Wave Equation Moveout - Part II

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Abstract

This paper presents a computational extension to the wave equation NMO technique presented by Thorson and Yedlin (SEP-25). It is demonstrated that by a simple change of variables, the p-stack can by converted into a convolution integral. What the wave equation NMO becomes, then, is simply the usual NMO stretch operation sandwiched between two linear filters which can be implemented essentially at little increased cost. That is, the process is one-dimensional and is applied separately to each trace.

The operator applied for the WENMO (wave equation NMO) is , as in SEP-25, a two-dimensional one. Therefore, upon application of the method, we shall see some of the types of distortion characteristic of wave propagation in two dimensions.

Theory

The equation to be derived begins with the equation of the moved out trace as given in SEP-25:

$$q(\tau) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \tilde{q}(\ \tau\sqrt{1 - p^2v^2} + 2ph\ )\, dp \tag{1}$$

where $\tilde{q}(t)$ is the rho-filtered trace, τ is migrated two-way travel time, p is the ray parameter, and $2h$ is the full offset, denoted by f. The above integral includes the evanescent zone over its integration limits. To put the integral in a more tractable form, a change of variables is made. The integral (1) becomes

$$q(\tau) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{dp}{dt}\tilde{q}(t)\, dt \tag{2}$$

where $t = \tau\sqrt{1 - p^2 v^2} + pf$, the two-way travel time. As the $t-p$ transformation has two branches, the integral has to be split into two parts. The integral is split at the (t, p) corresponding to the geometrical optics path, given by

$$p_0 = \frac{f}{v\sqrt{f^2 + \tau^2 v^2}} \qquad t_0 = \frac{\sqrt{f^2 + \tau^2 v^2}}{v} \tag{3}$$

Of course, at $p = \pm 1/v$, corresponding to $T = \pm f/v$, the square root in the $t-p$ transformation becomes imaginary. These limits for p, which represent a horizontally travelling ray, delineate the evanescent zone, which will be ignored in what follows.

With the above results in mind, the integral (2) can be written as

$$q(\tau) = \frac{1}{\pi} \int_{-f/v}^{t_0} dt\ \frac{dp_1}{dt} \tilde{q}(t) + \frac{1}{\pi} \int_{t_0}^{f/v} dt\ \frac{dp_2}{dt} \tilde{q}(t) \tag{4}$$

where

$$\frac{dp_{1,2}}{dt} = \frac{f}{f^2 + v^2\tau^2} \pm \frac{\tau}{f^2 + v^2\tau^2} \frac{v^2 t}{\sqrt{f^2 + v^2(\tau^2 - t^2)}} \tag{5}$$

and the plus sign is associated with p_1 (See figure 1).
Substitution of (5) into (4), after some algebra , results in

$$q(\tau) = \frac{1}{\pi} \int_{-f/v}^{f/v} \tilde{q}(t) \left[\frac{f}{f^2 + \tau^2 v^2} + \frac{\tau}{f^2 + \tau^2 v^2} \frac{v^2 t}{\sqrt{f^2 + v^2(\tau^2 - t^2)}} \right] dt$$

$$+ \frac{2}{\pi} \frac{v^2 \tau}{f^2 + \tau^2 v^2} \int_{f/v}^{t_0} \frac{t\ \tilde{q}(t)}{\sqrt{f^2 + v^2(\tau^2 - t^2)}} dt \tag{6}$$

For the rest of the discussion, we will consider only the second term in (6), as its contribution is by far the greatest to $q(\tau)$.

Now for the second integral in (6), a change of variables is required, to obtain the desired form for $q(\tau)$. We let

$$u = \sqrt{t^2 - f^2/v^2} \tag{7}$$

Then, the moved out trace is given by

$$q(\tau) = \frac{2}{\pi} \frac{v\tau}{f^2 + v^2\tau^2} \int_0^{\tau} \frac{u}{\sqrt{\tau^2 - u^2}} \tilde{q}(\sqrt{u^2 + f^2/v^2})\, du \tag{8}$$

