Split Backus Deconvolution Operators - Examples

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Introduction

In SEP-25 we outlined the principles of a predictive method for hard-bottom marine multiple suppression. The central idea was that the worst multiples can be suppressed by considering only the reverberatory effect of the seafloor. Limiting ourselves to this class of multiples has two advantages.

(1) No more than two subtractions are needed to remove the ringing from the data - one for the shot reverberation - the other for the geophone reverberation. A minimal number of subtractions means that the signal/noise ratio is not adversely affected.

(2) The wavefield does not have to be downward continued past the seafloor where velocity structure is not as well known as it is in the water layer.

The classical dereverberation operator which uses these ideas for the zero offset case is the Backus "three point operator" (Backus '59) given by

$$D_{Backus} = (1 + cz^{\tau})^2 \tag{1}$$

where τ is the two way water time and c is the local vertical incidence seafloor reflectivity. An intuitive extension of this model to the non-zero offset case gives the approximate decon operator:

$$D_{Split\ Backus} = (1 + c_s z^s)(1 + c_g z^g) \tag{2}$$

where z^s and z^g are the two way vertical traveltimes to the seafloor at the shot and geophone locations. The underlying assumption in this approximation is that the reverberation paths are nearly vertical at the shot and at the geophone (Fig. 1). For some types of pegleg multiples this is a useful assumption.

A more exact version of (2) that is faithful to the wave equation is given in Morley '80 by:

$$D_{Wave\ Eq.} = (1 + L_g c_g L_g)(1 + L_s c_s L_s) \tag{3}$$

where L_s and L_g are the (linear) wave operators in shot and geophone coordinates that continue the data between the seasurface and seafloor datum levels.

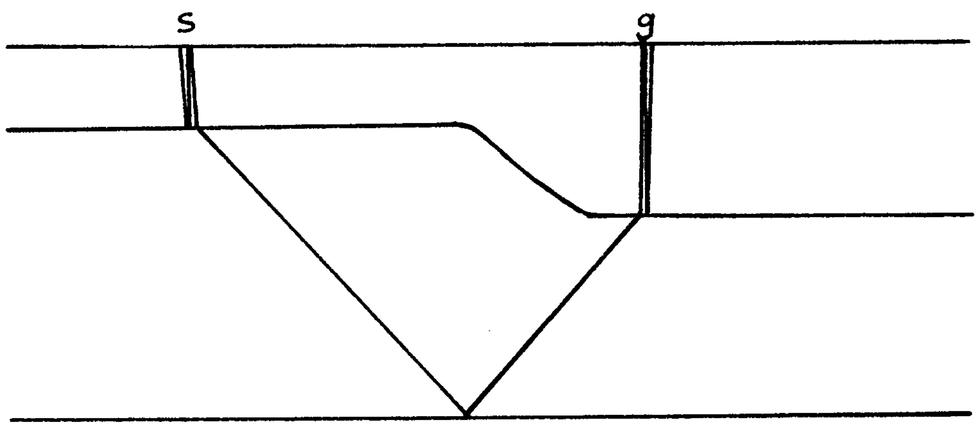

FIG. 1. Split Backus deconvolution operator assumes reverberation paths are near vertical.

This report contains two main sections. The first section deals with an implementation of (3) on synthetic data in midpoint-radial trace space. The motivation for working in this space will follow in the discussion. The second section looks at an application of the operator (2) to a real data example in which variations in seafloor topography split the pegleg multiples.

Section I - Wave Predictive Methods

(a) Constant Velocity Case

Taner (1980) first introduced the concept of radial traces for multiple suppression. For a constant velocity earth, each radial angle corresponds to a unique ray parameter for all depths. Because of this, the gather can be downward continued to depth 'd' by delaying each radial trace by a time of (d/v) secϑ. This method, however, fails when the velocity is not constant. Figure 2 demonstrates this fact.

Figure 2 shows an attempt to suppress water bottom and pegleg multiples by delaying radial traces. A synthetic seismogram was created for a model of a water layer of 0.5 seconds over a subsurface reflector at 2.0 seconds. The second layer has an interval velocity of 9000 ft/sec.

One hundred radial traces were sampled up to a maximum angle of 25^0. Each of these radial traces was then delayed by 0.5 secϑ seconds and subtracted from the non-delayed radial trace. The angle ,ϑ, was defined to be the ray angle in the water layer. Figure 2 is the Cartesian coordinate reconstruction of all the difference traces in radial trace space. The seafloor multiples are correctly predicted, but the *pegleg* multiple predictions are out by a wavelength - even at modest offsets. The events at 2.0 and 4.0 seconds are artifacts from the fact that only two water bottom multiples and two pegleg multiples were created in the original synthetic.

