable of the depth to the scatterer P. Depth and offset are fixed, while travel time is the function of midpoint given by equation 1. At wide offsets the response is no longer hyperbolic.

$$z' = z \left[1 - \frac{4r^2}{v^2} \right]^{-1/2} \tag{4}$$

For $r=0$, equation 3 reduces to the familiar hyperbolic equation seen on zero offset sections.

Migration Equation

Migration is a two step operation of converting a time recording of reflections into a depth image. First, the wavefield recorded at the surface is extrapolated into the earth. The extrapolation may be thought of as downward continuing the shots and geophones themselves. Second, the image is extracted when the shot and geophone are directly at the reflector. That is to say, the imaging condition is when both t and h equal zero.

The extrapolation equation for collapsing the hyperbolas seen on a zero offset section is well known (Claerbout 1976; Yilmaz and Claerbout 1980).

$$\frac{dP}{dz} = -i2\frac{\omega}{v}\sqrt{1 - Y^2}\, P \tag{5}$$

Y is the wavenumber ratio $vk_x / 2\omega$. This definition is consistent with earlier notation. Y equals zero for flat reflectors ($k_y = 0$) and equation 5 simplifies to one for depth-to-time conversion.

Equation 5 will extrapolate a wavefield with the hyperbolic point scatterer response of a Snell trace section given by equation 3 if z is replaced by the z' of equation 4. The chain rule expansion of dz' is used to convert this new extrapolation equation back into dz.

$$\frac{dP}{dz} = -i2\frac{\omega}{v}\left[\frac{1 - Y^2}{1 - \frac{4r^2}{v^2}} \right]^{1/2} P \tag{6}$$

Equation 6 is used to migrate Snell trace midpoint sections. As a consistency check, equation 6 reduces to the zero offset form, equation 5, when the ratio r is zero.

Depth Variable Velocity and the Snell Parameter P

Snell traces are generalized to depth variable velocity profiles by reformulating the ratio r in terms of the Snell parameter p. The motivation is that Snell's parameter is an invariant in such media. Results include equations for extracting Snell traces and a more general migration equation.

A Snell trace is redefined as the set of tangencies between a linear moveout trajectory and the hyperbolic expression of flat reflectors on a CDP gather. This geometry is shown in figure 4. These tangencies are exactly what a slant stack would select from a CDP gather for flat reflectors. The linear moveout trajectory is given by

$$t = t' - 2ph \tag{7}$$

where t' is the translated time coordinate. Hyperbolic moveout is given by

$$t = \left[t_0^2 + \frac{4h^2}{v^2} \right]^{1/2} \tag{8}$$

where t_0 is zero offset travel time. The tangency point is determined by eliminating t from equations 7 and 8 and solving for the zero of dt'/dh. This gives

$$h = \frac{pv^2 t_0}{2} \left[1 - p^2 v^2 \right]^{-1/2} \tag{9}$$

$$t = t_0 \left[1 - p^2 v^2 \right]^{-1/2} \tag{10}$$

The ratio of offset to time gives r in terms of p in constant velocity media.

$$r = \frac{h}{t} = \frac{pv^2}{2} \tag{11}$$

To find the Snell trace trajectories for depth variable velocity media, the depth invariance of p is used to integrate equations 9 and 10. Thin constant velocity depth slices are assumed. Zero offset time t_0 is measured in depth slices as $dt_0 = dz / v(z)$. The depth variable velocity Snell trace extraction trajectories are

$$h(z) = \frac{1}{2} \int_0^z dz \; pv \left[1 - p^2 v^2 \right]^{-1/2} \tag{12}$$

$$t(z) = \int_0^z \frac{dz}{v} \left[1 - p^2 v^2 \right]^{-1/2} \tag{13}$$

The extrapolation equation (6) is used to propagate a Snell trace wavefield across a thin constant velocity depth slice. This equation in terms of Snell's parameter is

$$\frac{dP}{dz} = -i2\frac{\omega}{v} \left[\frac{1 - Y^2}{1 - p^2 v^2} \right]^{1/2} P \tag{14}$$

