Velocity Analysis By Snell Trace Deformation And Stolt Imaging In Snell Midpoint Coordinates

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Abstract

In a stratified earth Snell Midpoint Coordinates are a suitable frame to put seismic data before velocity estimation. Main advantages are that we can image the data and estimate velocity exactly. Imaging is costly because it requires a phase shift method to handle a $v(z)$. However, if we do a Hyperbolic Stretch with some $\overline{v}(z)$, the data field will be quasi-hyperbolic, imaging could then be done using an Stolt algorithm and reduce its cost. The process can be applied iteratively to converge to the correct velocity function. In practice two or three iterations should suffice.

1. Velocity Analysis By Snell Trace Deformation And Stolt Imaging In Snell Midpoint Coordinates.

In previous *SEP*-reports, two new processes have been introduced separately for the problem of *velocity estimation* in an stratified earth. The first process was to transform seismic data into Retarded Snell Midpoint Coordinates, and read velocity information directly from the data (Claerbout, 1978; *SEP*-15, p. 57-71). The second process was to deform a *CMP*-gather with some first order estimation of the velocity $\overline{v}(z)$ into hyperbolas, using the definitions of Snell traces and Radial traces (González and Claerbout, 1981; *SEP*-26, p.137-156. Ottolini, 1981; *SEP*-26, p. 83-94). In this paper we want to explore the possibility of merging the two process in a velocity estimation routine. Our goal is to improve the resolution of the velocity function for early primary arrivals in a *CMP*-gather.

Several advantages result when we merge the two processes. By doing a hyperbolic stretch in the data, the mapping equations do three functions at once. First, all postcritical reflections and refraction data are muted smoothly by mapping them to infinity, thus avoiding the need of muting the data before the deformation. Second, it is easy to resample the data more densely (interpolation) while doing the stretch, and have better resolution for the precritical arrivals of the original gather, in particular for early arrivals. And third, the new space is hyperbolic. When we started imaging the data in Retarded Snell Midpoint Coordinates we wanted to handle stratified velocities, therefore we did the imaging using a phase-shift algorithm. This is an expensive process. However, if we preprocess the data to an hyperbolic space, an Stolt-type algorithm may be used instead, with a considerable savings in computations.

2. Retarded Snell Midpoint Coordinates.

The *Retarded Snell Midpoint Coordinates* were originally introduced by Claerbout (1978, *SEP*-15, p. 81-87), where advantages of processing seismic data in this coordinate frame were discussed, in particular the problem of accurate velocity estimation in an stratified earth. This coordinate system avoids the need to do small angle approximations in the expressions for velocity estimation. Besides, all transformations performed on the data are linear.

We can formulate the wave equation in Retarded Snell Midpoint Coordinates, and find wavefield extrapolators that enable us to image the data before velocity estimation. This was done using a phase-shift algorithm in the fk-domain (González and Claerbout, 1979, *SEP*-16, p. 181-204). In this article we explore the possibility of doing the imaging with an Stolt-type algorithm (Stolt, 1978). Our aim is to reduce the cost of the process while still getting the advantage of working with an imaged wavefield to estimate velocity.

Originally, the Snell Midpoint coordinate system was defined as a *retarded* frame:

$$
\begin{aligned}
t' &= t - p(g-s) + 2\int_0^z \frac{\cos\vartheta}{v(\xi)}\,d\xi \\
y &= \frac{g+s}{2} \\
h &= \frac{g-s}{2} + \int_0^z \tan\vartheta\,d\xi \\
\tau &= 2\int_0^z \frac{\cos\vartheta}{v(\xi)}\,d\xi
\end{aligned}
$$

where (s,g,z,t) = (shot,geophone,depth,travel-time) are the recording coordinates, and $pv = \sin\vartheta$.

In this paper, to formulate the dispersion relation in the fk-domain for an Stolt algorithm, we need to redefine the coordinate system as a *non-retarded* one. The definitions become:

$$
\begin{aligned}
t' &= t - p(g - s) \\
y &= \frac{g + s}{2} \\
h &= \frac{g - s}{2} + \int_0^z \tan\vartheta \, d\xi \\
\tau &= 2 \int_0^z \frac{\cos\vartheta}{v(\xi)} d\xi
\end{aligned}
$$

In this system the imaging conditions become $s = g$, and $t' = 0$.

The dispersion relation in this non-retarded Snell midpoint frame is:

$$
\frac{k_\tau}{\omega} = -\frac{pv}{1 - p^2v^2} H - \left[1 - \frac{2pvH + H^2}{1 - p^2v^2} \right]^{1/2} \tag{1}
$$

$$
H = \frac{k_h v}{2\omega}
$$

Transformation of field data into Snell coordinates requires two steps: 1. Sorting the data to CMP-gathers. 2. At $z = 0$ we obtain a *linear movout correction*:

$$
t'_{z=0} = t - p(g - s)
$$

This correction needs to be applied to the CMP-gathers.

3. Stolt Imaging In Snell Coordinates.

To formulate the Stolt-imaging of the data from (h,t') space to (h,τ) space, we need to change variables in the following integral:

$$
P(h,\tau,t'=0) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(k_h,\tau=0,\omega) e^{i\int_0^\tau k_\tau d\tau} e^{ik_h h} dk_h \, d\omega \tag{2}
$$

to resemble a $2D$-fft of the form

$$
P(h,\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(k_h,k_\tau) e^{-i(k_h h + k_\tau \tau)} dk_h \, dk_\tau \tag{3}
$$

Using the dispersion relation (1), we can solve for ω to get

$$
\omega = -\frac{pv^2k_h}{2(1 - p^2v^2)} + \left[k_\tau^2 + \frac{k_h^2 v^2}{4(1 - p^2v^2)} + \frac{pv^2 k_h k_\tau}{1 - p^2v^2} + \frac{p^2v^4k_h^2}{2(1 - p^2v^2)^2} \right]^{1/2} \tag{4}
$$

Next, assuming constant velocity, and changing variables in (2) from $\omega \rightarrow k_\tau$ we obtain

$$P(h,\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(k_h, k_\tau = k_\tau(k_h,\omega))\, e^{-i(k_h h + k_\tau \tau)} \left| \frac{d\omega}{dk_\tau} \right| dk_h\, dk_\tau \tag{5}$$

where the obliquity factor is given by

$$\frac{d\omega}{dk_\tau} = \frac{k_\tau + \dfrac{pv^2 k_h}{2(1-p^2v^2)}}{\left[k_\tau^2 + \dfrac{k_h^2 v^2}{4(1-p^2v^2)} + \dfrac{pv^2 k_h k_\tau}{1-p^2v^2} + \dfrac{p^2v^4k_h^2}{2(1-p^2v^2)^2} \right]^{1/2}} \tag{6}$$

4. Snell Trace Deformation.

To take advantage of the Stolt algorithm when the velocity structure is stratified, we need to deform the data into hyperbolas. The process of transforming seismic data from *Snell trace* space to *Radial trace* space has the equivalent effect of mapping non-hyperbolic seismic arrivals into hyperbolic arrivals, (González and Claerbout, 1981; *SEP*-26, p.181-204). This process consists of two steps:

i) Transformation to (p,z) space.