(b) Z-Variable Velocity - 0 Dip Case

Morley and Claerbout (1978) showed that slant stack traces can be used to overcome the problems of pegleg multiple nonstationarity on radial traces. For a flat layered earth, the multiple timing relationships of a slant stack trace are stationary. If we were to transform the original synthetic created for figure (2) to (p,τ) space, delay each p-trace by $d\cos\vartheta / v$[1] and inverse slant stack we would obtain another predictive model of the multiple reflections. Equivalently, we could transform the gather to the 2D Fourier space (ω,k_x), phase rotate by $exp(i\omega d\cos\vartheta / v)$ and inverse Fourier transform to get a multiple model. Figure (3) shows a result equivalent to figure 2 with the multiple model created by this F-K technique. Both the sea-floor multiples and peglegs are now properly predicted. The high dip segments of the seafloor multiples are not well cancelled because the multiple model was

(1) for geometry - see, e.g., figure (3) of Morley and Claerbout '78.

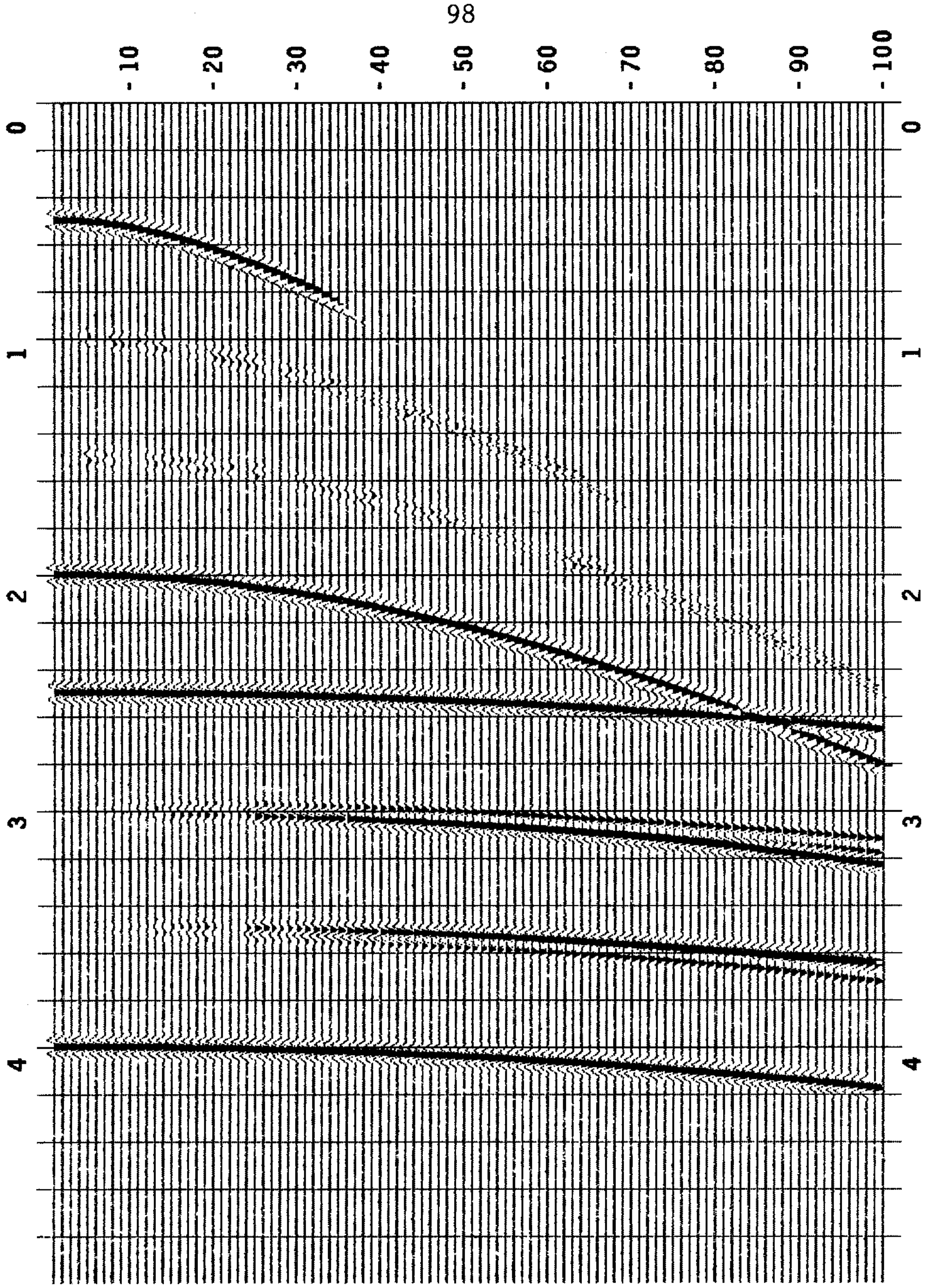

FIG. 2. Attempt to suppress water bottom and pegleg multiples by delaying radial traces for two layer model. Seafloor multiples are correctly predicted. Pegleg multiples are not well predicted because the subsea velocity differs from water velocity.