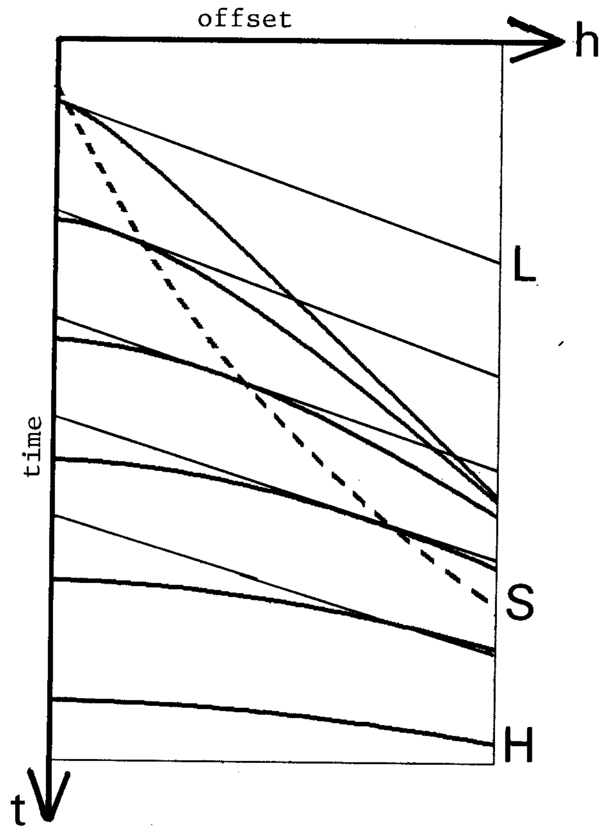

FIG. 4. This is how a Snell trace trajectory (S) is determined. It is the set of tangencies of a linear moveout trajectory (L) with the hyperbolic expression of flat reflectors (H) on a CDP gather. The slope of the linear moveout trajectory is Snell's parameter. These set of tangencies are also what a slant stack along the same linear moveout trajectory would select from a CDP gather. In this particular example, velocity increases with depth, so the Snell trace trajectory bows upward. The time axis is drawn at a non-zero offset. Otherwise the Snell trace trajectory would begin at zero time for zero offset.

Velocity Analysis

The basic principle of velocity analysis is the same as for conventional processing - velocity is a function of moveout with offset. In the case of Snell traces, offset has been translated into the Snell parameter. Velocity analysis is performed using a *Snell trace gather*. This is the set of all Snell traces extracted from the same CDP gather for different p values. Equation 9 is the moveout on a Snell trace gather. As p increases, so does the time of an event. This is the same sense of moveout as on a CDP gather. However, the moveout is like an ellipse rather than a hyperbola. An actual data example is shown in figure 6.

Velocity analysis may be done before or after migration. However we have not found a nice way to approximate the moveout equation (9) using an average velocity as is done when using root-mean-squared hyperbolas to analyze CDP gathers. Therefore it is necessary to use a stripping method similar to what Schultz (1980) uses on slant stack gathers. Unmigrated Snell trace gathers also suffer from the same dip problems as CDP gathers. Dipping events have higher apparent velocity and are incorrectly located along midpoint. Migration corrects for dip and enhances signal-to-noise, thereby improving velocity analysis.

Velocity analysis after migration works on the basis that if the wrong migration velocity was used, then residual moveout will remain on the unmigrated gather. The extrapolation equation (14) should produce the *same* image independent of choice of Snell parameter if the migration velocity is correct. The exceptions to this property are caused by missing offsets on CDP gathers and angle-dependent reflectivities. Thus, if the migration velocity is correct, then events will have the same travel time independent of p and thus no moveout.

A formula for measuring velocity from residual moveout can be derived from equations 13 and 14. Consider the time shift Δt_i due to a single, thin, constant velocity depth slice at z_i with velocity v_i. According to equation 13, the unmigrated moveout is then

$$\Delta t_i = \frac{\Delta z}{v_i}\left(1 - p^2 v_i^2\right)^{-1/2} \tag{15}$$

The migration equation (14) has a time shift which has been Fourier transformed into ω. Consider flat events where Y=0. Calling the migration moveout $\Delta\tilde{t}_i$ for migration velocity $\tilde{v}_i$, the migration time shift is

$$\Delta\tilde{t}_i = \frac{\Delta z}{\tilde{v}_i}\left(1 - p^2 \tilde{v}_i^2\right)^{-1/2} \tag{16}$$

Eliminating Δz from equations 15 and 16 leaves

$$\frac{\Delta t_i}{\Delta\tilde{t}_i} = \left(\frac{1 - p^2\tilde{v}_i^2}{1 - p^2 v_i^2}\right)^{1/2} \tag{17}$$

If $\tilde{v}_i$ equals v_i, the migration time shift is the same magnitude as the original moveout. If the migration velocity $\tilde{v}_i$ is zero, no migration is done and equation 17 reduces to the equation (15) for measuring moveout on unmigrated gathers. Otherwise, equation 17 can be solved for the actual velocity v_i after the left hand side has been measured from migrated and unmigrated Snell trace gathers.

Field Data Example I

The previous discussion suggests the processing sequence below. Figures 5 though 10 illustrate this processing sequence used on an actual growth fault dataset (see figure 11). Implementation details are discussed in the appendix.

(1) Measure a velocity depth function from CDP gathers for use in Snell trace processing.

(2) Extract Snell trace gathers. (figures 5 & 8)

(3) Sort into Snell trace midpoint sections. (figure 6)

(4) Migrate Snell trace midpoint sections.. (figure 7)

(5) Sort back a few migrated Snell trace gathers. (figure 8)

(6) Measure velocity after migration. (figure 9)

(7) If the original velocity estimate is bad, remigrate Snell trace midpoint.

(8) Stack the migrated Snell trace midpoint to increase signal-to-noise. (figure 10)

(Note: further examples are being processed and will be presented at the 1981 SEG and Ottolini's PhD thesis.)

