Effect of Reflection Coefficients on Synthetic Seismograms II. Results

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We are going to show the results obtained in the preceding article to generate synthetic seismograms. We will discuss the problems that have occurred in our trials, and will give the solutions which have been employed.

1. Green's Function Problem.

We have seen that the Green's function for our problem was of the form

$$\frac{e^{-ik_z \Delta z}}{k_z}$$

If we do not take any further preventions and calculate the synthetic seismogram for the primary reflection given by

$$f(x,t) = \frac{i}{2(2\pi)^2} \int\int S(k_x,\omega)\ C(k_x,\omega)\frac{e^{-2ik_z\Delta z}}{k_z} e^{-ik_x x + i\omega t}\ dk_x\, d\omega$$

with

$$S(k_x,\omega) = \text{source term,}$$
$$C(k_x,\omega) = \text{reflection coefficient,}$$
$$k_z = \frac{\omega}{v}\left[1 - \left(\frac{vk_x}{\omega}\right)^2\right]^{1/2},$$

we obtain figure (1).

This effect comes from the fact that k_z can become *null*, and the particular frequency when this occurs becomes dominant.

The technique used to get rid of these artifact is not by solving the exact inverse Fourier transform, but rather a modified version for which the Greens's function is

$$\frac{e^{-ik_z \Delta z}}{k_z + \frac{\omega}{v}\varepsilon} \tag{1}$$

with ε being a small constant parameter with respect to $\left[1 - \left(\frac{vk_x}{\omega}\right)^2\right]^{1/2}$. This form keeps the causality of the Green's function.

In fact, since $\left[1 - \left(\frac{vk_x}{\omega}\right)^2\right]^{1/2} = \cos\vartheta$, we cannot have $\varepsilon \ll \cos\vartheta$ for every ϑ. As ϑ is approaching 90^0, $\cos\vartheta$ approaches 0 and thus ε can become bigger than $\cos\vartheta$.

If ε is too small, it does not take off all the effects of k_z becoming *null* (*cf* figure 2). If ε is small enough, it gives a very clean seismogram (*cf* figure 3).

Figures 2 and 3 are the seismograms obtained for a reflection coefficient $C(k_x,\omega) = 1$ with the corrections applied also in the evanescent region.

2. Evanescent Region Problem.

We have defined in our preceding paper the reflection coefficient for $0 \leq \sin\vartheta \leq 1$. In the evanescent region one can consider "$\sin\vartheta$" > 1. In this region, the plane wave theory we are using is no longer valid, and formally the only way to deal with it correctly is by using *conical waves* (Cagniard's waves).

The way we are approaching the solution is by calculating the *reflection coefficient* in the evanescent region using the equations valid in the propagating region. Now the only difference is that

$$\left[1 - \left(\frac{vk_x}{w}\right)^2\right]^{1/2} = i\left[\left(\frac{vk_x}{w}\right)^2 - 1\right]^{1/2}$$

and this will guarantee causality like for the propagating region.

On the other hand, the Green's function is becoming a real exponential and as we do not want to increase the energy of the wave as it propagates, we must take a decreasing exponenetial, that is

$$\frac{e^{-ik_z \Delta z}}{k_z}$$

where $k_z = \frac{|\omega|}{v}\left[\left(\frac{vk_z}{\omega}\right)^2 - 1\right]^{1/2}$.

The problem with taking a Green's function like the one above, is that it is no longer causal and therefore will give anticausal events. These anticausal events will be of importance only about the region $k_z = 0$, because everywhere else the decaying exponential becomes negligible.

This problem will be solved in a satisfactory way when we will deal with viscoelastic media. In these media, the evanescent zone does not exist anymore and everything keeps its causality.

There is also the problem presented in the first paragraph ($k_z \to 0$), and we will handle it using the same implementation, namely replacing the Green's function by

$$\frac{e^{-k_z \Delta z}}{k_z + \frac{|\omega|}{v}\varepsilon}$$

The results are presented for a constant reflection coefficient in figure 3. One can see that the anticausal events are not bothering us at all at the clip values used for plotting.