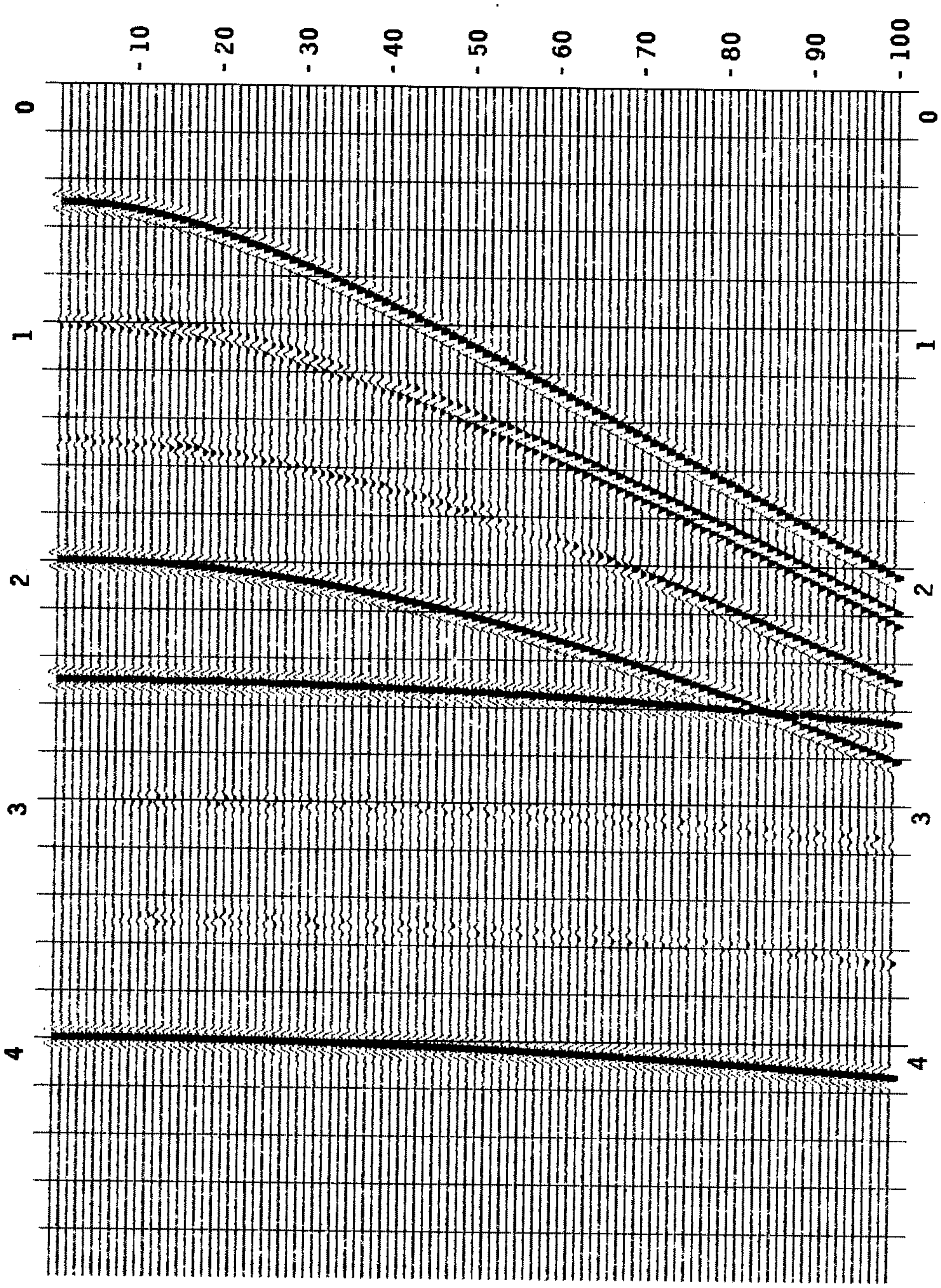

FIG. 3. Suppression of multiples for the same model as figure 2 with the multiple model created by F-K downward continuation (at water velocity) of the data to the seafloor. Both seafloor and pegleg multiples are now well predicted.

low-pass dip filtered to suppress spatial truncation artifacts. The artifact events at 2 and 4 seconds remain for the same reasons explained in the part (a) example. This method is the basis for CGG's "WEMUL" multiple prediction/suppression process.