Prestack Partial Migration

Conventional processing approximates migration-before-stack by dividing it into two operations. First is an offset only operation - NMO stacking. Second is a midpoint only operation - migration of a zero offset section. Yilmaz and Claerbout (1980) showed that these approximations become increasingly inaccurate as the dip of a reflector steepens. They also derived a compensatory migration operation to be applied before stack. This was done by taking the difference between the phase shifts of the extrapolation equations for migration-before-stack and conventional processing. However, they ran into problems applying this compensatory migration operator to constant offset sections because of non-hyperbolic nature of the point scatterer response at wide offsets. A Snell trace coordinate reformulation avoids this problem.

The phase shift from the equation (14) for extrapolating Snell trace midpoint sections is

$$\sqrt{\frac{1 - Y^2}{1 - p^2 v^2}}$$

The portion of this this phase shift which converts time-to-depth is unity. The portion which migrates zero offset sections only (see equation 5) is

$$\sqrt{1 - Y^2} - 1$$

The portion that corrects for moveout on Snell trace gathers without considering the effects of midpoint is (see equation 16)

$$\frac{1}{\sqrt{1 - p^2 v^2}} - 1$$

The residual phase shift after the zero offset and zero dip portions are subtracted from the full phase shift is

$$\sqrt{\frac{1 - Y^2}{1 - p^2 v^2}} - 1 - \left(\sqrt{1 - Y^2} - 1\right) - \left[\frac{1}{\sqrt{1 - p^2 v^2}} - 1\right] \approx -\frac{p^2 v^2 Y^2}{4} \qquad (18)$$

The right hand side was obtained by expanding the square roots to first order. For zero offset sections where $p=0$ there is no phase shift as expected. Combining the result of equation 18 with a time-to-depth conversion gives the prestack partial migration equation in Snell trace coordinates.

$$\frac{dP}{dz} = -i2\frac{\omega}{v}\left[1 - \frac{p^2 v^2 Y^2}{4}\right] P \qquad (19)$$

Field Data Example II

The following processing sequence was used for prestack partial migration on the same dataset as before.

(1) Measure a velocity depth function from CDP gathers for use in Snell trace processing.

(2) Extract Snell trace gathers. (figure 5 & 8)

(3) Sort into Snell trace midpoint sections. (figure 6)

(4) Partially migrate Snell trace midpoint sections.

(5) Sort back into Snell trace gathers.

(6) Invert Snell traces gathers into CDP gathers (figure 11).

(7) Revised CDP velocity analysis.

(8) CDP stack.

(9) Zero offset migration.

(Note: further examples are being processed and will be presented at the 1981 SEG and Ottolini's PhD thesis.)

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Appendix: Implementation

Below is a discussion of how the Snell traces were extracted in the examples shown and how the migration was performed.

Snell trace extraction is primarily a two-dimensional interpolation problem. First three traces are interpolated between each existing offset. This is done as a one dimensional interpolation along the predicted dip of a hyperbolic event (suggested by Dave Hale). Next a table of offset as a function of time for each Snell parametered is computed from equations 12 and 13. This table is applied to each CDP gather at about the same cost as NMO.

The distribution of p values is an important consideration. One should plot the Snell trace trajectories before chosing the distribution of p values. The Snell trace trajectories for low or high p values go off the edges of the CDP gather. It is desirable to have the Snell trace trajectories uniformly sample the CDP gather. The separation of trajectories is dependent on the velocity model. Evenly distributed p values tend to bunch up at high p values for depth increasing velocities. A geometrically increasing p value separation at a 1.3 power was found to give a good separation of trajectories.

The frequency domain phase shift method (Gazdaz 1979) was used to implement the migration equations. Assume an extrapolation equation of the form

$$\frac{dP(\omega,z,k)}{dz} = -i\frac{\omega}{v}\Phi(\omega,z,k)\,P \tag{A1}$$

Equation A1 is rewritten in extrapolation time coordinates $\tau = v(z)dz$.

$$\frac{dP}{d\tau} = -i\omega\,\Phi\,P \tag{A2}$$

A good, but not exact, exponential solution to equation A2 in depth variable velocity media is

$$P(\omega,\tau,k) = e^{-i\omega\int_0^\tau d\tau\,\Phi}\,P(\omega,\tau=0,k) \tag{A3}$$

Finally, the imaging condition $t=0$ is applied. Because there is an inverse Fourier transform involved, this is simply a sum over omega.

$$P(t=0,\tau,k) = \sum_\omega \left\{ e^{-i\omega\int_0^\tau d\tau\,\Phi}\,P(\omega,\tau=0,k) \right\} \tag{A4}$$

Increases in speed were obtained by only computing the frequencies present in the seismic data, assuming Φ changes only after several $d\tau$ intervals, and constructing a look-up table for computing the exponential.