3. Synthetic Seismogram

The parameters used for generating the synthetic seismogram (figure 4) are the following:

First medium:

water

P-velocity $= \alpha = 1500\ m/s$

density $= \rho = 1$

Second medium:

solid

P-velocity $= \alpha' = 2500\ m/s$

S-velocity $= \beta' = 1200\ m/s$

density $= \rho' = 2$

The plots of the amplitude, phase, real-part and imaginary-part of the reflection coefficient are shown in figures (5) (6) (7) and (8) respectively.

The values chosen for the P-velocity and the S-velocity in the second medium are such that there exists only one head wave, as can be seen on figure 4.

We must also remark on figure 4 the π phase shift after critical angle, which has been predicted on figure (6).

ACKNOWLEDGMENTS

We wish to thank M. Yedlin for suggesting the source waveform function.

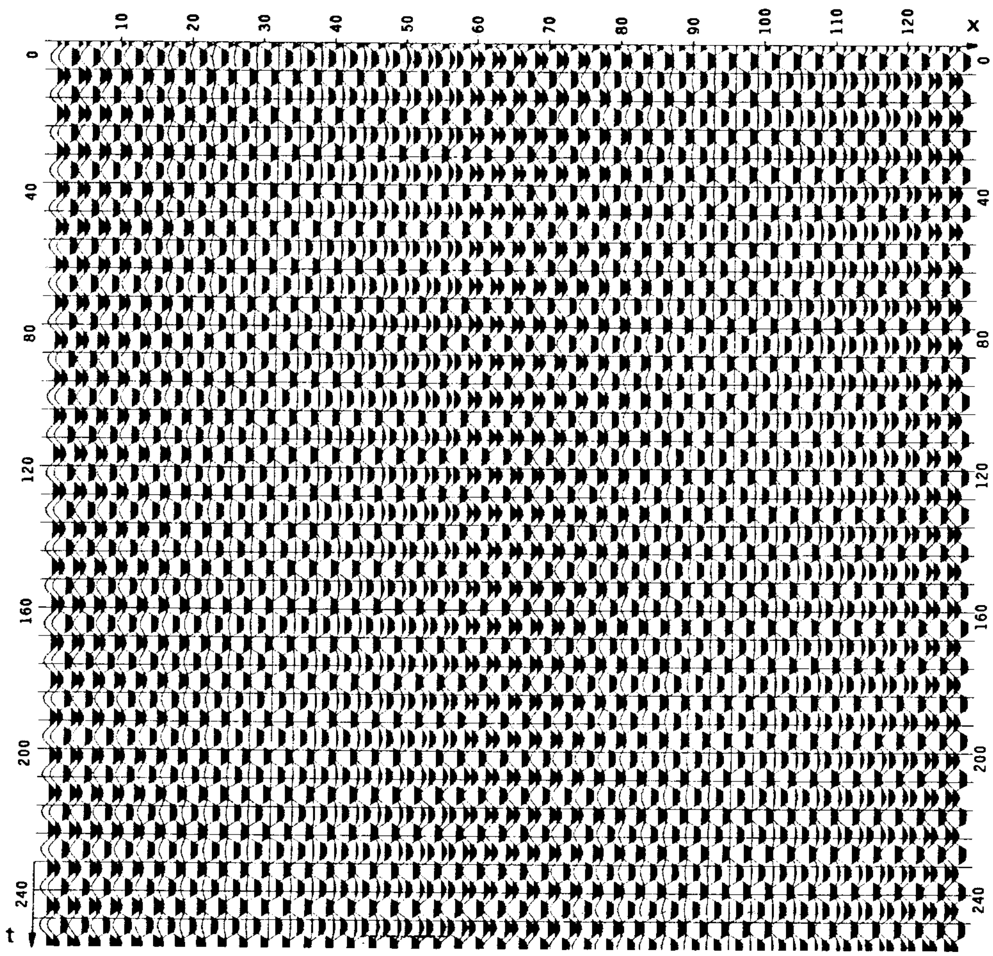


FIG. 1. This figure shows the effect of k_z becoming *null* in the Green's function. In this case a particular frequency becomes dominant.