(c) Z-Variable Velocity - First Order Dip:

Let us rewrite (3) as

$$D_{W.E.} = (1 + L_g L_s c_s L_s L_g^{-1})(1 + L_s L_g c_g L_g L_s^{-1}) \tag{4}$$

This is legitimate since L_s and L_s^{-1} commute with c_g, the seafloor coupling operator in shot coordinates. Similarly, L_g and L_g^{-1} commute with c_s[(2)]. Physically, this is saying that the depth of the shots has no bearing on whether or not the upcoming and downgoing waves in geophone coordinates happen to be coupled and vice-versa.

For a flat seafloor of depth d, the operators L_s and L_g amount to multiplication by $exp(i(\omega/v)d\sqrt{1-(Y \pm H)^2})$ in the frequency domain. (v=water velocity, Y= $vk_y/2\omega$,H= $vk_h/2\omega$, k_y and k_h are Fourier duals of midpoint,y and half-offset,h). Expanding the square root arguments of $L_g L_s$ and $L_s L_g^{-1}$ we note that:

$$\sqrt{1-(Y+H)^2} + \sqrt{1-(Y-H)^2} = 2\sqrt{1-H^2}(1 - Y^2/2(1-H^2)^2 + OY^4) \tag{5a}$$

and

$$\sqrt{1-(Y+H)^2} - \sqrt{1-(Y-H)^2} = 2HY/\sqrt{1-H^2} + OY^3 \tag{5b}$$

From 5a we see that the $L_s L_g$ operator is dip independent to first order in dip. Furthermore, if H is approximated by $\hat{H} = 2h/vt$ then the first order dip approximation to $L_s L_g^{-1}$ is just a lateral shift in midpoint-radial trace space. This is because '2h/vt' is a constant for any radial trace section. Hence, to first order in dip the only wave operators we need apply are zero-dip diffraction and a spatial shift on radial trace sections. We can remark in passing that the whole scheme can go to second order in dip if we are willing to add a "Devilish" type operator (Yilmaz '79) as indicated by (5a).

The wave predictive multiple suppression scheme just outlined is in some sense a hybrid of Taner's radial trace deconvolution and WEMUL. Table I compares it with these other two methods.

Figure 4 gives a synthetic example of this algorithm. Figure 4a is one of a collection of common midpoint gathers taken over a two layer model consisting of a flat seafloor and a

(2) L_s does not commute with c_s unless the seafloor is both flat and of uniform reflectivity.

TABLE I

	Velocity	Dip
Radial Trace	Constant	5°
Wemul	v(z)	5°
Wave Eq.-Radial Tr.	v(z)	15°

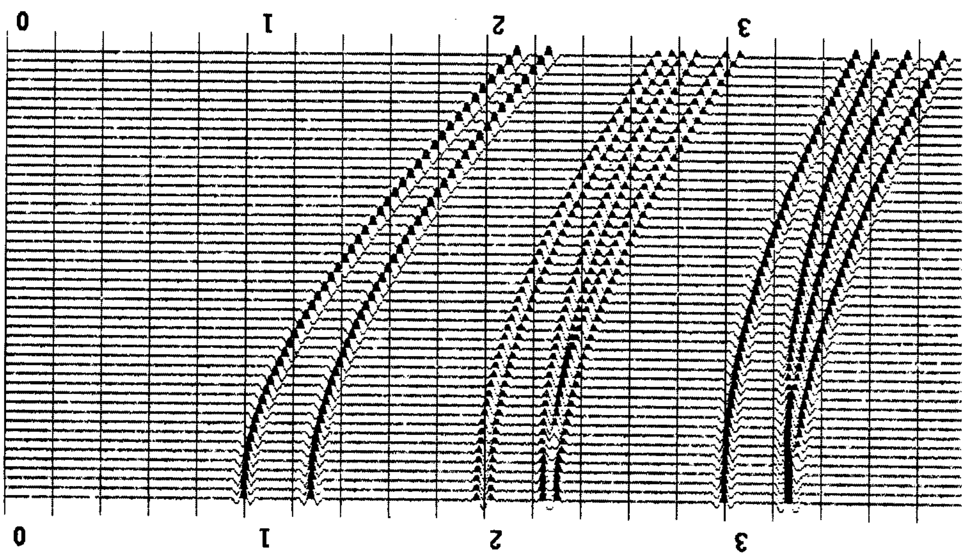

FIG. . (4a) One of a suite of synthetic CMP gathers generated for a two layer model. The flat seafloor is at 1.0 seconds and the dipping-bed primary is at 1.25 seconds. The splitting of the pegleg multiples is caused by the subsurface dip.

subsurface primary dipping at 8°. The model velocity is a constant 5000 ft/sec.