For the first transformation we need to take the seismic *CMP* gather and determine a velocity function $\bar{v}(z)$. This velocity function does not need to be extremely accurate, but should include the most predictable changes in the velocity function, such as the sea-floor sediment interface for marine data. Using this velocity function we use ray tracing equations to transform the data into (p,z) coordinates.

$$\begin{bmatrix} t \\ \\ h \end{bmatrix} = \begin{bmatrix} \displaystyle\int_0^z \frac{2\, v(\xi)^{-1}}{\left[1 - p^2 v(\xi)^2\right]^{1/2}} d\xi \\ \\ \displaystyle\int_0^z \frac{pv(\xi)}{\left[1 - p^2 v(\xi)^2\right]^{1/2}} d\xi \end{bmatrix} \tag{7}$$

where t is two-way traveltime, h is half-offset, v is velocity, and p is the Snell parameter.

ii) Transformation to $(\hat{h},\hat{t})$ space.

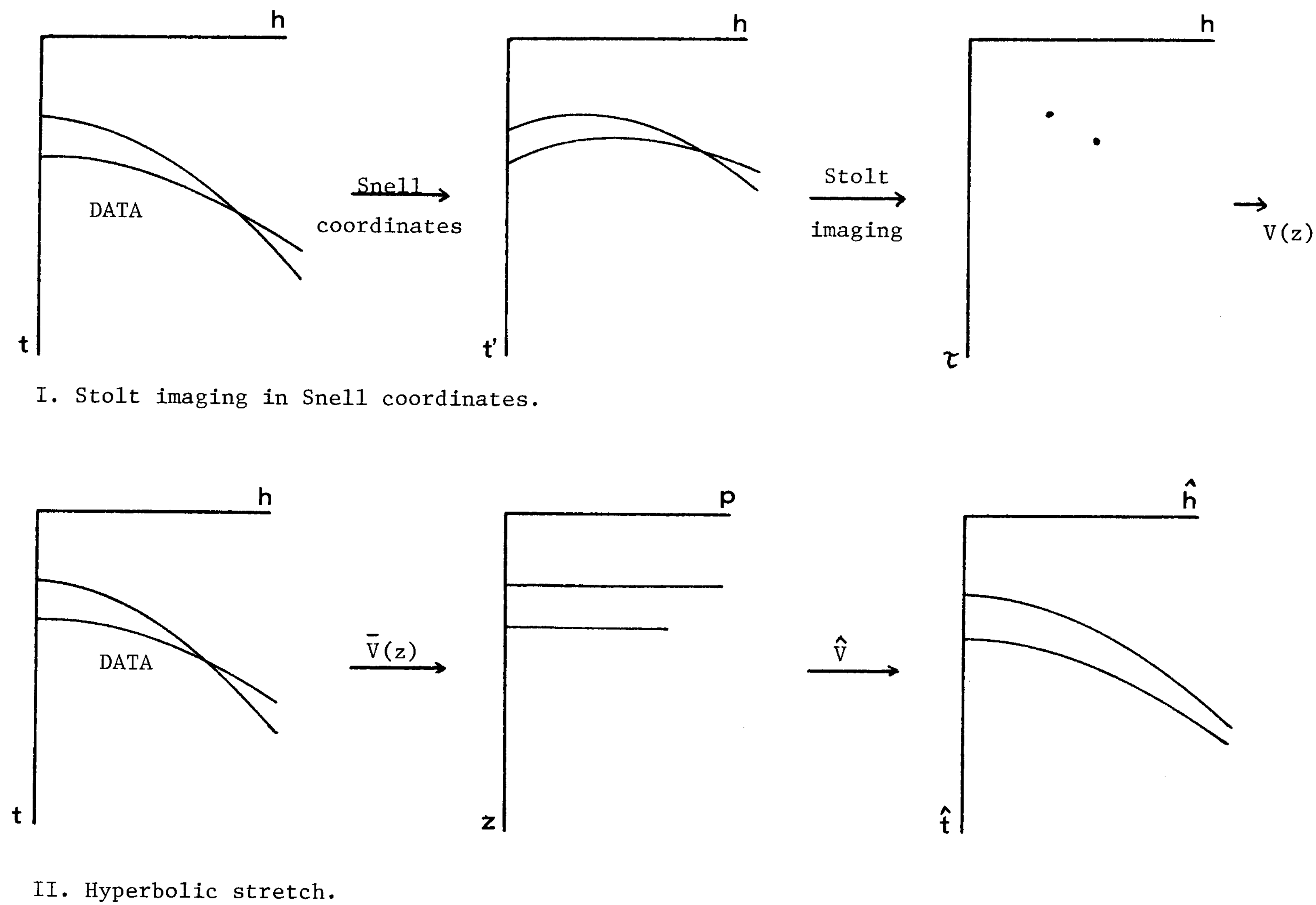


FIG. 1. **Velocity estimation using Stolt Imaging and Hyperbolic stretch. The process consists of iteratively applying this two processes. Stolt imaging in Snell midpoint coordinates transforms the data into an space where we can read velocities directly from the imaged data. Hyperbolic stretch uses the current velocity function to map the data into a quasi-hyperbolic space, this space is required by Stolt imaging.**

The second transformation consists of taking the data from the (p,z) space into new *CMP* $(\hat{h},\hat{t})$ coordinates. This second transformation is done at a constant velocity $\hat{v}$. For constant velocity we can invert the ray-equations (7) to get

$$\begin{bmatrix} p \\ \\ z \end{bmatrix} = \begin{bmatrix} \dfrac{\hat{h}}{\hat{v}^2\hat{t}} \\ \\ \dfrac{1}{2}\left(\hat{v}^2\hat{t}^2 - \hat{h}^2\right)^{1/2} \end{bmatrix} \tag{8}$$

We stress that for computation the intermediate step is not necessary. Since the ray equations are invertible for the second transformation, we may substitute directly equation (8) into (7) to get the result in a single pass.

5. Velocity Estimation.

A relationship to estimate a velocity function using the *LMO* (*LinearMoveout*) method is found substituting the imaging conditions $s = g$ and $t' = 0$ into the definition of Snell coordinates, to get:

