Wave Equation Normal Moveout Using a Stolt Algorithm

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Abstract

Wave equation normal moveout is a special case of migration applied to a single trace. Besides the computational efficiency inherent to the Stolt algorithm, additional economies accrue due to this special form of migration. Examples of normal moveout processed gathers by both conventional and Stolt wave equation algorithms are included.

Algorithm

In SEP-25 Ottolini demonstrated the workability of *wave equation stacking*. Wave equation stacking used migration algorithms to collapse hyperbolas on gathers to their apexes at zero offset. Furthermore, Ottolini established that wave equation stacking works even for strongly aliased and truncated cable spreads. *Wave equation normal moveout* (WENMO) evolves from the wave equation stack of the minimal cable spread- a single offset.

The basic WENMO algorithm is a wave equation stack of a common midpoint gather with all but the desired offset zeroed out. Equation (1) gives Stolt's algorithm for such a wave equations stack.

$$P(x,t') = \int_{-\infty}^{\infty} d\omega' e^{i\omega' t'} \int_{-\infty}^{\infty} dk_h e^{-ik_h h} \frac{\omega'}{\sqrt{\omega'^2 + v^2 k_h^2}} \tilde{P}(k_h, \omega = -\sqrt{\omega'^2 + v^2 k_h^2}) \qquad (1)$$

In this integral, h and k_h are the lateral coordinate and wave number, the exponentials are the inverse Fourier transform kernals, $\tilde{P}$ is the double Fourier transform of the source gather,and the primed variables are the moved out coordinates. When migrating just a single trace, the lateral Fourier transforms can be simplified. The double Fourier transformed data $\tilde{P}$ becomes

$$\tilde{P}(k_h,\omega) = \tilde{P}_{h_0}(\omega)\, e^{ik_h h_0}$$

where h_0 is the trace offset. We only wish to know the result at zero offset, so that h in equation (1) can be replaced by zero. The the k_h exponential reduces to unity and the inverse lateral Fourier transform simplifies to a sum. Therefore, equation (1) simplifies to

$$P_{h_0}(t') = \int_{-\infty}^{\infty} d\omega e^{i\omega' t'} \int_{-\infty}^{\infty} dk_h \frac{\omega'}{\sqrt{\omega'^2+v^2k_h^2}} \tilde{P}_{h_0}(\omega = -\sqrt{\omega'^2+v^2k_h^2})\, e^{ik_h h_0} \qquad (2)$$

Equation (2) suggests the following computational algorithm for implementing wave equation normal moveout:

(1) Perform a real to complex Fourier transform on the trace to be moved out.

(2) For each k_h frequency chosen, stretch and scale the transformed trace according to the rightmost part of equation (2). From practical experience, about 20 k_h frequencies evenly spaced from zero to pi seems to give adequate results.

(3) Sum together all of the traces of part 2.

(4) Finally, inverse Fourier transform this sum to obtain the WENMOed trace.

Examples

Two synthetic datasets are normal moveout processed by the conventional and wave equation algorithms (figures 1 and 2). The Stolt WENMO algorithm works at least as well as the conventional algorithm. Both exhibit moveout stretch however. The new algorithm may in fact work better than the conventional algorithm by accounting for amplitude and phase changes with offset which the conventional algorithm does not account for.