Figure 4b is a radial trace section in midpoint-time space. One trace was taken from each midpoint gather in the model. Figure 4c is a radial trace section taken from the zero-dip downward continued gathers. Theoretically, the radial angle chosen (about 13°) in figures

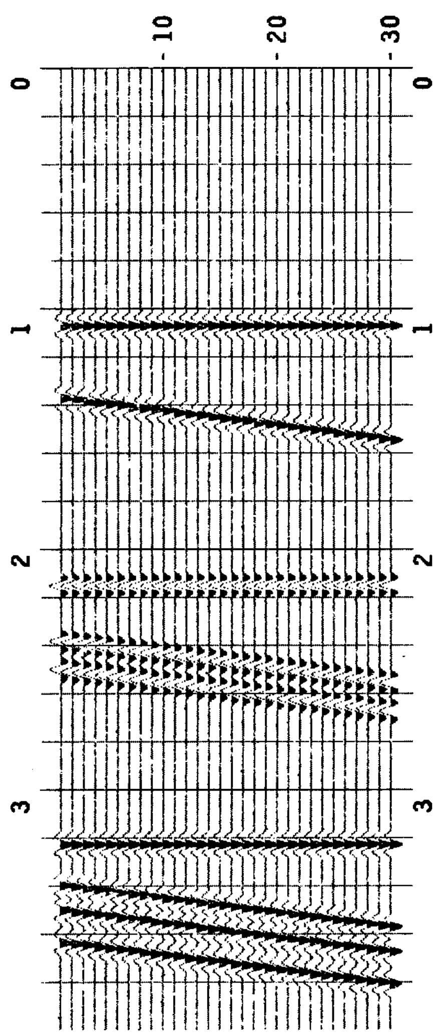

FIG 4b

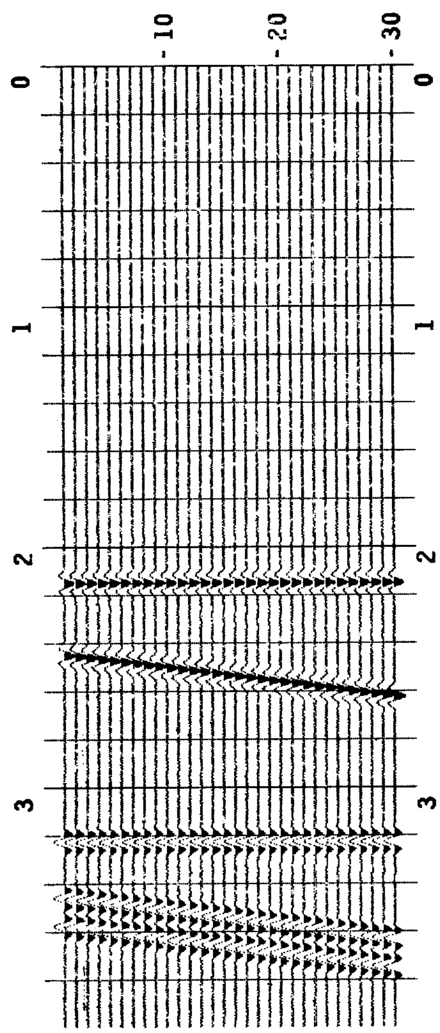

FIG 4c

4b and 4c requires a ten trace midpoint shift for the predicted pegleg to coincide with the actual pegleg. Inspection of these two figures shows that this is indeed the proper shift to cause optimal pegleg cancellation in the summed section.

Section II - Real Data Example

The method described in the last section was designed to handle the split pegleg problem in the case where the seafloor is flat and splitting is caused by subsurface dip. The water also has to be fairly deep (0.5-1.0 sec.) before noticeable splitting begins to develop. A more common situation is for the splitting to be caused by differences in seafloor topography at intermediate water depths (.3-.7 sec.). Figure 5 shows just such an example.