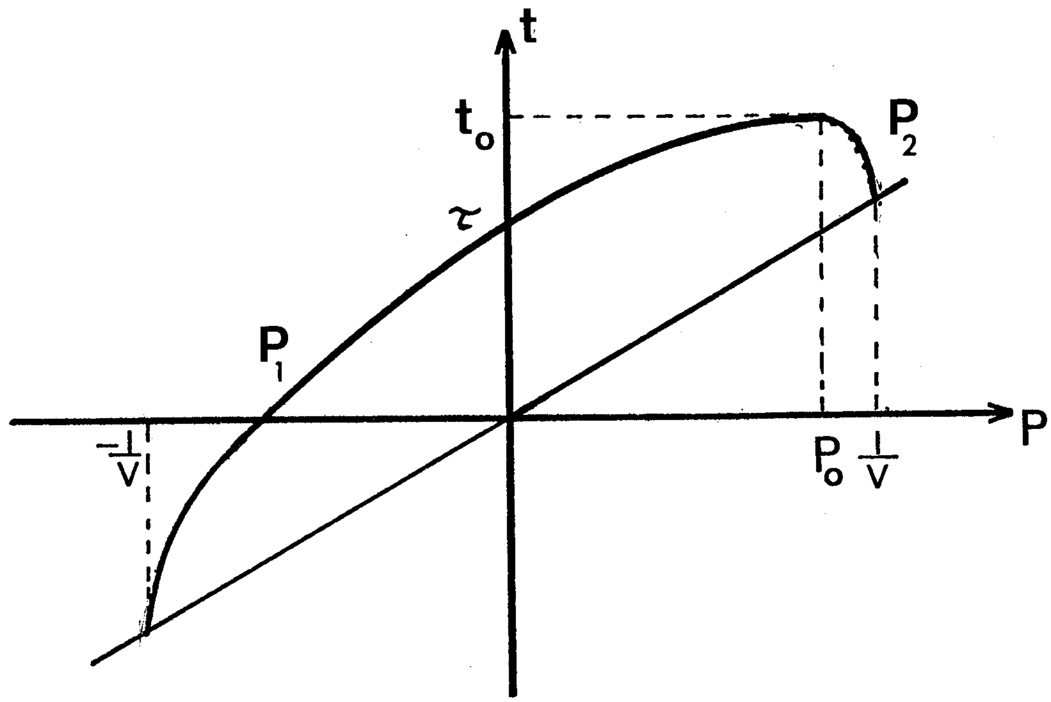

FIG. 1. The phase function $t(p)$. It is the inverse function $p(t)$ which is needed in equation 2. The two branches of the $p(t)$ transformation are indicated by p_1 and p_2. Consistent with equation (5), we see from the figure that p_1 is an increasing function of t, while p_2 is a decreasing function. The point (p_o, t_o) separates the two branches.

Equation (8) is the fundamental equation for WENMO. It is easily implemented as follows:

a) Rho filter the trace. In the frequency domain, this means multiplication by $|\omega|$.

b) Stretch each trace by the usual NMO transformation: $u = \sqrt{t^2 - f^2/v^2}$.

c) Multiply by u, and do something like a convolution, with τ as the running variable. Actually (8) may be converted into a convolution by the following argument. In actual practice, the arrival time of a seismic pulse is much greater than the duration of the pulse itself. This means that the factor $\sqrt{(\tau^2 - u^2)}$ can be replaced by the factor $\sqrt{2\tau(\tau - u)}$, since the major contribution to the integral in (8) comes near $u = \tau$. Thus (8) becomes

$$q(\tau) = \frac{\sqrt{2\tau}\, v}{\pi\,(f^2 + \tau^2 v^2)} \int_0^{\tau} \frac{u}{\sqrt{\tau - u}} \tilde{q}(\sqrt{u^2 + f^2/v^2})\, du \qquad (9)$$

For the examples which follow, (9) was actually used to compute $q(\tau)$. A discrete representation was used to do the convolution with the $1/\sqrt{t}$ tail, which commonly appears in two-dimensional wave propagation problems. Note that convolution with the $1/\sqrt{t}$ tail is an Abel transform.

d) Multiply the trace by an offset and time dependent gain factor.

The above procedure may be done also for the case when the velocity is τ dependent. The same change of variables may be used, although the final results will have to be obtained via table lookups.

Examples

In figure 2, six panels are plotted, to illustrate the difference between regular NMO and WENMO. Panel A is the original data gather, B is the regular NMO, and C is B multiplied by a gain factor. Panel D is the rho-filtered version of A, E is WENMO without proper gain, and F is the complete WENMO.

To analyze WENMO, it is appropriate to compare panel B, regular NMO, with panel F, WENMO. Of course, the most obvious difference in the two panels, is the absence in B of any form of time and offset dependent weighting, which is an intrinsic part of WENMO. The WENMO has less stretch distortion at the far offsets, due to the application of the zero phase rho filter. However, the convolution with the $1/\sqrt{t}$ tail does introduce wavelet distortion. First, the waveform is no longer symmetrical, and there exists a $\pi/4$ phase shift, which can be seen by comparing the location of the central peak on the first trace in F to the corresponding one in B. Also, at the far offsets, the central peak appears to arrive later in time. These wavelet distortions (also present in the paper "Wave Equation Normal Moveout Using a Stolt Algorithm" by Ottolini, this volume) arise from the mere two-dimensionality of our wave operators. A possible extension of the work is to look at the problem in three dimensions, where we shouldn't expect any tails and phase shifts.

It appears from this work that regular NMO as it is currently applied does a good enough job. However it is interesting to see that with a minimum of extra computational effort one can attain a moveout operation that has the following desirable property: The resulting stacked trace is an exact migration of the common-midpoint plane for a horizontal reflector model.

