Half-Plane Space-Time Prediction Filters

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Abstract

A 2D lattice filter is presented which extends the spatial range of a seismic dataset in a *stable* manner. The sampled region of the 2D autocorrelation depends on the *chosen* region of support of the reflection coefficients.

Stability of Prediction Filter

In the practical application of wavefield operators to seismic datasets, spatial truncation effects are often troublesome. One way to deal with such problems is to extrapolate these datasets off their side boundaries with a 2-D filter that is recursive in the spatial direction,x (see figure 1). Not all "reasonable" linear filters, however, will give stable predictions as we move along in x. An example is the standard least squares prediction filter, h, satisfying the design criterion:

$$\min_{h} \sum_{data} \| p_{t,x+1} - h ** p_{t,x} \|^2$$

where '**' means 2D convolution and p is the recorded data. At first glance this might seem to be a useful prediction filter, but closer examination shows that its stability is not guaranteed.

Stability will only be realized if our prediction error filter, $A(z_x,z_t)$ has all its poles outside the unit sphere in (z_x,z_t) space. A, need not be "2-D minimum phase" since we will require causality in the x direction only. A good discussion of 2-D minimum phase and causality as well as a wealth of references to the problems of 2-D linear prediction can be found in Marzetta, '80.

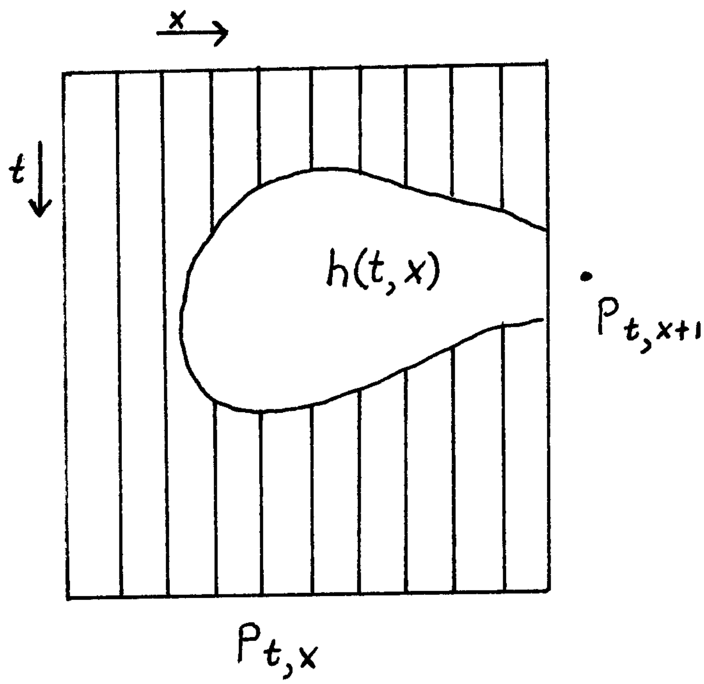

FIG. 1. Prediction of data $p_{t,x+1}$ by application of filter h(t,x) to known data

It is well known that in 1-D prediction the Levinson recursion can be used to construct a stable inverse filter to a time series. An analogous procedure can be used in 2-D. As in the 1-D Burg technique we can construct a prediction error filter $A(z_x,z_t)$ which minimizes the sum of forward and backward (in the x direction) prediction error energy. If this prediction error filter is constructed by a 2-D Levinson recursion of the form[1]

$$A^n = A^{n-1}(z_x,t) + c^n(t) * A^{(n-1)*}(z_x{}^{-1},t)\, z_x{}^{n+1} \tag{1}$$

then A^n will be a stable approximation to the true A as long as $c^n(z_t) \leq 1$ for z_t on the unit circle. The reader should note that the reflection coefficients, $c^n(t)$, are now functions of time instead of scalars as in the 1-D case.

Using an argument analogous to Claerbout '76 it can be seen that the minimization of forward and backward prediction error energy is equivalent to minimizing the error functional

$$E(c) = \sum_x \sum_t [\, | f - c * b |^2 + | b^* - c * f^* |^2] \tag{2}$$

(1) All convolutions (denoted by lower case *'s) are taken over time only.

with respect to the reflection coefficients, where

$$f^{n+1}=f^n-c^n*b^n \tag{2a}$$

and

$$b^{n+1}=b^n-c^{n*}*f^n \tag{2b}$$

are the 2D forward and backward prediction errors of order n+1.