$$v^2 = \frac{1}{p\left(p + \dfrac{1}{2}\dfrac{d\tau}{dh}\right)} \tag{9}$$

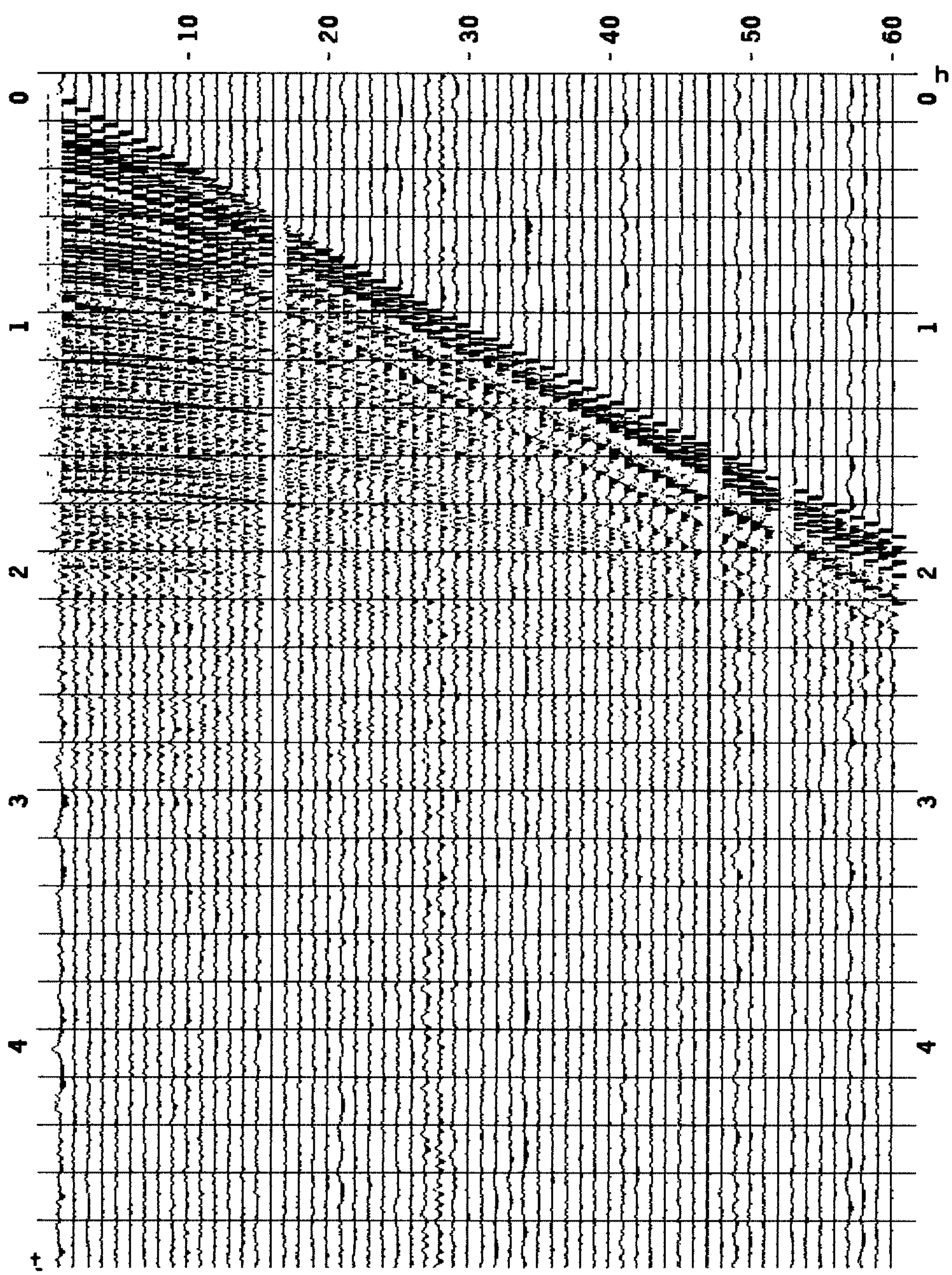
Figure(1) illustrates the process we propose to estimate velocity. It consists of imaging plus stretching of the data. The first step is to get an approximate velocity function $\bar{v}(z)$. In particular we are interested to resolve velocity for early arrivals close to the sea-floor. We can turn to conventional hyperbolic velocity estimation to get the first velocity function, but this process requires muting of refraction arrivals to get resolution for early arrivals, and then we get the problem of end effects. Instead we found the first estimate $\bar{v}(z)$ using the *LMO* method. Additional advantages are that the *LMO* method is less expensive than conventional hyperbolic scanning, and that it is exact.

Since our process is iterative, we need to write the velocity at a given depth as a series expansion about the $\hat{v}$ we are using in the hyperbolic stretch. Because we are going to read velocities in imaged (h,τ) space, it can be done as follows:

Defining as m the slope in imaged coordinates (h,τ)

$$m \equiv \frac{d\tau}{dh}$$

FIG. 2. Common midpoint marine gather. nh=60 nt=120 dt=0.004 s dh=25 m h0=68.5 m. No gain applied.



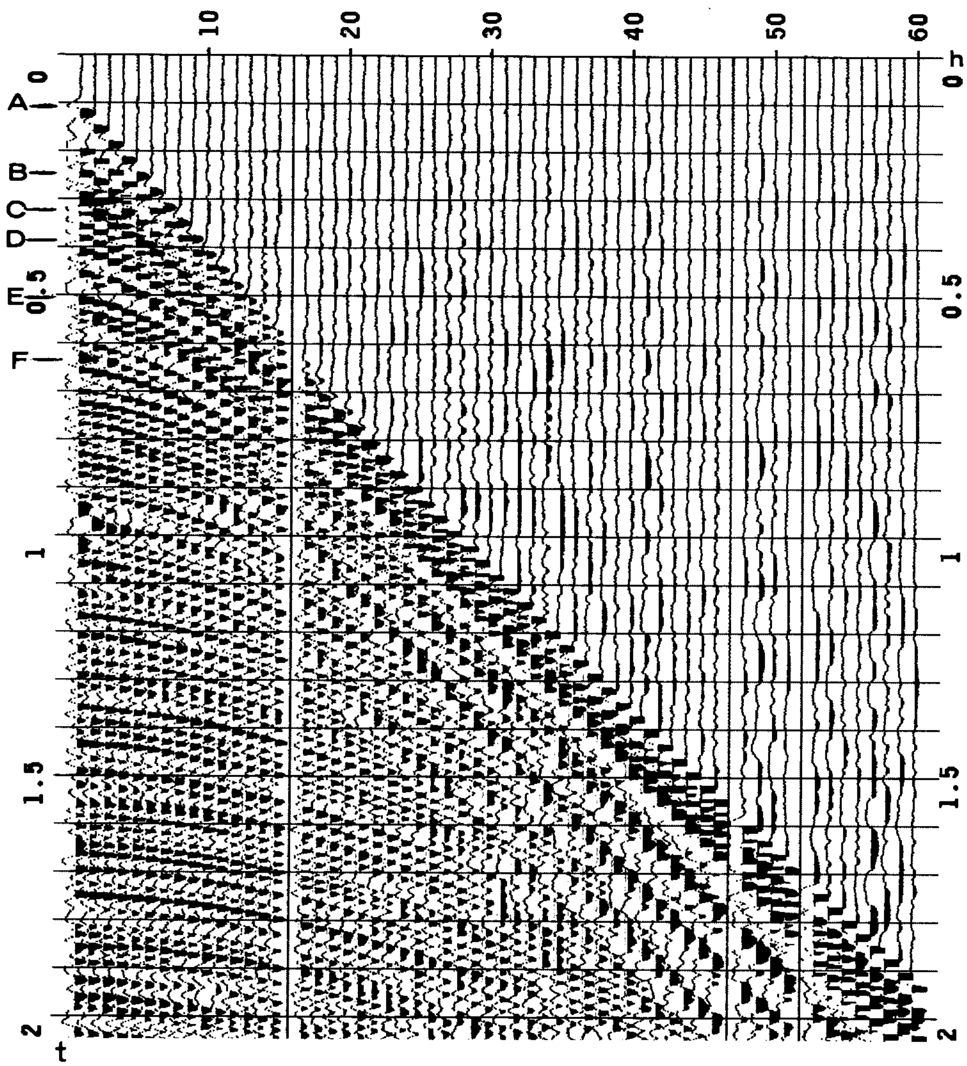


FIG. 3. Window of the data from figure 2. The data has been square-root gained. We will concentrate in the first half second of data to estimate velocity. The reference events are

