WAVEFIELD INVERSION METHODS FOR REFRACTION AND REFLECTION DATA

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Wavefield Inversion Methods For Refraction and Reflection Data

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Abstract

Two inversion methods are presented: one for refraction (post-critical angle) data, and one for reflection (pre-critical angle) data. Both methods utilize the recorded data in the form of a sampled wavefield, and consequently are applicable to data that is well-sampled spatially.

For refraction data, the process of wavefield continuation (migration) is used to produce velocity-depth models directly from the recorded data. The procedure consists of two linear transformations: a slant stack of the data produces a wavefield in the $p-\tau$ plane which is then downward continued. The result is that the data wavefield is linearly transformed from the time-distance domain into the slowness-depth domain, where the velocity profile can be picked directly. No traveltime picking is involved, and all the data are present throughout the inversion. The method is iterative because it is necessary to specify a velocity function for the continuation. Convergence is determined when the output wavefield images the same velocity-depth function as was input to the continuation. The inversion scheme is easily extendable to free-surface multiples. The method is illustrated with refraction lines from the Imperial Valley and Mojave Desert in California, and with data recorded with a marine streamer cable.

For reflection data, a method is presented for determining density and bulk-modulus variations in the earth from standard reflection surveys. Explicit formulas are given which utilize the amplitude-versus-offset information present in the observed wave fields. The method automatically accounts for dipping reflectors, but since it is based on a Born approximation of the scattering equation, it is restricted to subcritical reflections. For the inversion, the medium is considered to be composed of a known low-spatial frequency variation (the background) plus an unknown high-spatial frequency variation in bulk modulus and density (the reflectivity). The division between the background and the reflectivity depends on the frequency content of the source. Solutions are obtained for a constant background variation, a depth variable variation, and a smoothly varying background. The Born approach is also extended to two-dimensional elastic reflection data. The observations in this case are a linear combination of the scattering potential evaluated along four different shells, which may be interpreted as $P \rightarrow P$, $P \rightarrow S$, $S \rightarrow P$, and $S \rightarrow S$ scattering. If the source is either purely compressional or purely shear, then one experiment will suffice to invert the forward equation. If the source is a (known) mixture of P and S components, then two experiments with different combinations of P and S components are necessary for the inversion.

To extrapolate the wavefields, a formulation for one-way wave equations is given that is both accurate and stable. The scalar equations are generated from the full wave equation by a continued fraction square root recursion. The solutions are WKBJ-accurate in the extrapolation direction. The resulting operators are unconditionally stable. Scalar extrapolation is illustrated with the problem of computing Love wave modes in a laterally varying medium. For elastic problems, the choice of variables determines the form of the one-way extrapolation equations. Three sets of variables are considered: displacements, potentials, and a mixed set of variables. None of these three sets appear to satisfy the requirements of being both stable and accurate.

Approved for publication

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INTRODUCTION

An underlying problem in geophysics is determining the earth's properties from remote observations of related physical phenomena. The term inversion is commonly applied to this problem because in some sense, the solutions are obtained by inverting or "backtracking" the forward physical problem that relates the earth properties to the data. In this thesis two inversion methods are presented. The first is a procedure for determining vertical velocity variations from refraction data. The second is a method for determining the rapid variations in density and bulk modulus from subcritical reflection data.

The usual method for solving inverse problems is by model fitting. In simplistic terms, the procedure consists of computing a set of observations from a starting model and comparing these with the actual observations. The forward problem is usually cast as an algebraic system by discretizing the parameters of the model. The model is then updated either by trial and error or by linearizing and partially inverting the matrix of the system. The best fit is determined when the error in the system (the difference between actual and computed observations under some norm) is minimized. The common feature of the several variations of this approach to inversion is that the parameters in model space are transformed into data space. One advantage of this feature is that it only requires an understanding of the forward physical problem. The disadvantage to the approach is that certain features (such as traveltimes in seismic data) must be extracted from the observed data. There are two reasons for this. First, to predict the response for every observed data point will require an extremely large matrix in the algebraic system. Second, the data may contain effects that are not predicted by the model.

The inversion procedures presented in this thesis uses a reverse approach, that is, the observations are transformed from data space into model space. This approach was pioneered by Claerbout (c.f. 1976) in his work on wave equation migration. The chief difference is that all the recorded data is transformed into model space, and consequently no subjective feature extraction or "picking" of the data is required. Parts of the data that are not predicted by the model are mapped into artifacts under the transformation. The "picking" part of the inversion comes in extracting the model from its image in the model space. The interpreter of the inversion must learn to separate the real parts of the image from the artifacts. One advantage of delaying the picking is that errors in picking may be directly translated into error bounds on the model.

In Chapter I, the method for inversion of refraction data is presented. The procedure consists of two linear transformations which are applied to the entire recorded data set. The first is a slant stack which decomposes the wavefield into its plane wave components or ray parameters. The second transformation is a downward continuation which maps the time axis of the wavefield into a depth axis. The result of the two transformations is an image of the slowness versus depth curve. This solution must be iterated upon because the downward continuation step requires a knowledge of the velocity function.

The refraction inversion method makes two basic assumptions. The first is that the data is recorded with sufficient spatial density that it may be treated as a wavefield. The second is that the velocity function is laterally homogeneous. In Chapter II, two real data examples are presented which do not rigidly satisfy these assumptions. The result is that in model space the are several artifacts due to the aliasing of the data, and the slowness image is multivalued in some regions due to the lateral homogeneity. The inversion however, still produces a useable slowness image.

The inversion method for reflection data is presented in Chapter III. The method is based on the acoustic Born model which relates the rapid variations in bulk modulus and density to the amplitude-versus-offset dependence of the subcritical reflections. The inversion procedure consists of a migration of the data followed by a least squares fit of the reflectivity to the variations in bulk modulus and density. The migration removes the effects of propagation through the medium above a particular point and corrects for the dip of the structure. The least squares fit relates the reflection coefficient as a function of angle to the density and modulus variations. The inversion procedure is not an "exact" inversion method for several reasons. First the use of the Born model means the procedure is limited to sub-critical primary reflections. Consequently, multiple reflections and refracted waves will produce artifact that are similar to the ones found in standard migration. Second, the method assumes a knowledge of the slow variations in background velocity. The finite bandwidth the the typical seismic source prevents the method from estimating the slow variations the earth parameters.

The extension of the Born model to elastic waves to presented in Chapter IV. This allows one to include the effect of the shear modulus in the model for the reflectivity. Although the necessary field data for the inversion of elastic waves is probably not presently available, the analysis is useful for determining the effect of the shear modulus on the P to P reflection coefficients.

The primary tool used in the two inversion methods presented here is the downward continuation of the surface recorded wavefields. This step has to done in an accurate and stable manner if the inversion is to be meaningful. In Chapter V, the oneway operators for extrapolating scalar wavefields in a (known) laterally varying medium. This operators are unconditionally stable (in fact they are unitary) and are as accurate as WKBJ solutions in the direction of extrapolation. The fact that the operators are unitary means that signal to noise ratio in the inversion will remain constant.

In Chapter VI a discussion of the attempts to extend the scalar extrapolation theory to elastic wave fields is given. Stable and accurate extrapolation operators have not been found for the extrapolation of elastic wavefields.

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