Figure 5a is a near trace section from a line of offshore Labrador data on which two strong pegleg multiples can be seen cutting across the section between 2.5 and 3.5

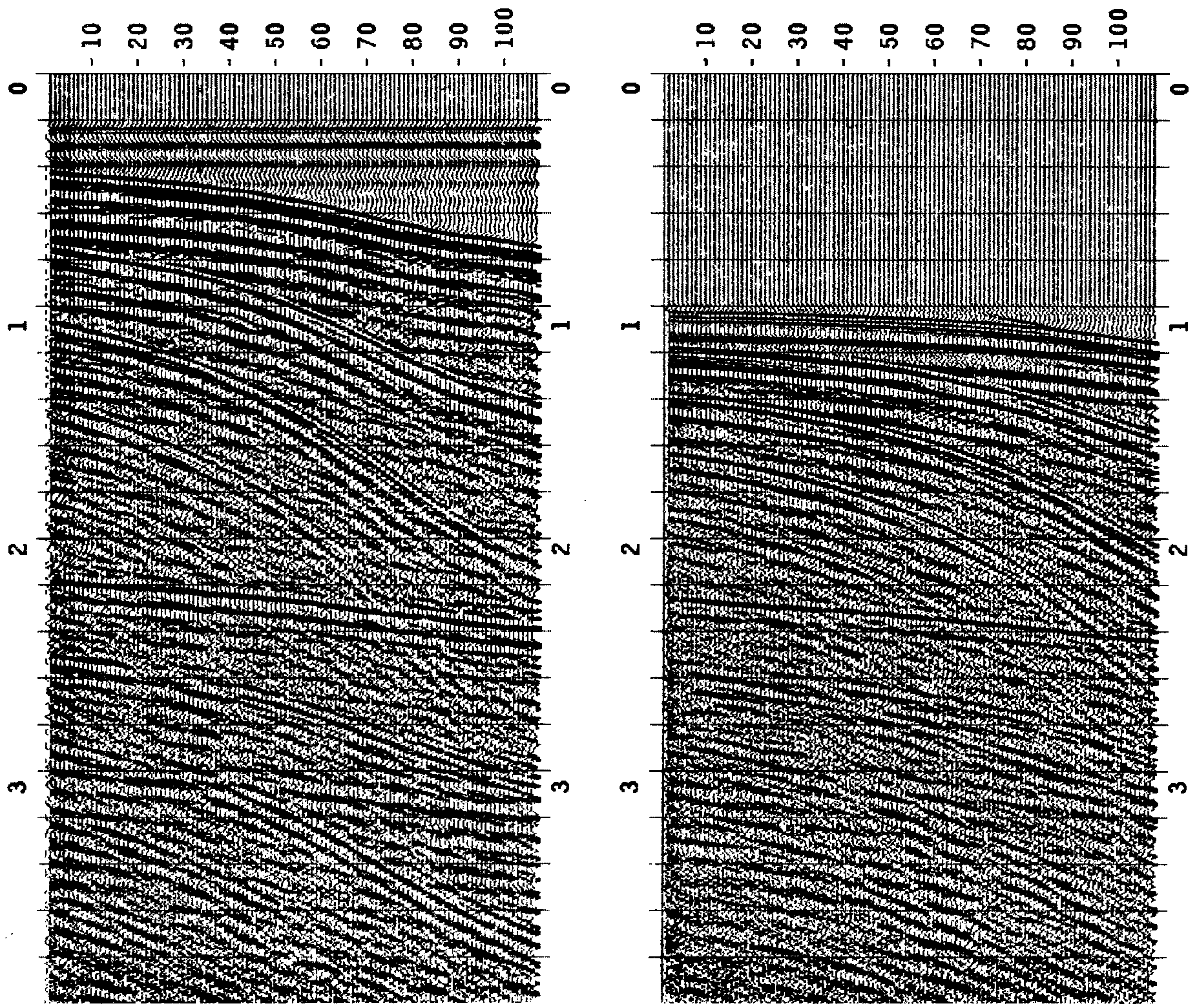

FIG. 5. 5a (left): Near trace section showing hard bottom multiples. Two peglegs from the primary at 2.2-2.4 seconds can be seen cutting across the section from 2.5-3.1 seconds and from 2.9-3.8 seconds.

5b (right): Constant offset section from an offset half-way down the cable. The first order pegleg (from 2.5-3.1 seconds on the near trace section) is now clearly split.

seconds. Figure 5b is a constant offset section of the same line for an offset half-way down the cable. The first order pegleg multiple starting at 2.5 seconds on the left and cutting across the section to 3 seconds on the right is now split. The maximum split is 200 mils around shot points 85-90. This correlates with the location on the near trace section where the seafloor has maximum dip. Even though there is a big pegleg split there is little total moveout (50 mils) on the primary just above it. This means that the pegleg reverberation raypaths are very close to vertical.