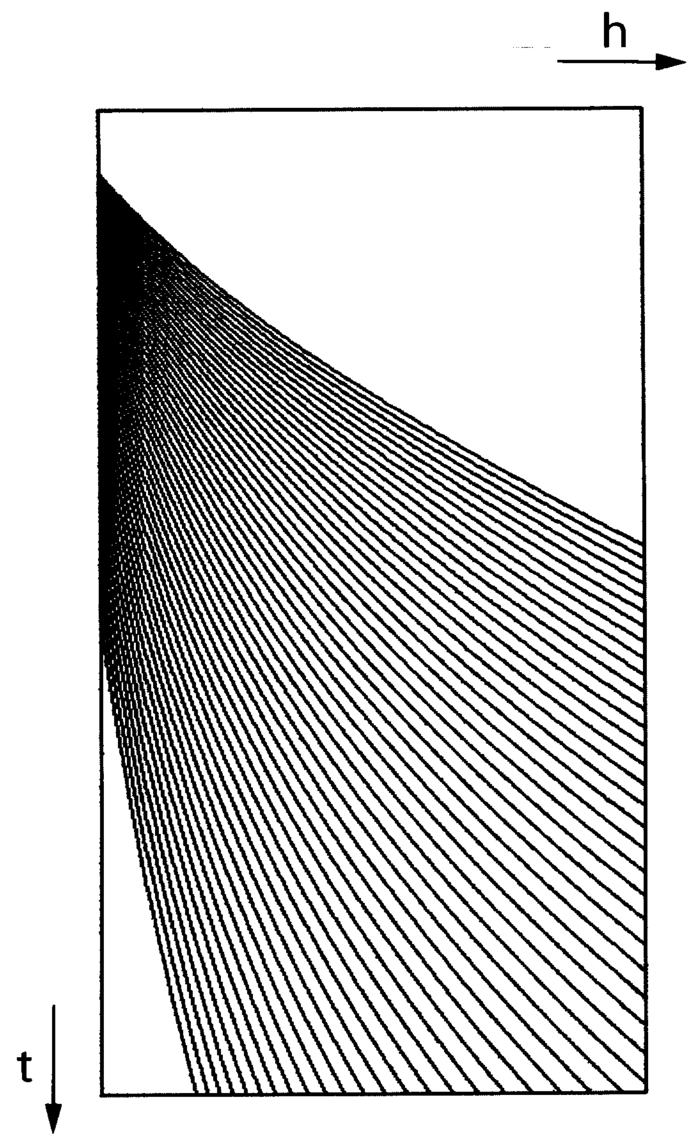

FIG. 5. Radial trace trajectories used for Snell trace transformation of CDP gathers of dataset in this report. Distribution of p values chosen to evenly sample gather. Parameters are: nt=1500 dt=.004 noff=48 off1=1037 doff=166-332 np=48 p1=2.5e-6 p=p1+ip*dp**1.03