Proceeding with the minimization, we obtain:

$$\nabla_{(c^*)}E = \sum_x[b^**(f-c*b)+f*(b^*-c*f^*)]=0 \tag{3}$$

or

$$c*\sum_x[b^**b+f*f^*] = \sum_x[b^**f+f*b^*] \tag{4}$$

Transforming (4) from (x,t) space to (x,z_t) space, it is apparent that c(z_t) is bounded by unity for *all* z_t. In particular it is less than unity on the unit circle and our prediction filter is therefore stable. We can also see from the (x,ω) transformed version of (4) that the 2-D prediction problem can be reduced to a series of 1-D Burg prediction problems - one for each temporal frequency in the data. Thorson (this report) shows some examples of seismic datasets extended with this frequency domain algorithm.

Choice of Reflection Coefficients

In practice it is not necessary to solve for an infinite number of values of c with each iteration. If we formulate the minimization problem with the assumption that the only non-zero c's are c_0 through c_{m-1} and that data exists only for time values 0 to n-1 then equation (4) reads:

$$\sum_{i=0}^{n-1}\sum_x(b_{i-j}{}^*f_i+f_{i-j}b_i{}^*)=\sum_{k=0}^{m-1}c_k\sum_{i=0}^{n-1}\sum_x(b_{i-j}{}^*b_{i-k}+f_{i-j}f_{i-k}{}^*) \quad ; j=0,...,m-1 \tag{5}$$

where the x indices have been suppressed for notational simplicity.

Having a finite length c vector limits the region of support of the prediction error filter in (x,t) space. Figure 2 shows the shape of the 2D prediction error filters (PEF's) up to order 4 for the case of a two point c. The PEF's are, in general, diamond shaped for a compact choice of c's. The inverse autocorrelation island of the PEF's (figure 3) is also diamond shaped but centred on the origin.

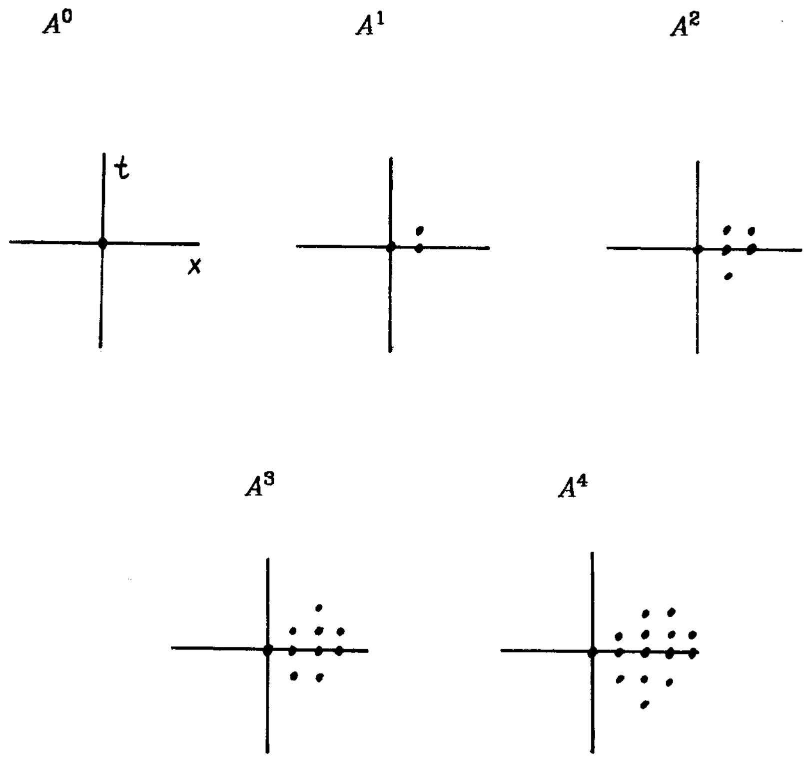

FIG. 2. Shape of 2D PEF's up to order four for the case of two-point reflectivity.

From figure (3) we can see that the choice of reflection coefficient structure determines the region of 2D lag space that is effectively sampled. If the data contains high dips it is important to have a large number of c's so that the PEF design is sensitive to these events. In such cases a two point reflectivity would clearly be a naive choice of c-structure.