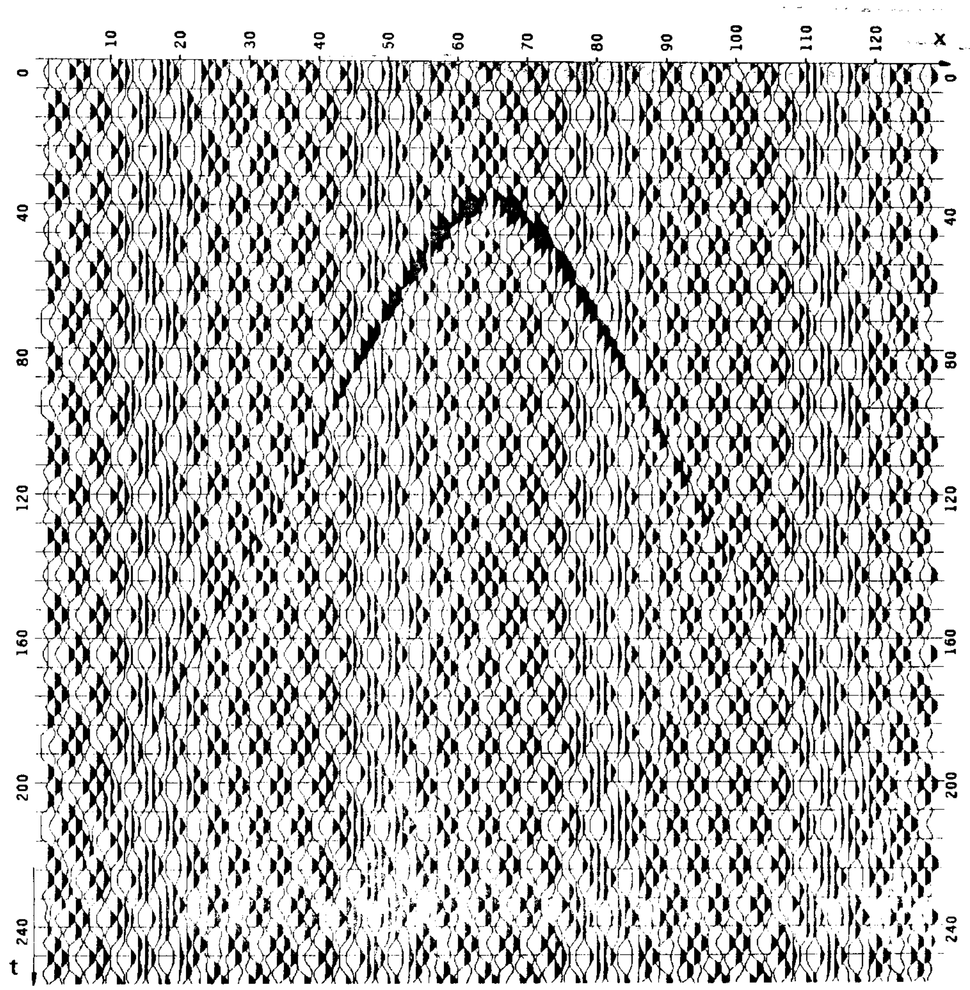


FIG. 2. In this figure we show the effect of the ε parameter defined in equations (1) and (2). Here we used a value of $\varepsilon = 0.02$. If we consider we are dealing correctly with angles for which the ε term is less than $\frac{vk_z}{\omega} = \cos\vartheta$, then we are speaking in this case of an accurate representation for angles up to 88.9^0. On the other hand we can see that the smoothing effect of ε is not enough.