The water depth cannot be obtained from the first break times on the near trace section since there is a substantial gun delay of 100 mils - varying by as much as 15 mils from shot to shot. The autocorrelation of the near trace section, however, (fig. 6) does provide a good depth estimate. Using the depths picked from figure 6 gapped prediction filters were designed for each shot location by Wiener filtering. The split Backus prediction error filter obtained from this is plotted in figure 7. The terms of order c^2 have been dropped for this display. Figure 8 is the pegleg multiple model obtained after convolving figure 7 onto figure 5b. The arrival times of the predicted peglegs match the data peglegs to within one quarter wavelength of the dominant seismic period (i.e.-about 8-10 mils) across the section. They appear to consistently precede the data peglegs by this amount. This is to be expected, however, since the actual reverberation paths are at a slight angle to the vertical. Before a useful subtraction can be made, an offset filter has to be estimated to account for this effect.

REFERENCES

Backus, M.M. (1959), Water Reverberations - Their Nature and Elimination, Geophysics 24, pp.233-261.

Morley, L.C. (1980), Suppression of Hard-Bottom Marine Multiples with the Wave Equation, SEP-25

Morley, L.C. and Claerbout, J.F. (1978), Wave Equation Deconvolved Time Sections, SEP-15, Oct. '78., pp 191-203.

Taner, M.T. (1980), Long Period Sea-Floor Multiples and Their Suppression, Geophysical Prospecting 28, 30-48.

Yilmaz, O. (1979), Stanford Ph.D. Thesis - SEP 18.

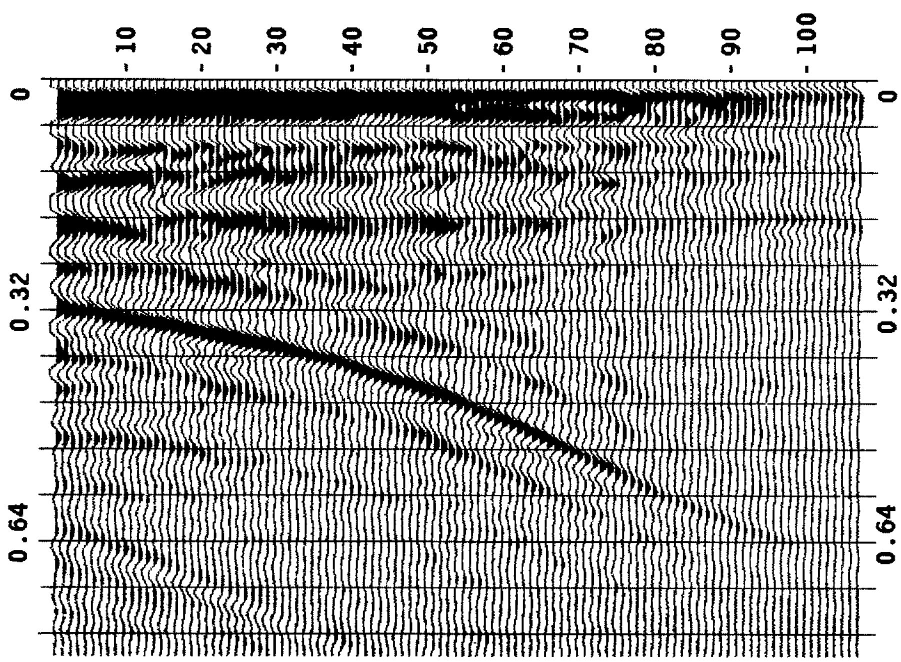

FIG. 6. Negative autocorrelation of near trace section (fig-5a). The prominent dipping event is the expression of the seafloor.