event	t_0
A	0.10
B	0.24
C	0.32
D	0.38
E	0.50
F	0.63

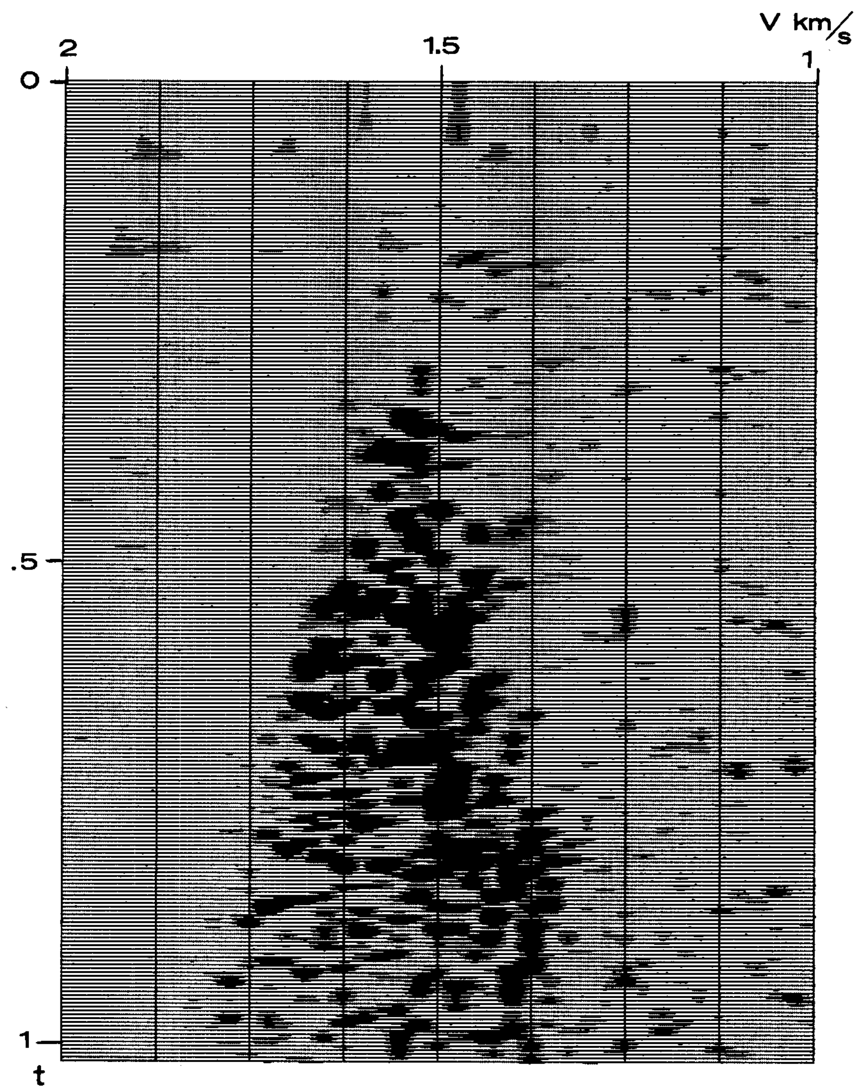


FIG. 4. Conventional velocity estimation for the gather of figure 3. Note in particular the poor resolution for the first .35 s of data. We did *not* use this panel to estimate $\bar{v}(z)$.

we can write an expansion for the velocity as

$$v = \hat{v} + \left[\frac{\partial v}{\partial m} \right]_{v=\hat{v}} dm + \left[\frac{\partial^2 v}{\partial m^2} \right]_{v=\hat{v}} dm^2 + \cdots \tag{10}$$

We are making the expansion about $\hat{v}$. This is the inverse mapping velocity in the hyperbolic stretch, and is also the imaging velocity. We are interested in measuring departures from this reference velocity to correct $\bar{v}(z)$.

We can use equation(9) to find $\partial v / \partial m$

$$\left(\frac{\partial v}{\partial m} \right)_{v=\hat{v}} = - \frac{p}{4 \left(p^2 + m_{v=\hat{v}} \right)^{3/2}} \tag{11}$$

$$m_{v=\hat{v}} = 2 \frac{1 - \hat{v}^2 p^2}{p \hat{v}^2}$$

From this equation we can find the desired dv correction. Keeping the linear term only

$$dv_{v=\hat{v}} = \bar{v}_{v=\hat{v}} - \hat{v} = \left(\frac{\partial v}{\partial m} \right)_{v=\hat{v}} dm = - \frac{p \hat{v}^3}{4} dm \tag{12}$$

In figure(2) we plot the marine seismic gather we will be using to test the method. Figure(3) shows the first 2 sec of data, where an *square –root* gain (Claerbout, 1981; *SEP*-26, p. 75-78) has been applied. In this figure we show the reference events we will use to test our method. For reference we did a conventional hyperbolic velocity estimation to this data in figure(4). Note in particular the poor resolution for the first $1/2$ second because we did not mute the data.

The next step is to transform the data into Snell midpoint coordinates. At the sea-surface $z = 0$ we only need to apply a linear moveout correction to achieve this. Figure(5) shows the data after *LMO*. In this correction we need to decide what value of the ray parameter p to use. This should be done remembering that the smallest value of p is a function of the near-offset in the gather. If the first trace has a large offset, a choice of too small p will place the stationary point of the hyperbolas in the zone where we have no data. On the other hand, a too large p value will restrict the range of velocities we can estimate, and will enhance the asymmetry of the skewed-hyperbolas, introducing a bias when we estimate velocity. In practice we probably want to divide the data into several regions, then we can use different p in each region according to some rule, for instance

$$p \approx 0.5 / v_{min}$$

In figure(5) we used $p = 1/3333 \ s/m$ for the first $1/2$ sec of data.

Figure(6) is a reference grid to estimate velocity generated using equation(9). We can use it directly on figure(5) to get the first velocity function. However, we can image the data at this point using equation(5).

To test our process we are mostly interested in early arrivals, a choice of water velocity $\approx 1500 \ m/s$ for the first imaging of the data is reasonable. We need to avoid artifacts from end effects, and errors because we are using a constant velocity algorithm in

