The Smile Equation for Velocity Estimation

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Abstract

Downward continuation of both shots and geophones using the double square root equation is called SG migration. A proper SG migration leaves no energy at non-zero offsets. If the wrong velocity structure is used then smiles are generated. From a measurement of the curvature of these smiles (and frowns) it is possible to construct a correction to the velocity model used in the migration.

Introduction

Velocity estimation is an important and still unsolved problem in seismic data acquisition and processing. If a good methods of estimating acoustic velocity were available then better estimates of reservoir size and structure could be made, stratigraphic trap identification would be less tricky, and maps of velocity in the subsurface could be made for interpretation.

When a data set is migrated with the wrong velocity the output is more complicated than the output of a migration with the right velocity structure. This phenomenon is clearest in SG migration (at least in theory) because a proper migration leaves no energy in the non-zero offsets.

This does not tell us how to estimate velocity. One way might be to use a gradient technique which maximizes the energy at zero offset. This does not use all of the information in the non-zero offsets because we know something about the shapes in a migration in which a velocity error has been made. In particular, we expect that there will be smiles and frowns where events have been migrated poorly. It turns out that something like a velocity analysis can be done on the data at non-zero offset which will yield estimates of the velocity corrections. The partial differential equation which governs the evolution of the smiles and frowns will be called the smile equation (if you're a pessimist, you can call it the frown equation).

Imaging Conditions and Velocity Estimation

Migration has usually been explained in terms of two sets of concepts: one-way wave equations and imaging conditions. For a zero-offset section the imaging condition is that the image is equal to the wavefield at $t = 0$. For simultaneous migration of shot and geophone wavefields the imaging condition is that the image is equal to the wavefield at $t = 0$ if the shots and geophones are coincident.

Unfortunately, these statements of the various imaging conditions which we use in migration are only partly true. If a mistake is made in migration velocity then the data set may be fuzzily focused. In fact, interpreters have known this for a long time and may insist on several migrations with different velocity imputs. The migrated data which is the most sharply focused is that which is best interpreted. A further logical step is taken when it is hypothesized that the velocity structure which gives the sharpest focus is very close to the true acoustic velocity structure for the earth.

In practice, imaging is treated as a handle on the velocity estimation problem. If this is accepted, a good way to proceed is to define what we mean by a good focus and then to look for good places in the processing sequence at which to make this measurement.

Derivation of the Smile Equation

Neglecting multiples, the partial differential equation which governs the seismic experiment is the double square root equation. With the Fourier transform conventions

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega F(\omega)\, e^{i\omega t} \qquad F(\omega) = \int_{0}^{\infty} dt f(t)\, e^{-i\omega t}$$

and a convention that the square root function be understood to be positive real the double square root equation for an upward traveling wavefield P of frequency ω is given by

$$\frac{\partial P}{\partial z} = -\left\{\left[-\omega^2\Lambda_g^2 + |D_g|^2\right]^{1/2} + \left[-\omega^2\Lambda_s^2 + |D_s|^2\right]^{1/2}\right\} P + S\,\delta(h) \qquad (1)$$

In this equation Λ_g is the reciprocal of the acoustic velocity at the geophone position (x_g, z). A similar definition holds for Λ_s. Between the two there is the consistency relation $\Lambda_s(x,z) = \Lambda_g(x,z)$. The operator $|D_s|^2$ is a non-negative symmetric operator which is the negative of the second derivative with respect to the lateral shot coordinate. Similarly,

$|D_g|^2$ is the negative of the negative of the second derivative with respect to the lateral geophone coordinate. S is the source function which governs the way in which reflectivity enters into the wavefield. Finally, h is equal to half the shot-receiver offset.

Any migration will make two sorts of errors, if the truncation effects due to finite offsets are ignored. First, the velocities used will be $\Lambda_g + \delta\Lambda_g$ and $\Lambda_s + \delta\Lambda_s$ rather than Λ_g and Λ_s, respectively. Second, the sources subtracted out of the wavefield will be $S + \delta S \neq S$. The double square root equation for the migration of the wavefield P will be

$$\frac{\partial P}{\partial z} = -\left\{\left[-\omega^2\left(\Lambda_g + \delta\Lambda_g\right)^2 + |D_g|^2\right]^{1/2} + \left[-\omega^2\left(\Lambda_s + \delta\Lambda_s\right)^2 + |D_s|^2\right]^{1/2}\right\} P$$

$$+ (S + \delta S)\,\delta(h) \qquad (2)$$

The boundary condition for this wavefield is the condition that P equal the recorded wavefield at the surface. If $f(x_s, x_g, \omega)$ is this record then

$$P(x_s, x_g, \omega, z = 0) = f(x_s, x_g, \omega)$$

The equation that governs migration error is obtained by subtracting equation (1) from equation (2).

$$\frac{\partial P}{\partial z} = \left[-\omega^2\left(\Lambda_g + \delta\Lambda_g\right)^2 + |D_g|^2\right]^{1/2} P + \left[-\omega^2\left(\Lambda_s + \delta\Lambda_s\right)^2 + |D_s|^2\right]^{1/2} P$$

$$- \left[-\omega^2\Lambda_g^2 + |D_g|^2\right]^{1/2} P - \left[-\omega^2\Lambda_s^2 + |D_s|^2\right]^{1/2} P + \delta S\,\delta(h) \qquad (3)$$

This partial differential equation is useless as it stands because the square roots make it computationally intractable. Applying the 15-degree approximation to the roots changes the situation.