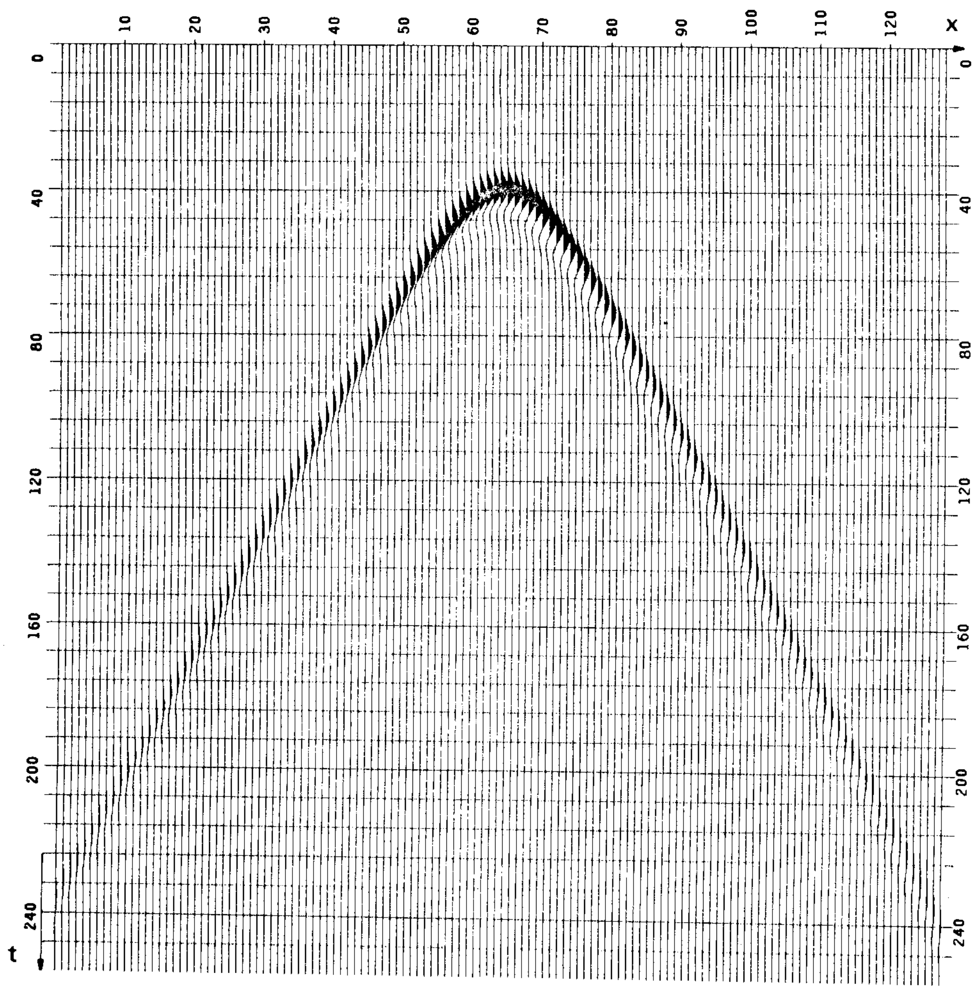


FIG. 3. In this figure $\varepsilon = 0.2$, which, using the criteria of figure (2) gives an accurate representation of angles up to 78.5^0. Here the reflection coefficient is 1 for all angles of incidence. The parameters used to generate this synthetic seismogram are: $dx = 86\ m$, $dt = 0.032\ s$, $nx = 128$, $nt = 256$, $nkx = 256$, $n\omega = 256$ and $\alpha = v_p = 1500\ m/s$. The density $\rho = 1$ in the first medium.