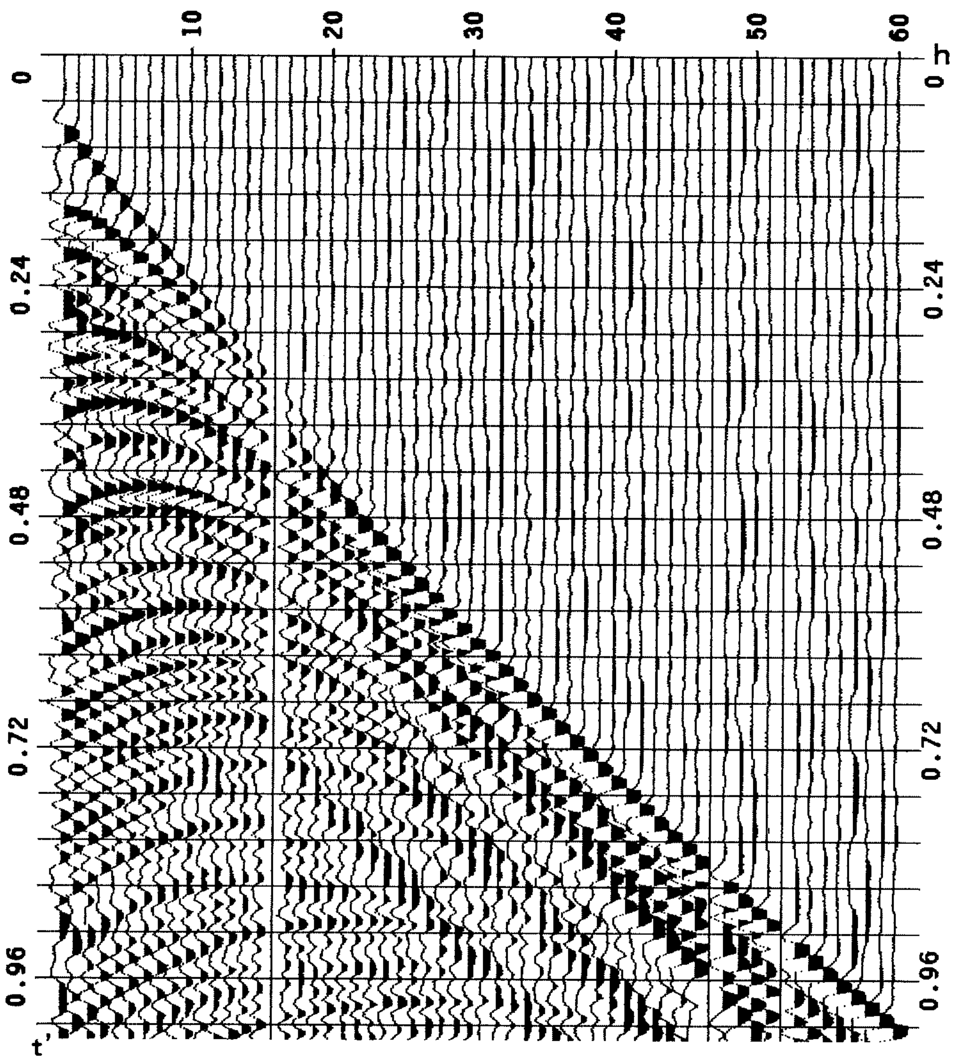


FIG. 5. The data after Linear Moveout Correction. The ray parameter is $p = 1/3333 s/m$. We could estimate velocity directly in this panel identifying coordinates of hyperbola-tops. However we prefer to image the data first.

a variable velocity wavefield. For these reasons we may restrict ourselves to a range of 15^0 about the stationary point of the skewed-hyperbolas to image the data. This is done easily remembering that by definition

$$H = \frac{k_h v}{2\omega} = \sin\vartheta' \tag{13}$$

where ϑ' is measured about the Snell trajectory associated with the ray parameter p used in the LMO-correction. About the stationary point, the data should be less sensitive to

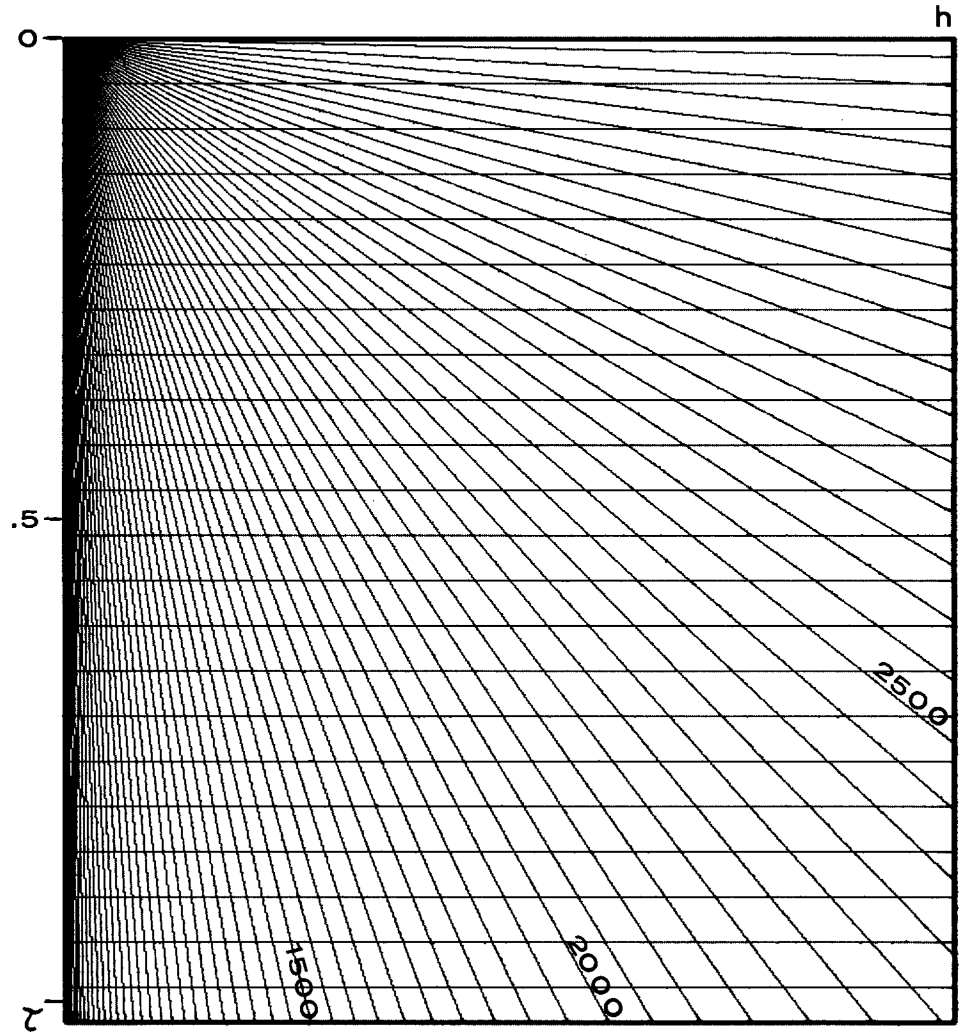


FIG. 6. Reference grid to estimate velocity. The slope about the origin gives the *RMS velocity*, the slope between two consecutive primary events gives the *Interval velocity*. This grid has the same dimensions as figs (5) and (7), the ray parameter is $p = 1/3333 \ s/m$.

velocity compared with wide offsets. Figure(7) shows the imaged data, as well as the Snell trajectory for the background velocity $\hat{v}$. The obliquity function given by equation(6) was not implemented because it becomes important at wide angles.

A useful feature of imaging the data in Snell Coordinates is the separation of events with angle. Multiple reflections stay aligned below their associated primaries (at water velocity slope) becoming easy to discriminate. Refractions remain at high angles, without strong interference with primaries. This decoupling is an advantage in estimating velocity.