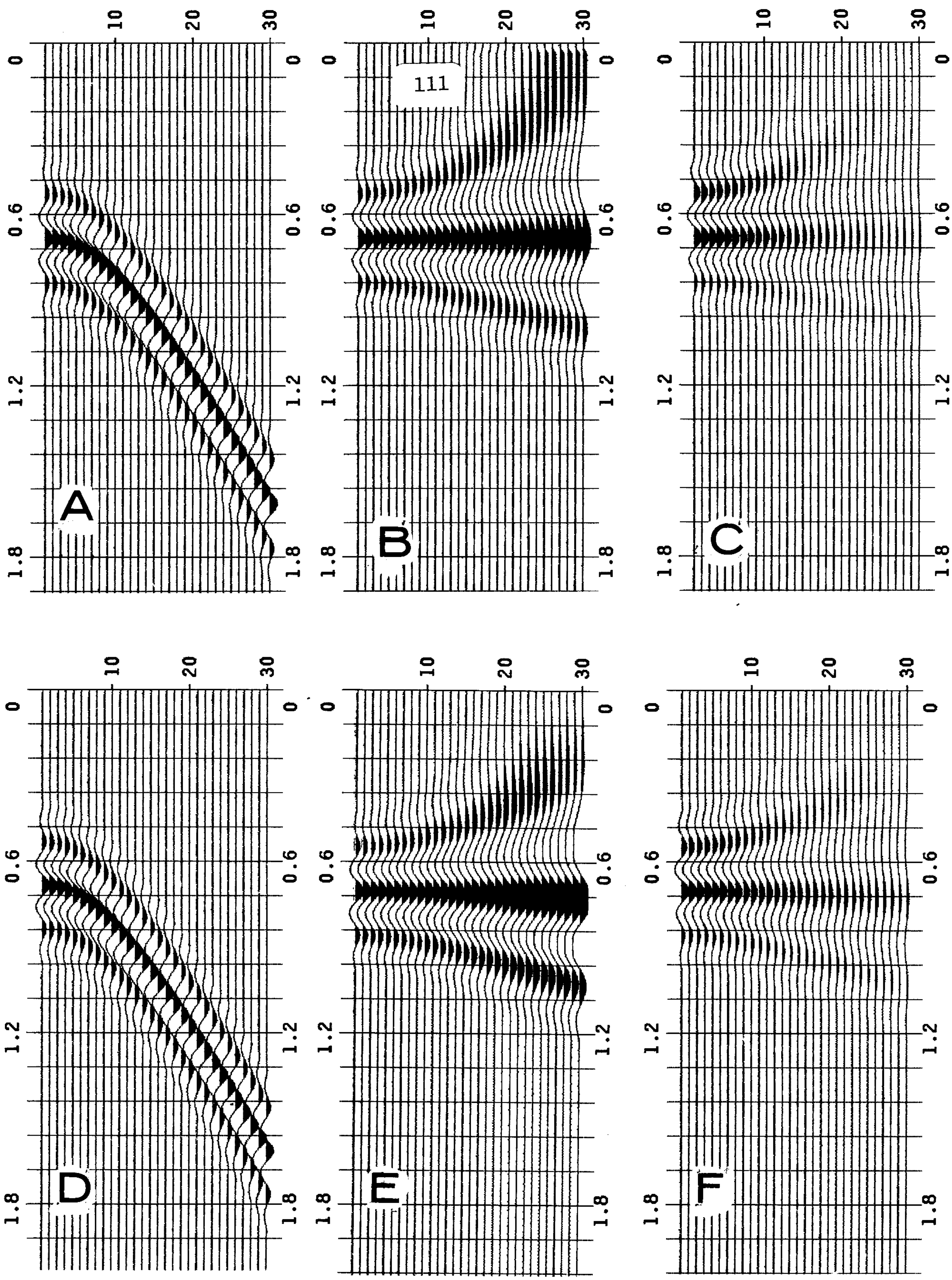

FIG. 2. Six panels (see text) illustrating the differences between regular NMO and WEMNO. The artificial common midpoint gather was generated with a constant velocity of 1500 m/sec , with a constant waveform superimposed on a hyperbolic trajectory. The parameters chosen for this model were: .02 sec sample interval, 75 m trace separation, 30 traces, and 128 samples per trace. Zero offset travel-time of the event is .67 sec.

Alliance: In international politics, the union of two thieves who have their hands so deeply inserted in each other's pocket that they cannot separately plunder a third.

Life is a yo-yo, and mankind ties knots in the string.

Did you know that clones never use mirrors?

Year: A period of three hundred and sixty-five disappointments.

Like so many Americans, she was trying to construct a life that made sense from things she found in gift shops.
-- Kurt Vonnegut, Jr.

The chicken that clucks the loudest is the one most likely to show up at the steam fitters picnic.

"I don't have any solution but I certainly admire the problem."
-- Ashleigh Brilliant

Computers are not intelligent. They only think they are.

Stop searching. Happiness is right next to you. Now, if they'd only take a bath...

Once Law was sitting on the bench
And Mercy knelt a-weeping.
"Clear out!" he cried, "disordered wench!
Nor come before me creeping.
Upon you knees if you appear,

Then Justice came. His Honor cried:
"YOUR states? -- Devil seize you!"
"Amica curiae," she replied --
"Friend of the court, so please you."
"Begone!" he shouted -- "There's the door --
I never saw your face before!"

"Horse sense is the thing a horse has which keeps it from betting on people."
-- W.C. Fields