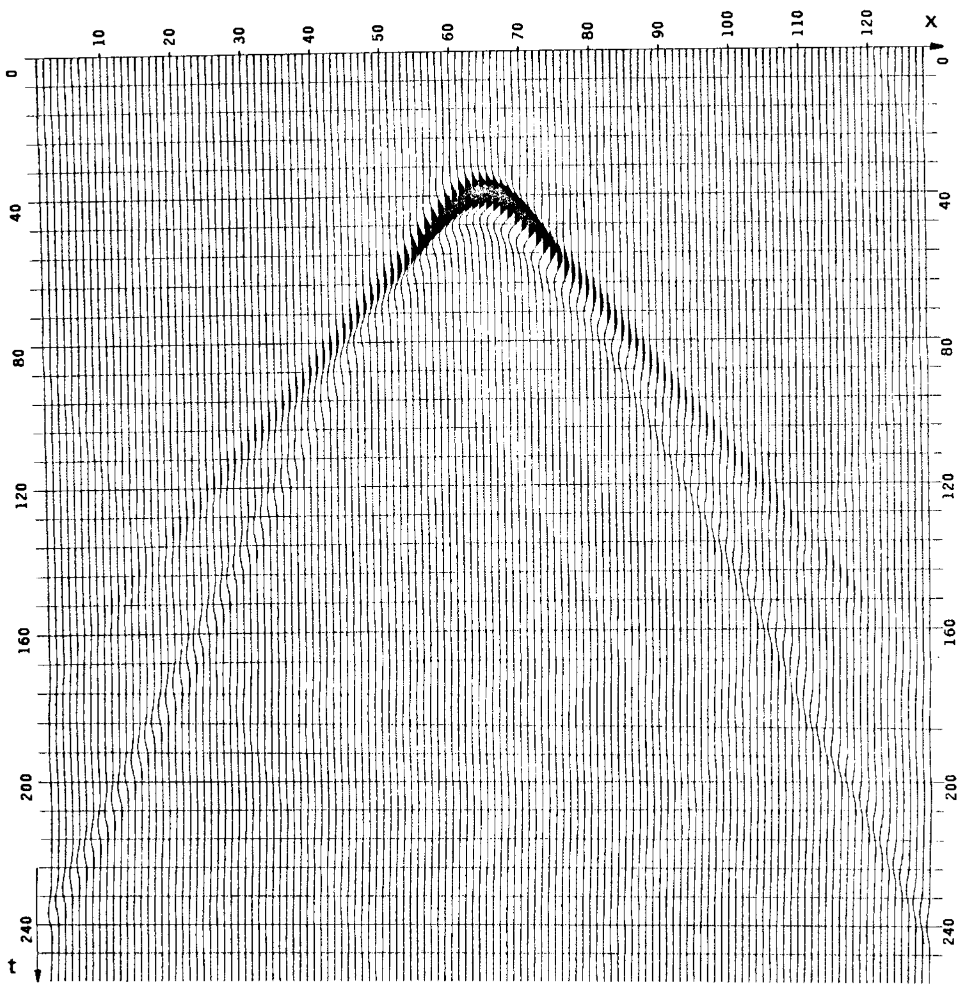
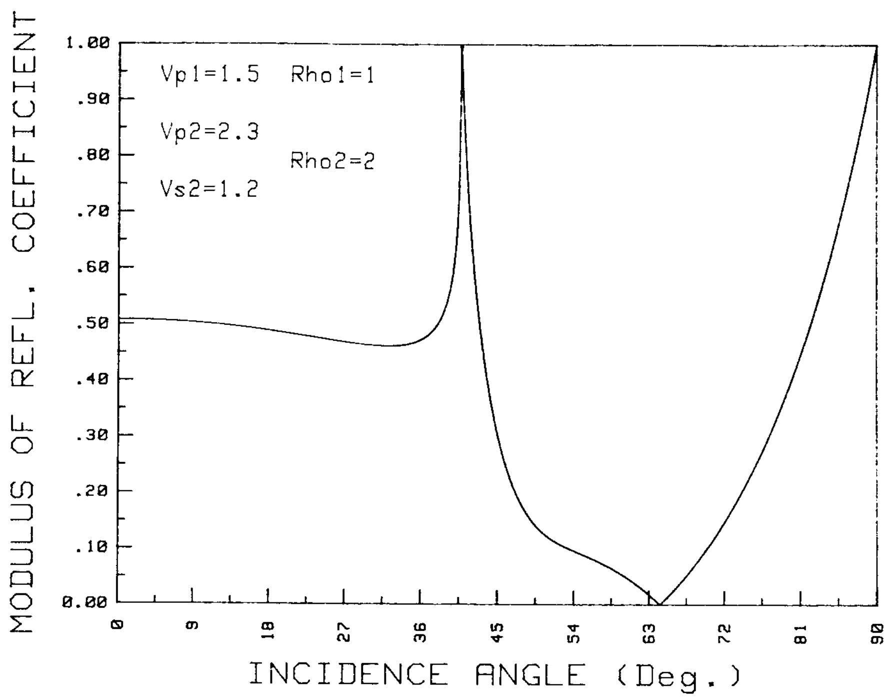