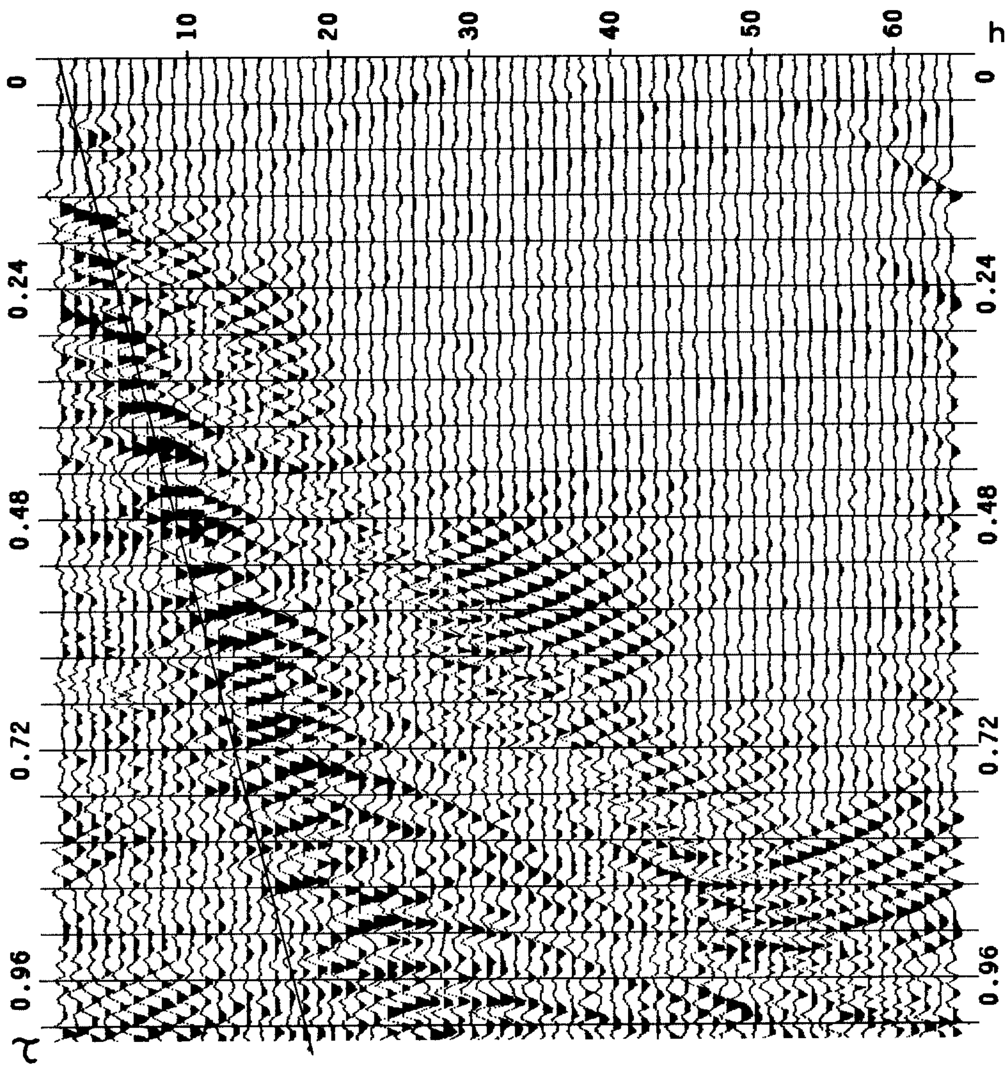


FIG. 7. The data after imaging using an Stolt algorithm. For this step we did not know the velocity function, thus we used water velocity v = 1500 m/s for the extrapolation. Also there is no need to do the imaging to wide angles. We used a window of 15^0 measured about the reference Snell wave (characterized by its ray-parameter p). The line drawn is for water velocity, the tops of the hyperboloids are right-shifted because events have higher velocity than water.

The next step is to use $\bar{v}(z)$ in our equations for the hyperbolic stretch. For early arrivals, and for the particular data set we are using, with prescence of strong refractions, the critical angle is reached within a few offsets. This is a severe drawback in conventional velocity estimation. However we can use our hyperbolic stretch to interpolate the data more densely, while at the same time muting-out refractions by sending them to infinity. In figure(8) we display the data after deformation ($\hat{v}$ = 1700 m/s), where we resampled the

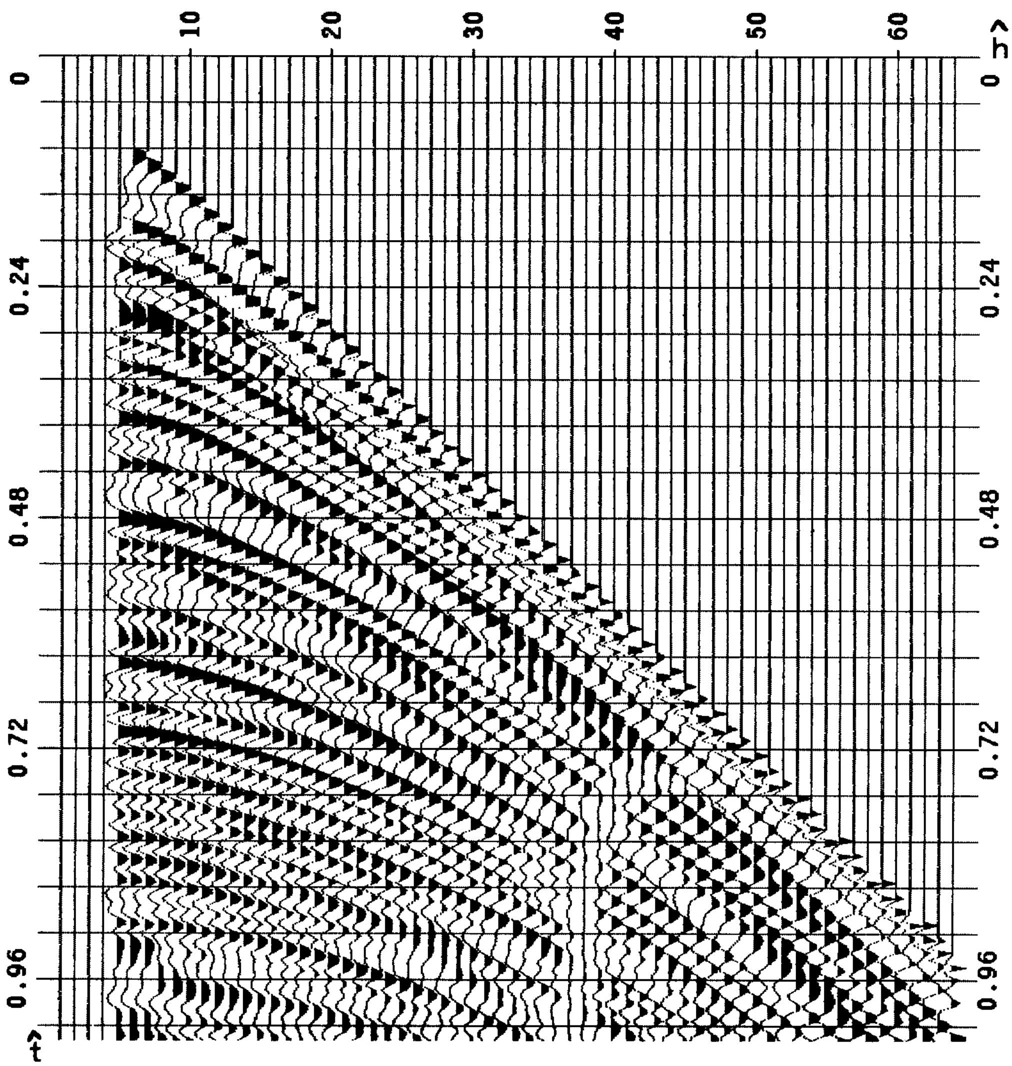


FIG. 8. This figure shows the mapping to Hyperbolic stretch for the data of figure (3). We are interested in estimating velocity for only the first half second of data, therefore in the mapping we resampled the gather to duplicate the number of traces dt_{new} = 12.5 m. Note in particular the location of the first missing trace in the gather. The velocity function $\bar{v}(z)$ is plotted in figure (13).

gather to duplicate the density of precritical data. $dh_{old} = 25.\ m \rightarrow dh_{new} = 12.5\ m$. Figure(9) is a conventional velocity estimation with this stretched data. Compare the resolution of the first 0.5 sec with that in figure(4).