Further Work

Further work needs to be done to get a practical algorithm for the recursive prediction problem. It is desirable to avoid doing a 2D convolution of the PEF on the data since the PEF size grows exponentially with prediction order. Hopefully a knowledge of $f^{(n)}$, $b^{(n)}$ and $c^{(n)}$ will be sufficient for the prediction problem as well as the PEF design problem.

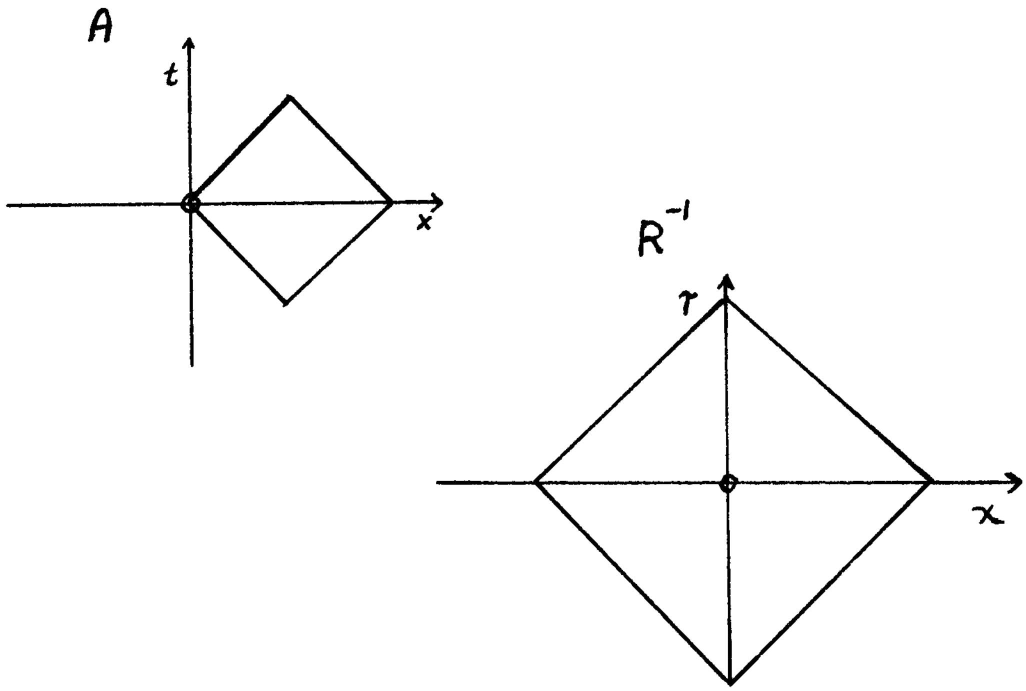

FIG. 3. General shape of A(x,t) and $R^{-1}(\chi,\tau)$ for a finite and compact c-structure.

REFERENCES

Claerbout,J.F. Fundamentals of Geophysical Data Processing. McGraw-Hill, 1976, p. 135

Marzetta, T.L. Two - Dimensional Linear Prediction: Autocorrelation Arrays, Minimum-Phase Prediction Error Filters, and Reflection Coefficient Arrays. IEEE vol. ASSP-28,No. 6, Dec. 1980, p.725

Money is the root of all evil, and man needs roots

Do what comes naturally now. Seethe and fume and throw a tantrum.

Remember that whatever misfortune may be your lot, it could only be worse in Cleveland.

If you think last Tuesday was a drag, wait till you see what happens tomorrow!

The brain is a wonderful organ; it starts working the moment you get up in the morning, and does not stop until you get to school.

Il brilgue: les tõves libricilleux
   Se gyrent et frillant dans le guave,
Enmîmés sont les gougebosquex,
   Et le mõmerade horgrave.

Tonights the night: Sleep in a eucalyptus trees.

If all be true that I do think,
There be Five Reasons why one should Drink;
Good friends, good wine, or being dry,
Or lest we should be by-and-by,
Or any other reason why.

Troubled day for virgins over 16 who are beautiful and wealthy and live in eucalyptus trees.

Shaw's Principle:
   Build a system that even a fool can use, and only a fool will want to use it.

I really hate this damned machine
I wish that they would sell it.
It never does quite what I want
But only what I tell it.

Captain Penny's Law:
   You can fool all of the people some of the time, and some of the people all of the time, but you Can't Fool Mom.