$$\frac{\partial P}{\partial z} = i\omega(\Lambda_g + \delta\Lambda_g)\,P - \frac{1}{2i\omega(\Lambda_g + \delta\Lambda_g)}|D_g|^2 P - i\omega\Lambda_g P + \frac{1}{2i\omega\Lambda_g}|D_g|^2 P$$

$$+ i\omega(\Lambda_s + \delta\Lambda_s)\,P - \frac{1}{2i\omega(\Lambda_s + \delta\Lambda_s)}|D_s|^2 P - i\omega\Lambda_s P + \frac{1}{2i\omega\Lambda_s}|D_s|^2 P + \delta S\,\delta(h)$$

This can be approximated to within second order in $\delta\Lambda/\Lambda$ by the equation

$$\frac{\partial P}{\partial z} = \left[i\omega(\delta\Lambda_g + \delta\Lambda_s) + \frac{\delta\Lambda_g}{2i\omega\Lambda_g^2} + \frac{\delta\Lambda_s}{2i\omega\Lambda_s^2}\right] + \delta S\delta(h)$$

Since $\delta\Lambda/\Lambda^2 = -\delta V$ to first order

$$\frac{\partial P}{\partial z} = \left[i\omega(\delta\Lambda_g + \delta\Lambda_s) - \frac{\delta V_s|D_s|^2 + \delta V_g|D_g|^2}{2i\omega}\right] P + \delta S\delta(h) \qquad (4)$$

Near the midpoint axis $\Lambda_s = \Lambda_g$ and $\delta\Lambda_s = \delta\Lambda_g$, so we can define functions Λ, $\delta\Lambda$, V, and δV with domains consisting of the zero offset plane.

$$\Lambda = \delta\Lambda_s \qquad V = V_s \qquad \delta\Lambda = \delta\Lambda_s \qquad \delta V \qquad \delta V_s$$

so that equation (4) changes to

$$\frac{\partial P}{\partial z} = \left[2i\omega\Lambda - \frac{\delta V(|D_s|^2 + |D_g|^2)}{i\omega} \right] P + \delta S\delta(h)$$

Introducing the midpoint coordinate y

$$\frac{\partial P}{\partial z} = \left[2i\omega\Lambda - \frac{2\delta V(|D_h|^2 + |D_y|^2)}{i\omega} \right] P + \delta S\delta(h)$$

Assume for now that $|D_y|^2 P \ll |D_h|^2$. This is probably a good assumption when the data obeys a reciprocity relation. It also simplifies our differential equation and its analysis. We will not be able to estimate the first term either so we will drop it, too. This term will also be small if we have migrated with the correct velocity and have extrapolated off the ends of the data sufficiently well. If we have done a good job of subtracting out the sources (removing wraparound) then δS will be small compared to the wavefield amplitude.

$$\frac{\partial P}{\partial z} = -\frac{4\delta V|D_h|^2}{i\omega} P \tag{5}$$

This equation is the equation which will be referred to as the smile equation. It governs the behavior of the smiles and frowns due to migration velocity errors as long as 1) the wavefield is in the vicinity of the zero offset plane, 2) the migration velocity error is small, 3) wraparound has been removed almost perfectly, 4) all propagation angles are small, 5) multiples are low enough in amplitude to be neglected, and 6) truncation artifacts do not swamp the smiles and frowns in the vicinity of the zero offset plane.

Velocity Updating Using the Smile Equation

Equation (5) looks like a 15-degree equation with an effective velocity of $8\delta V$. It is the contention of this paper that this effective velocity V_{eff} can be measured using semblance techniques in the offset direction after an SG migration (or some cheap approximation to an SG migration). The correction to the migration velocity, δV can then be computed using

$$\delta V = 8V_{eff}$$

Given a velocity correction at an event in the zero offset plane the next thing to do is to find where in space it is distributed. So far we can only guess at the solution to this problem. Taking a hint from conventional velocity analysis, our first guess as to where a given velocity anomoly at an event in the zero-offset plane comes from is somewhere along the normal ray path to that point. This necessitates an estimate of local dip, but for once this measurement is easy, for at the time the velocity corrections are known the data has been more or less migrated.

Now all we have to do is determine where along the normal ray path a given velocity correction lies. There does not seem to be any easy solution to this problem. It will therefore be proposed that the solution can be found by using a back-projection algorithm like that used in medical imaging systems.

Good day for overcoming obstacles. Try a steeplechase.

"If while you are in school, there is a shortage of qualified personnel in a particular field, then by the time you graduate with the necessary qualifications, that field's employment market is glutted."
-- Marguerite Emmons

Succumb to natural tendencies. Be hateful and boring.

Cynic: A blackguard whose faulty vision sees things as they are, not as they ought to be. Hence the custom among the Scythians of plucking out a cynic's eyes to improve his vision.

This fortune cookie program out of order. For those in desperate need, please use the program "randchar". This program generates random characters, and, given enough time, will undoubtedly come up with something profound. It will, however, take it no time at all to be more profound than THIS program has ever been.

According to the latest official figures, 43% of all statistics are totally worthless.

I'd rather have a bottle in front of me than a frontal lobotomy.

Screw up your courage! You've screwed up everything else.

Laws of Computer Programming:

1. Any given program, when running, is obsolete.
2. Any given program costs more and takes longer.
3. If a program is useful, it will have to be changed.
4. If a program is useless, it will have to be documented.
5. Any given program will expand to fill all available memory.
6. The value of a program is proportional the weight of its output.
7. Program complexity grows until it exceeds the capability of the programmer who must maintain it.

Celebrate Hannibal Day this year. Take an elephant to lunch.