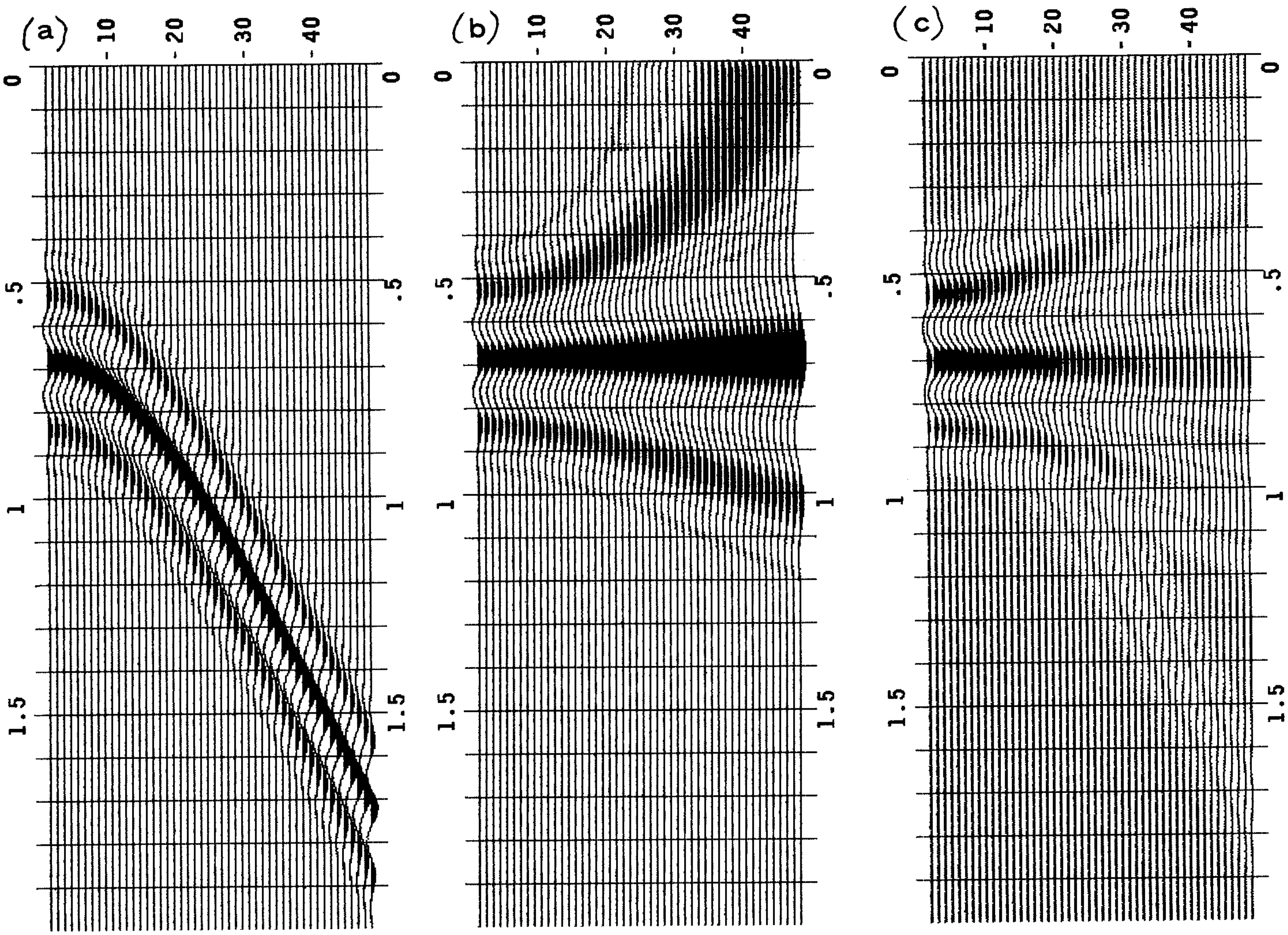

FIG. 1. Normal moveout of a synthetic gather created with raytracing by Jeff Thorson. The raytracing algorithm does not account for changes in amplitude and phase with offset which occur in real data. Parameters are dt=.02sec, dh=25m, and v=1500m/s. A fairly wide waveform has been convolved onto the single event at .666 seconds. Part (a) is the source gather, part (b) the conventional algorithm, and part (c) the Stolt wave equation algorithm. The conventional algorithm used a four point cubic spline interpolation. The wave equation algorithm used 20 frequencies equally spaced from zero to pi. The artifact in the lower right corner is probably to frequency domain wraparound. The wave equation result looks much like the conventional result, except that wider offsets have lower amplitudes. Both posses moveout stretch.