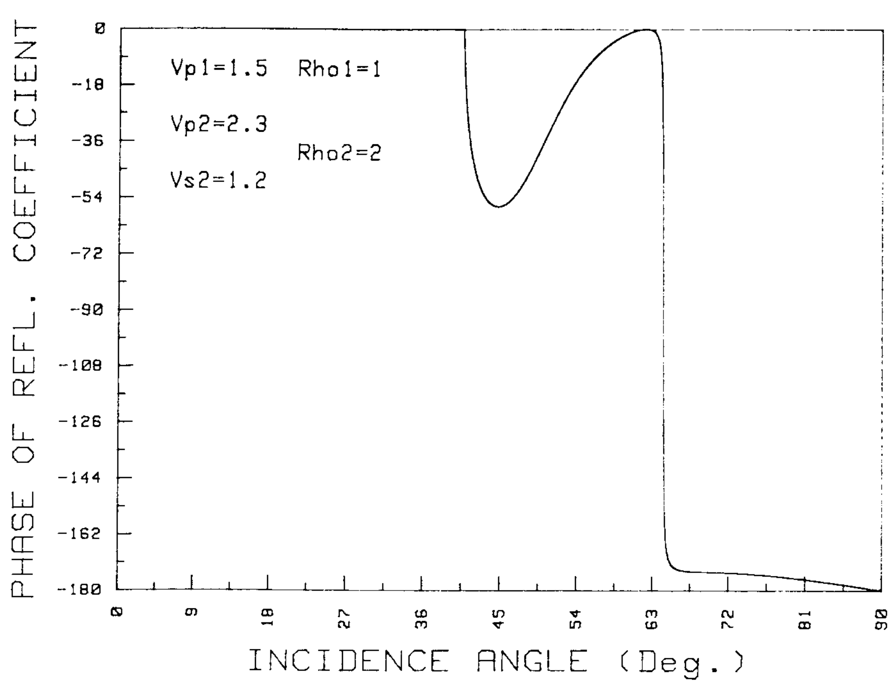