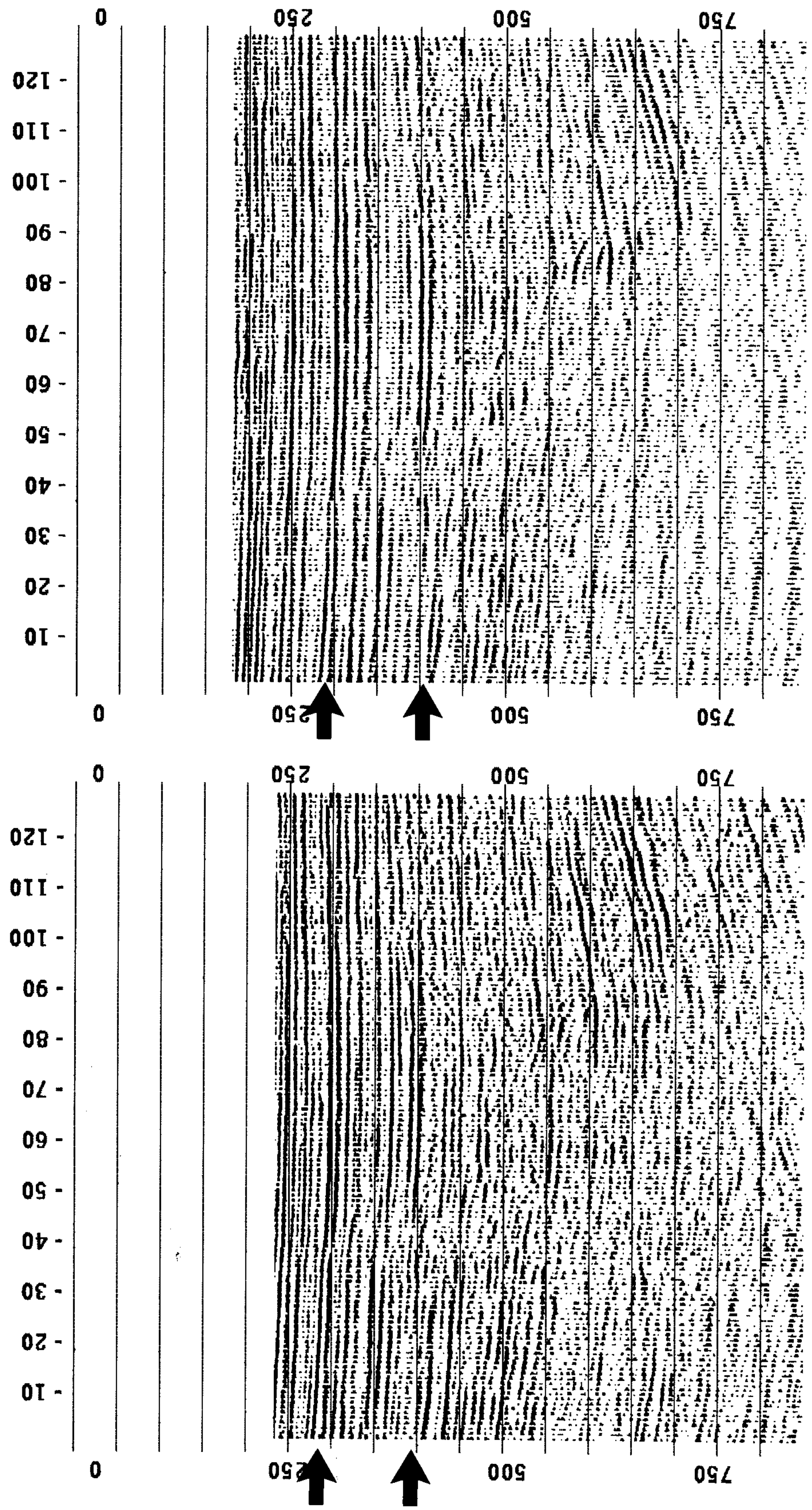

FIG. 6. Snell trace midpoint sections of growth fault dataset in figure 11 at two different p values. Left p=2.5e-5; Right p=3.5e-5; The white area at the top is the missing near trace washout. This washout region decreases with increasing p. This is not to be confused with event moveout which increases with increasing p. Event moveout obeys equation 13. Arrows point to same event on two different midpoint sections. Parameters are: nt=850 dt=.004 nx=128 dx=83

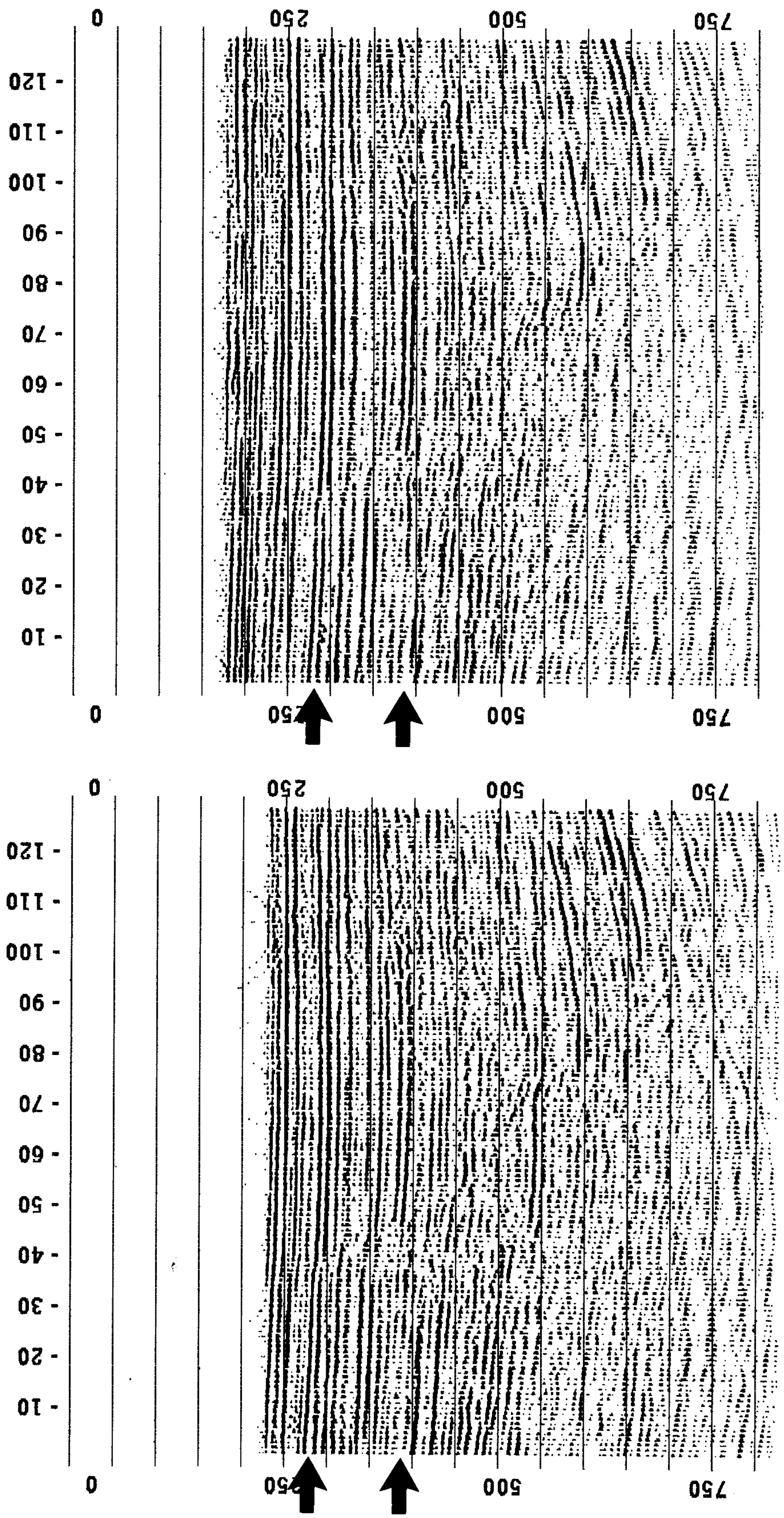

FIG. 7. Migrated Snell trace midpoint sections from figure 6. The migration velocities were CDP stacking velocities converted to depth interval velocities. Migration corrects moveout and sharpens growth fault image.