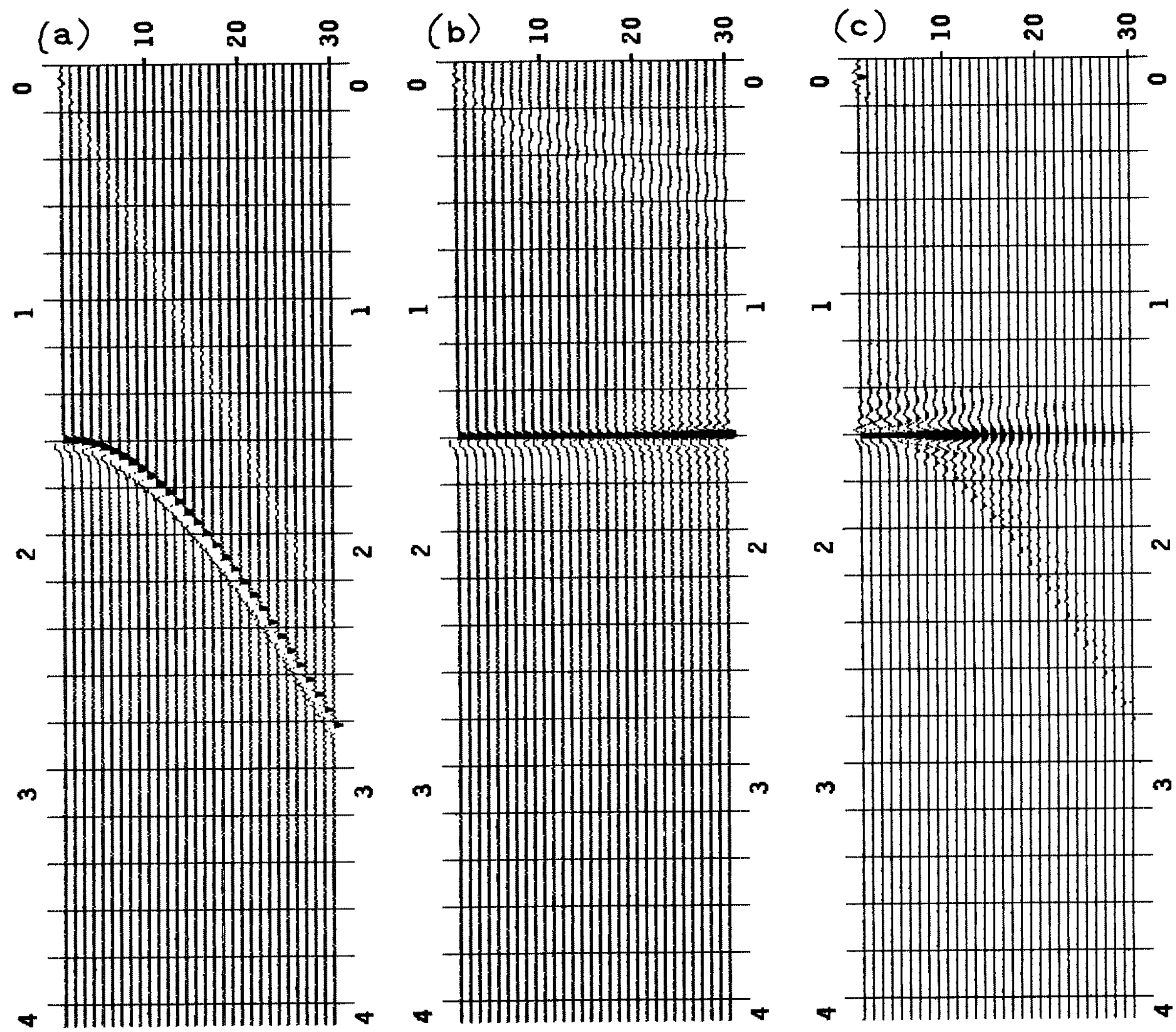

FIG. 2. The same processing as in figure 1, except beginning with a different source gather. The source gather was generated with a Stolt diffraction algorithm. The amplitudes and frequency content of the reflector waveform then changes with offset. Parameters are dt=.008sec, dh=100m, v=2500m/s. The wave equation result is comparable to the conventional result.