FIG. 4. In this figure $\varepsilon = 0.2$ as in figure (3). The reflection coefficient as a function of the angle of incidence is given in figures (5-8). The parameters used to generate this synthetic seismogram are the same as in figure (3) with velocities in the second medium $\alpha' = v_{p'} = 2500\ m/s$ $\beta' = v_{s'} = 1200\ m/s$. The density $\rho' = 2$. Note the prescence of the refracted wave, and the phase shift after critical angle.



MODULUS OF REFL. COEFFICIENT
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
0.00
0
9
18
27
36
45
54
63
72
81
90
INCIDENCE ANGLE (Deg.)
Vp1=1.5 Rho1=1
Vp2=2.3
Rho2=2
Vs2=1.2

FIG. 5.



PHASE OF REFL. COEFFICIENT
0
-18
-36
-54
-72
-90
-108
-126
-144
-162
-180
0
9
18
27
36
45
54
63
72
81
90
INCIDENCE ANGLE (Deg.)
Vp1=1.5 Rho1=1
Vp2=2.3
Rho2=2
Vs2=1.2

FIG. 6.

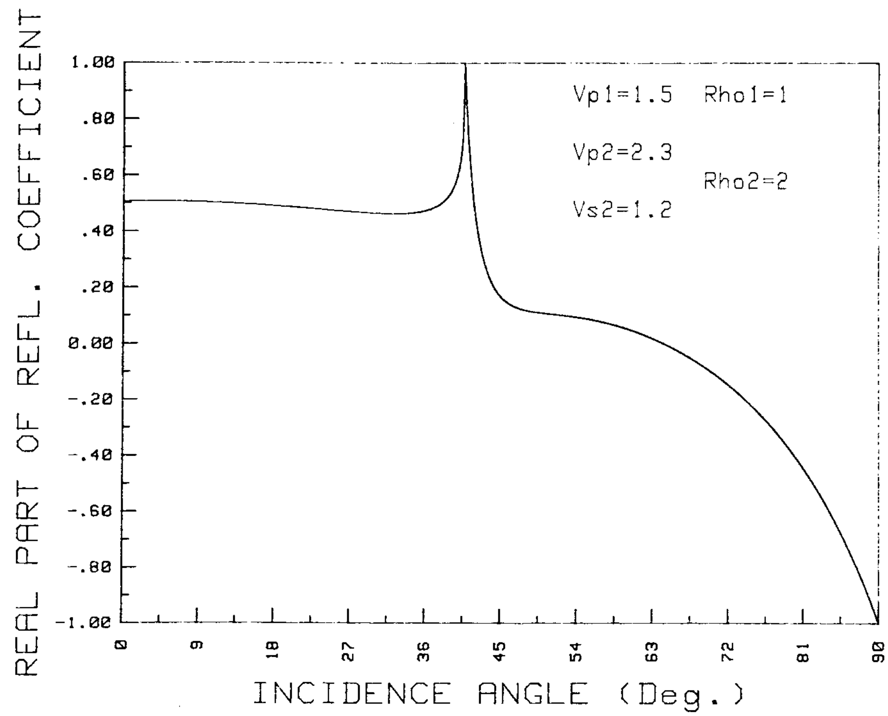

REAL PART OF REFL. COEFFICIENT
1.00
.80
.60
.40
.20
0.00
-.20
-.40
-.60
-.80
-1.00
0
9
18
27
36
45
54
63
72
81
90
INCIDENCE ANGLE (Deg.)
Vp1=1.5 Rho1=1
Vp2=2.3
Rho2=2
Vs2=1.2

FIG. 7.

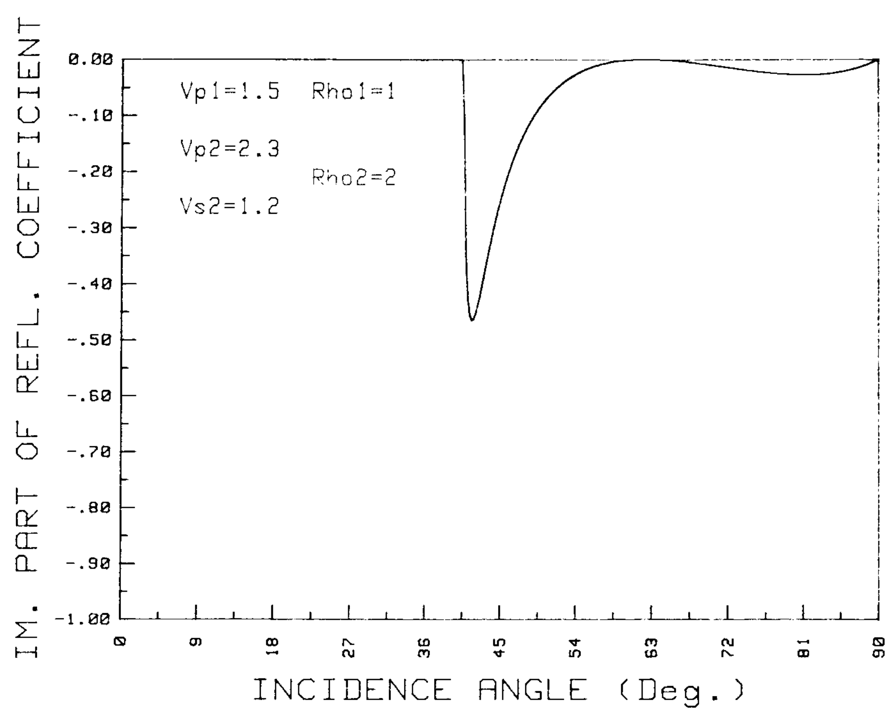

IM. PART OF REFL. COEFFICIENT
0.00
-.10
-.20
-.30
-.40
-.50
-.60
-.70
-.80
-.90
-1.00
0
9
18
27
36
45
54
63
72
81
90
INCIDENCE ANGLE (Deg.)
Vp1=1.5 Rho1=1
Vp2=2.3
Rho2=2
Vs2=1.2

FIG. 8.