The last step of the process is to find the correction dv to our initial $\bar{v}(z)$. We use again an Stolt imaging of the data, but now we can go a little further than 15^0 because the data has a first order correction to hyperbolicity. For this imaging we must be careful to

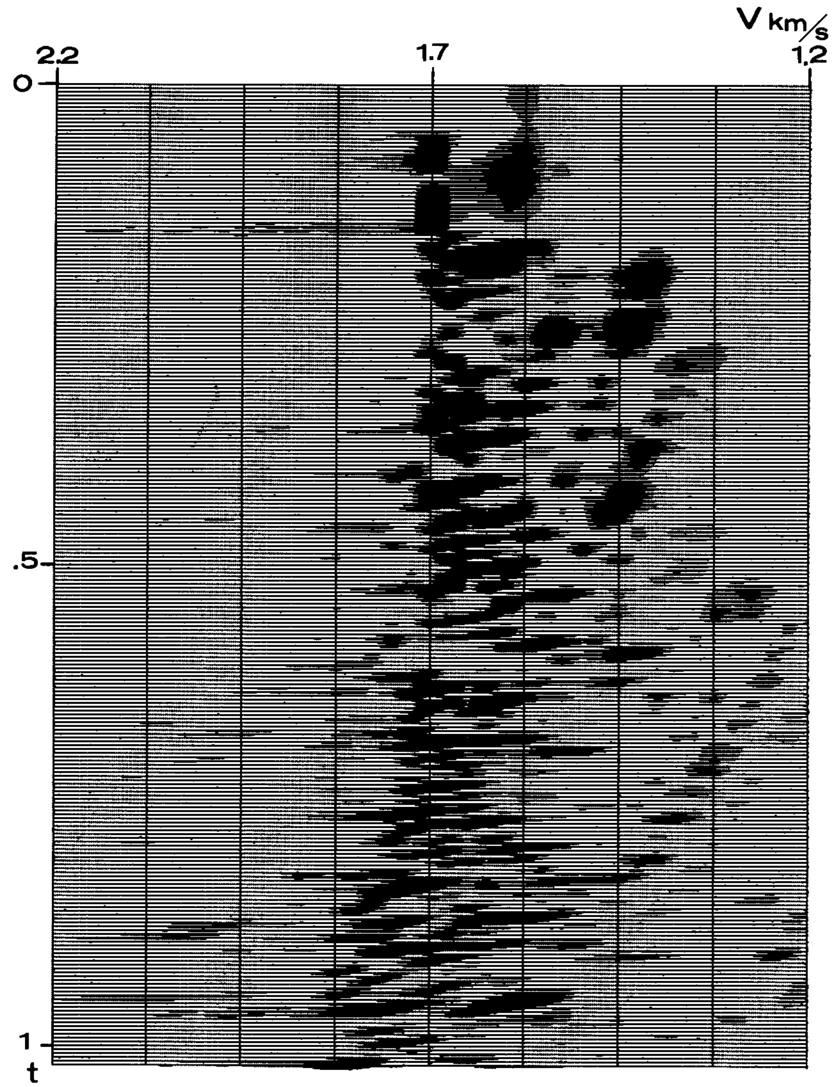


FIG. 9. Conventional velocity estimation for the stretched data. The events for the first half second of data are aligned along the background velocity used in the Stolt imaging and the Hyperbolic stretch. $\hat{v}$ = 1700 m/s. Compare the resolution for the first half second of data with the non-stretched velocity analysis of figure 4. We did *not* use this panel to estimate velocity corrections.

choose the same velocity $\hat{v}$ that was employed in the hyperbolic stretch. The representation of the velocity function is a series expansion about this velocity. In figure(10) we display the stretched data with a *LMO*-correction (p = 1/3333 m/s). Figure(11) is the reference grid for velocity estimation with the new offset space dh = 12.5m. Figure(12) is the image of the data. Note all reference events are now aligned along the slope corresponding to the background velocity. Figure (13) shows the final velocity function, it

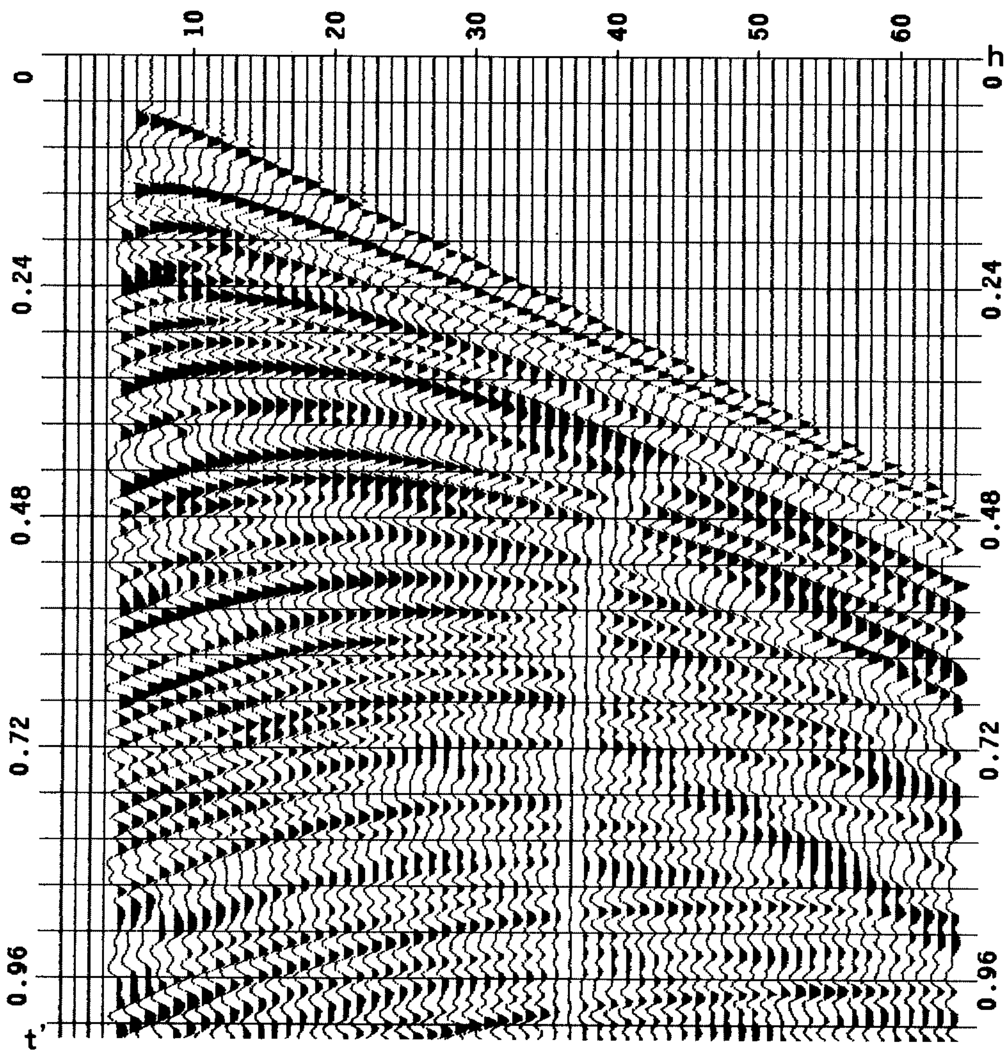


FIG. 10. The stretched data of figure (8) with Linear Moveout correction. $p = 1/3333\ s/m$.

took 3 iterations to align the reference events. With experience doing velocity corrections we can reduce the number of iterations. Velocity corrections were done using equation(12).