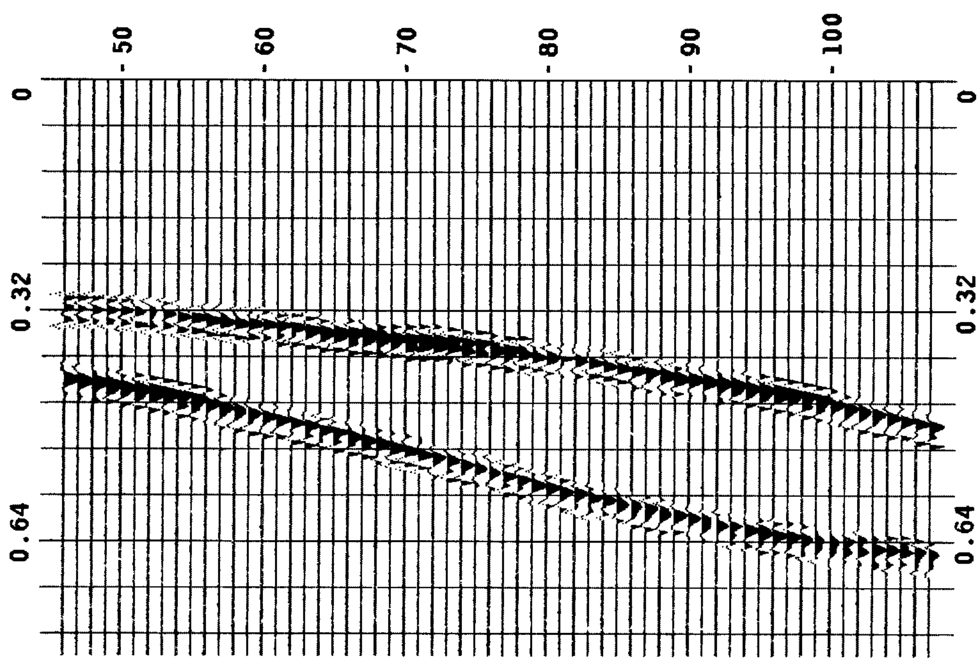

FIG. 7. Split Backus prediction filters obtained by superposing gapped Wiener prediction filters designed on the neartrace section.

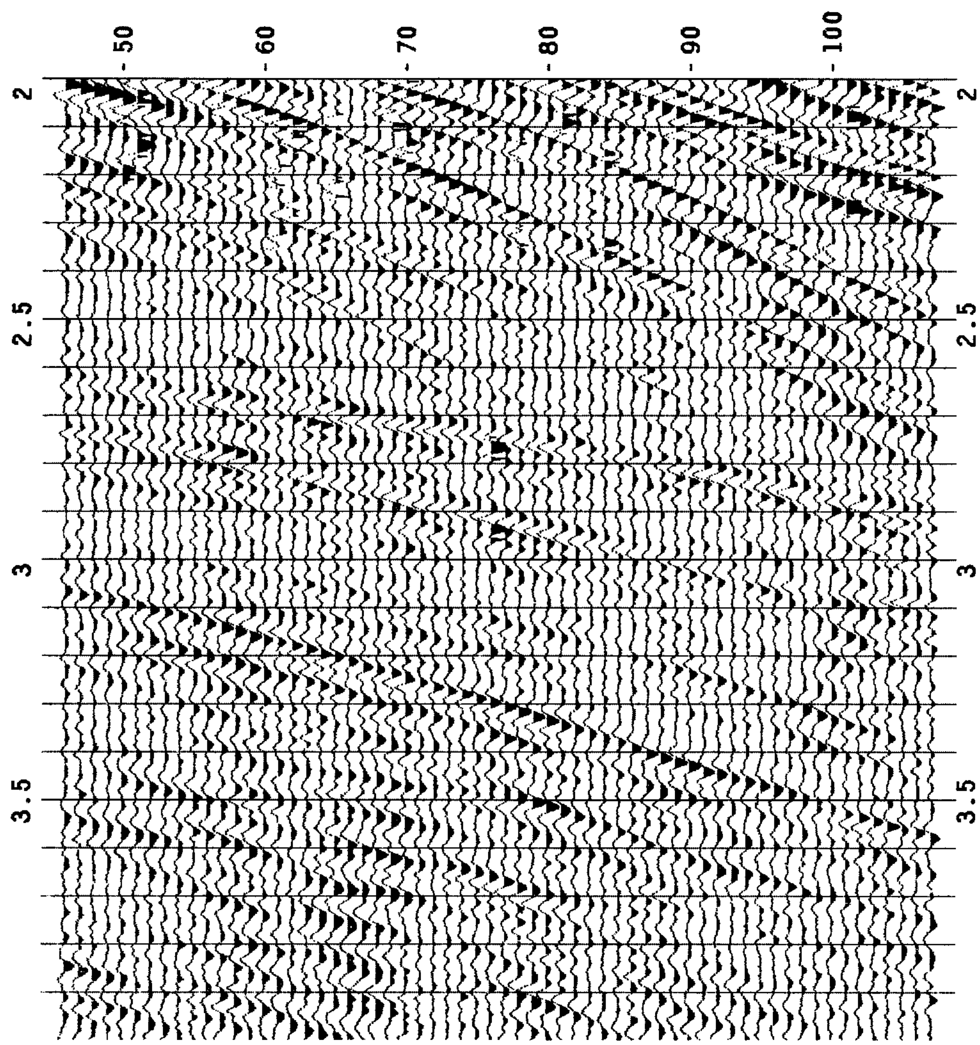

FIG. 8. Window of pegleg multiple model obtained after convolving split Backus predictor onto constant offset section in figure (5b). The first order pegleg between 2.6 and 3.2 seconds matches the data pegleg to one quarter the dominant period.