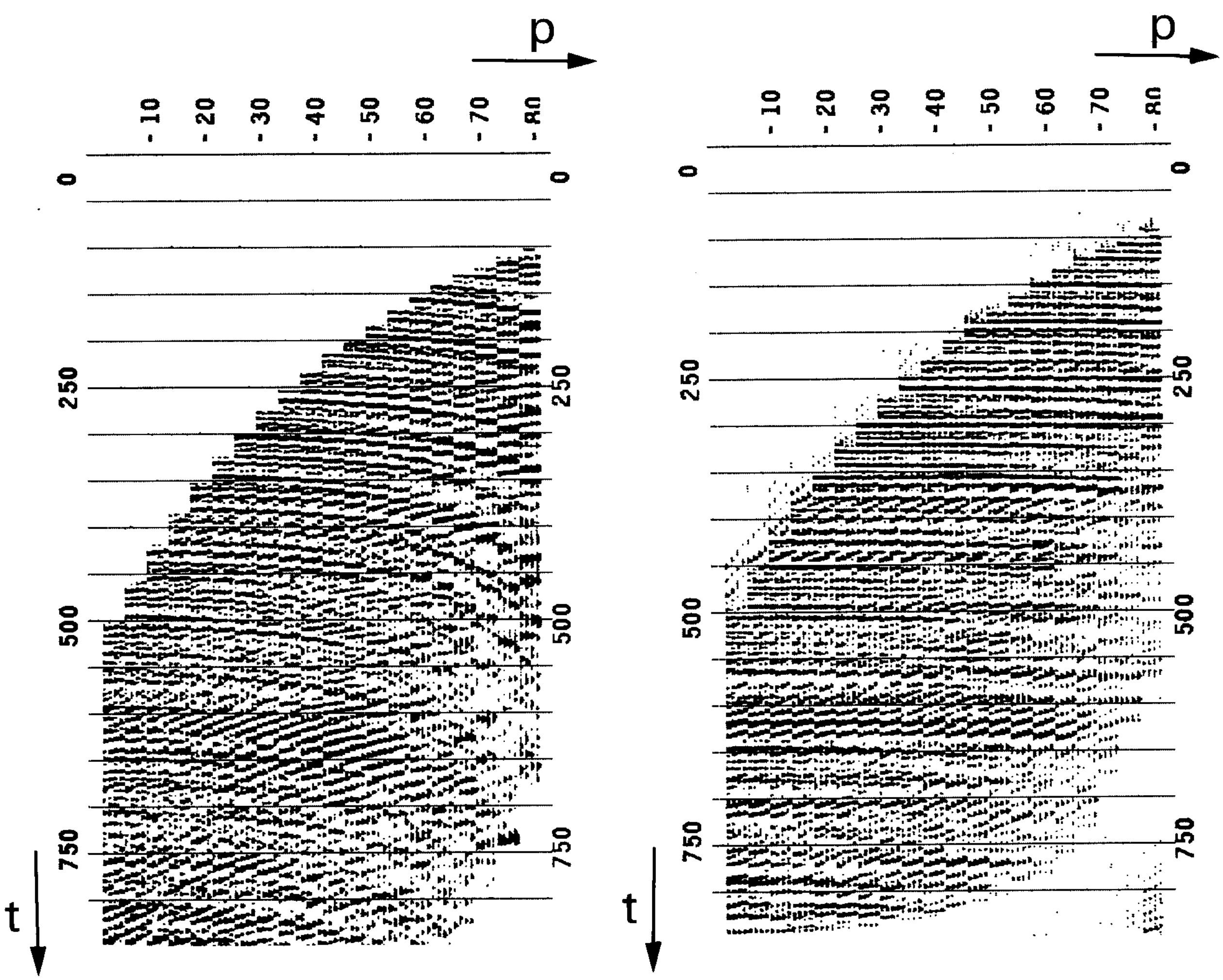

FIG. 8. Migrated and unmigrated Snell trace gathers. The last four traces of midpoint sections for 20 p values were assembled into a gather. Both flat and dipping events can be distinguished. In the unmigrated gather on the left p moveout is shown. In the migrated gather to the right p moveout has almost been entirely removed. The migration velocity was a little bit too low to completely correct for moveout.