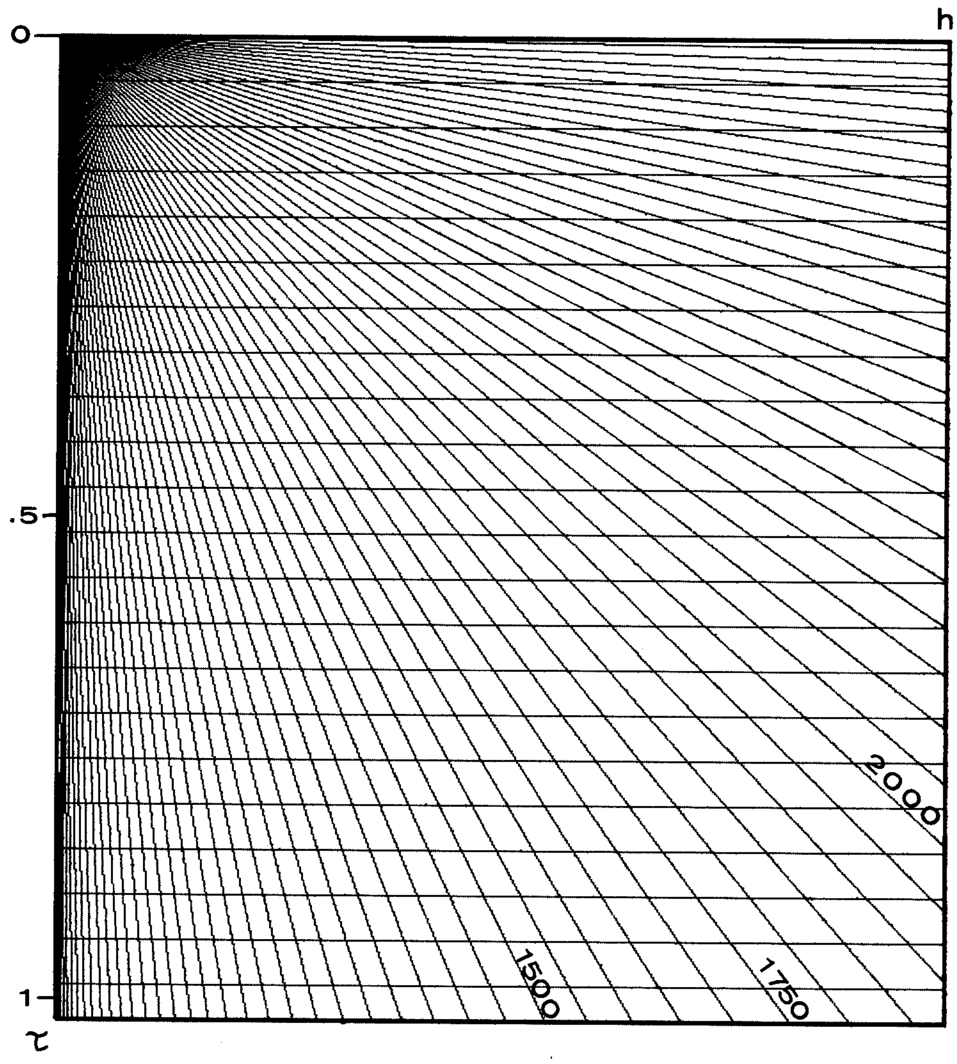


FIG. 11. Reference grid to estimate the velocity corrections in the stretched data, with the new range of offsets. $p = 1/3333\ s/m$.

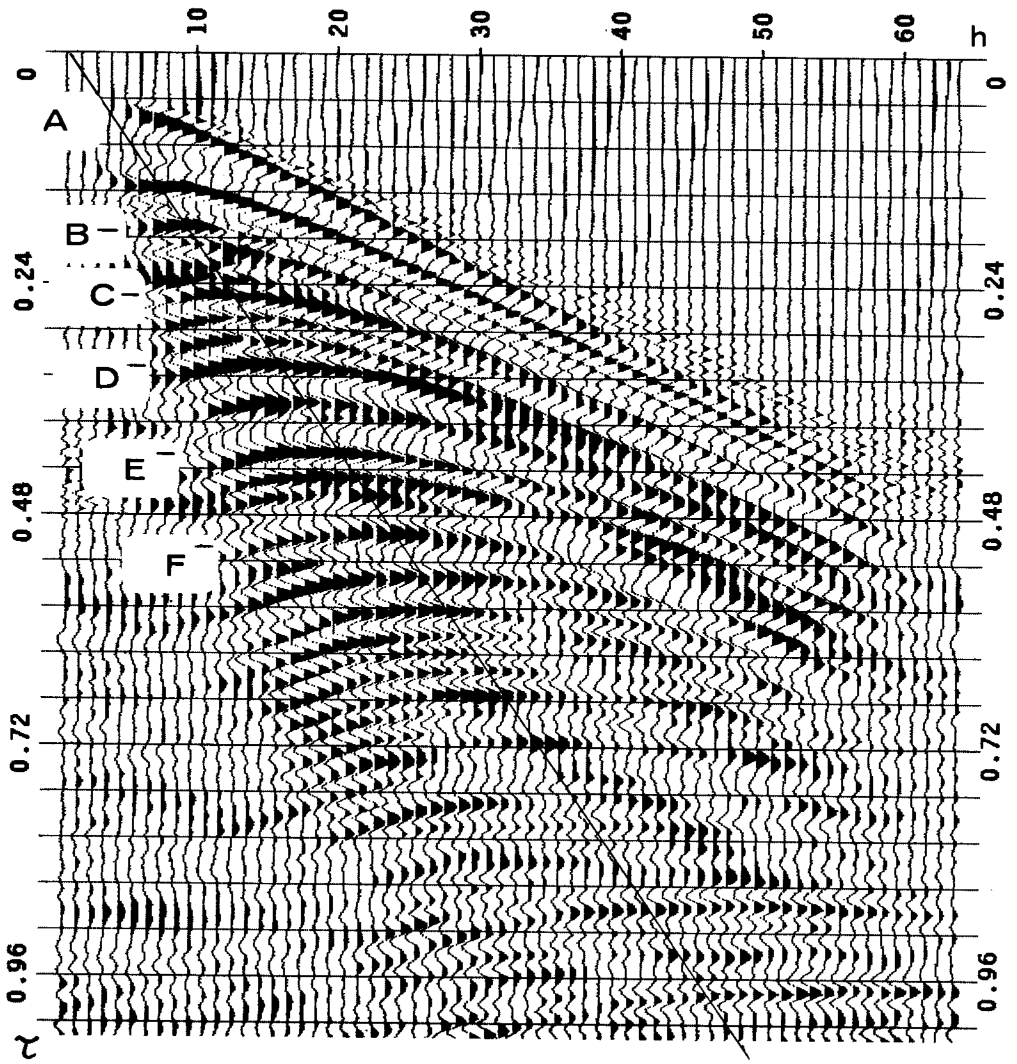


FIG. 12. Image for the stretched data. We used the velocity function plotted in figure 13. Reference events are shown in figure (3). The image was done with background velocity $\hat{v}$ = 1700 m/s, the window was 30^0 about the reference Snell parameter p = 1/3333 s/m. Note how we have aligned all reference events along the background velocity. Departures from the Radial trace with ray parameter p give the desired velocity corrections. It took 3 iterations to get this image.

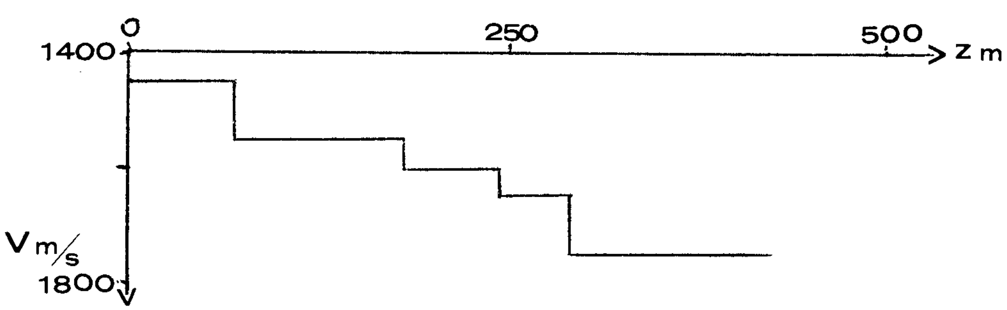


FIG. 13. Velocity function for the first half second of data obtained using the Hyperbolic stretch plus Stolt imaging. This velocity functions was obtained after 3 iterations.