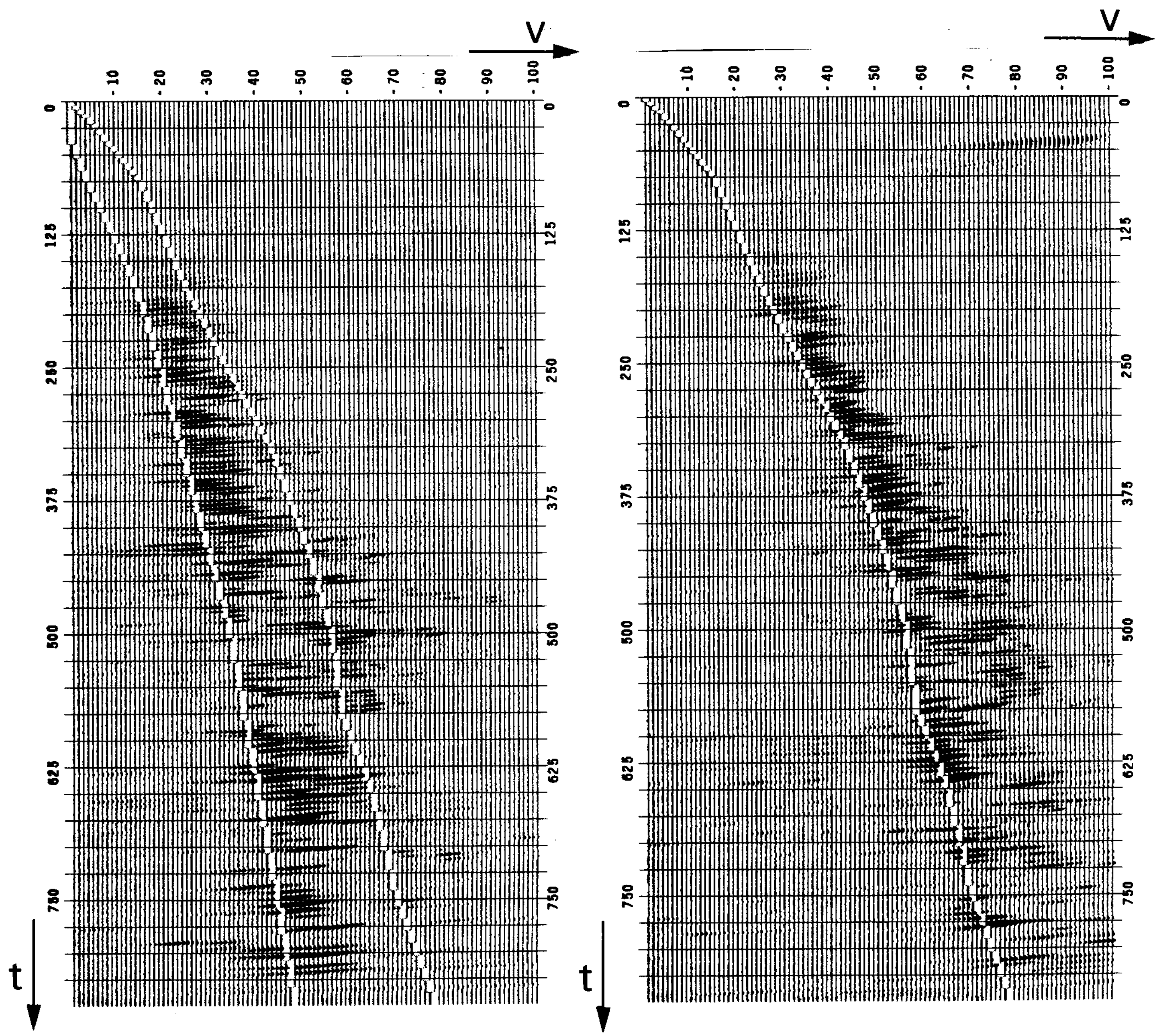

FIG. 9. Velocity scans of migrated and migrated Snell trace gathers from figure 8. Semblence values were measured along the moveout trajectory given by equation 17 for constant velocities between 5000 and 12000 feet/sec. Rms and interval CDP stacking velocities are also plotted for comparison. We do not yet know how to interpret the average velocities measured from the unmigrated gathers. The velocities measured from migrated gathers are depth interval velocities. The migrated semblence plot has a little higher resolution than the unmigrated semblence plot. As confirmed by in figure 8, the migration velocities were a bit low.

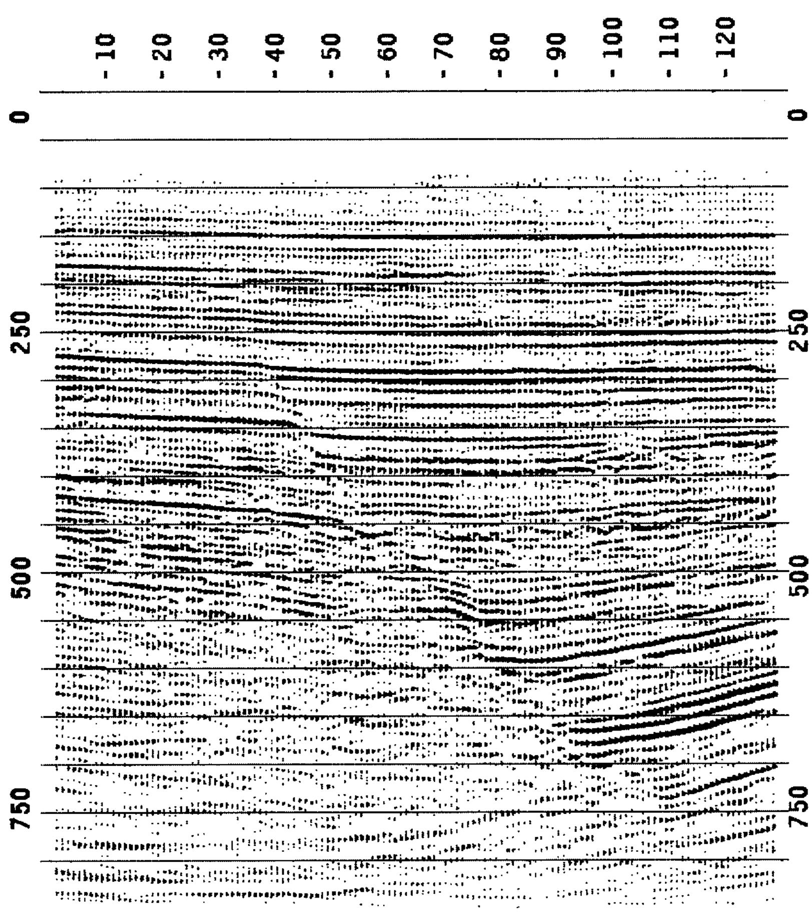

FIG. 10. Stack of 21 migrated Snell trace sections. A growth fault runs through the center of the image. It is slightly undermigrated.

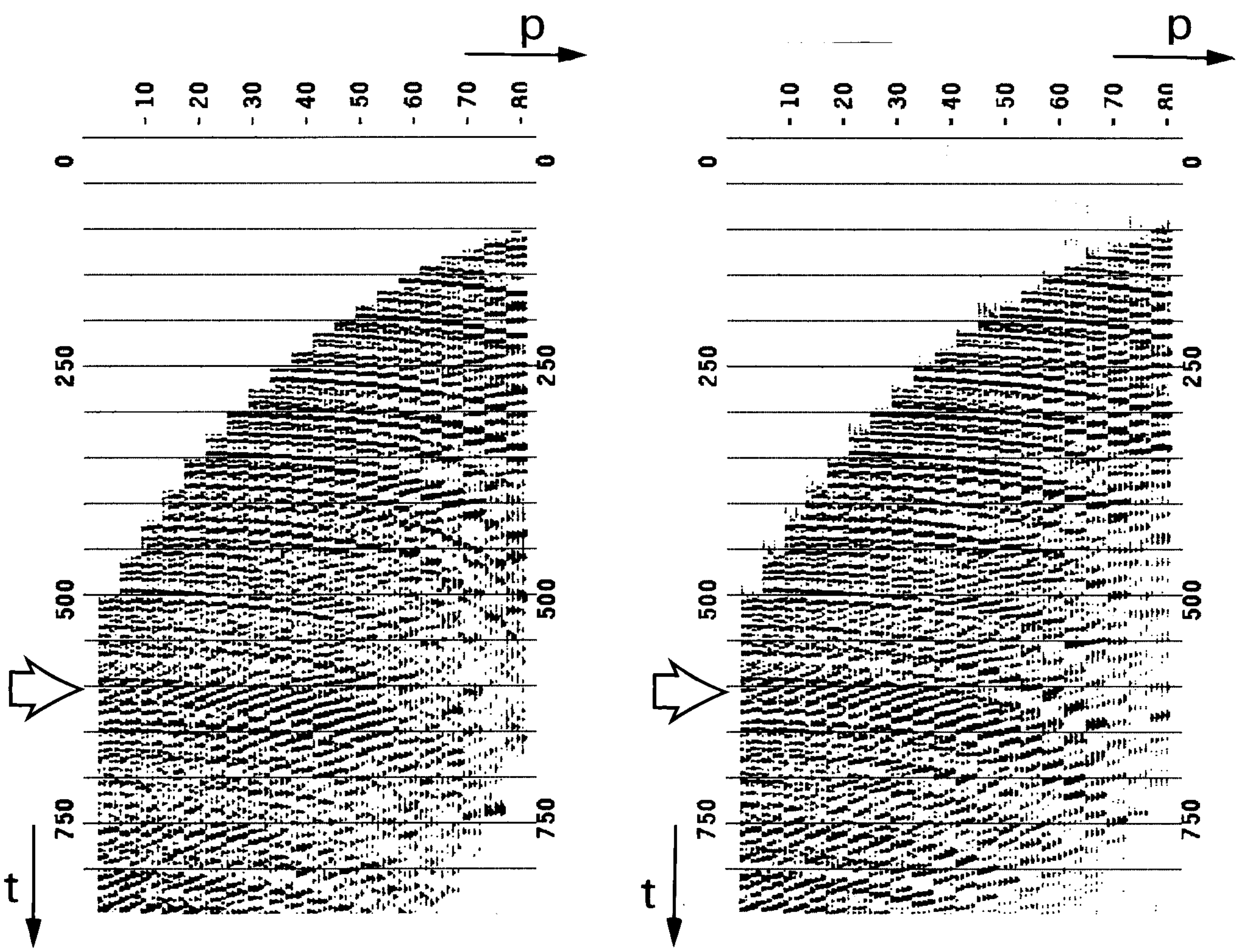

FIG. 11. Unmigrated and pre-stack partial migrated Snell trace gathers of same data from figure 9. As expecteed, there is little change between the two gathers, except for dipping events. The moveout of dipping events has been increased to bring them into line with flat events.