Brief Notes on the Detection of Frequency Dispersion

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In SEP-25, Yedlin and McMechan discussed a method of finding the phase velocity versus frequency of a dispersive wave (SEP-25, pp. 101-113). They demonstrated that a velocity versus frequency curve could be directly obtained from the original traces by transforming the original data field from $x-t$ space into $p-\omega$ space. In this paper I show, using synthetic examples, how various parameters such as group interval and cable length affect the final result. By using synthetic data I can isolate inherent problems, such as truncation effects and spatial aliasing, from external factors, such as noise.

My approach was fairly simple-minded. I began with a data set of synthetic Love waves, shown in Figure 1. The traces represent the wave seen at receivers ranging from 5.0 to 17.7 km from the source, at intervals of .1 km, 128 traces in all. Although it isn't really relevant to the qualitative features that I will be looking at, these synthetic traces were generated using the same program that Yedlin and McMechan used, and there were three dispersion modes present. More relevant is the fact that the source wavelet was a triangle-shaped pulse, which maps into a sinc-squared function in the frequency domain. For this reason, all Figures for this paper that happen to lie in the frequency domain will have blanks at certain frequencies corresponding to zeroes of the sinc-squared function. In order to compensate for the weak high-frequency component of this source function, all traces were rho-filtered twice.

I created three subsidiary data sets from the main data set. The first data set was composed of the 64 near traces, from 5.0 to 11.3 km from the source. The second was composed of the 64 far traces, from 11.4 to 17.7 km from the source. The last was made up of every other trace from the main data set, so it had traces from receivers from 5.0 to 17.6 km, with intervals of .2 km, again 64 traces in all.

Once the data sets were created, I slant-stacked and Fourier-transformed them in the way suggested by McMechan and Yedlin (thus transforming them into the $p-\omega$ domain),

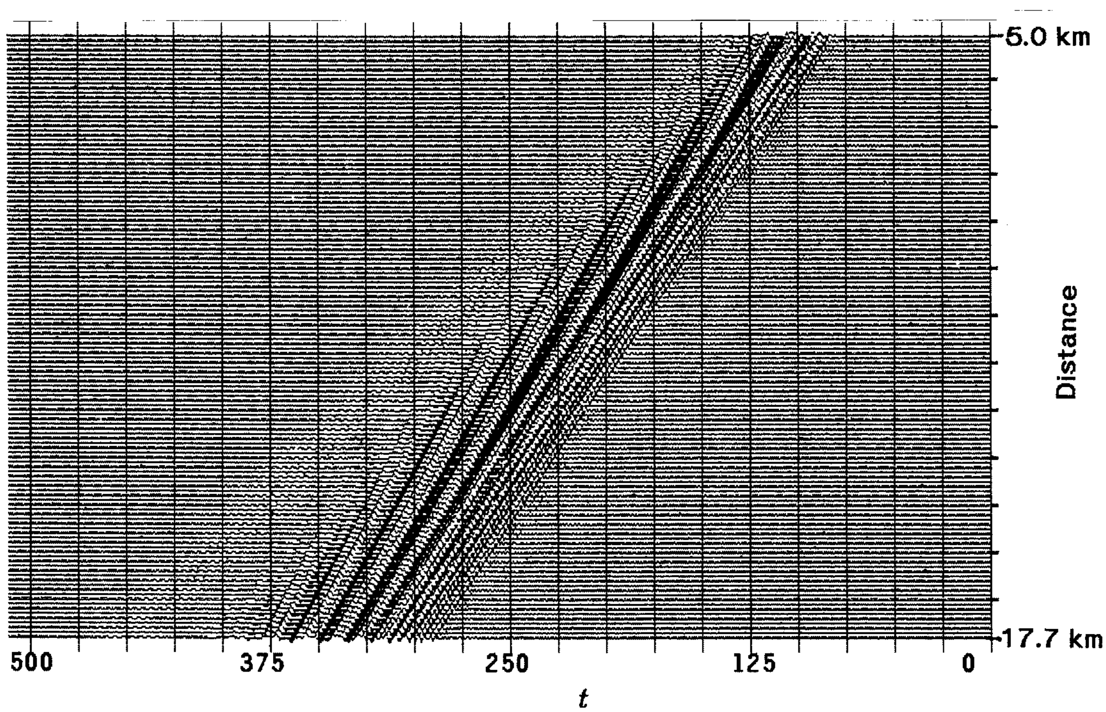

FIG. 1. Synthetic three-mode Love waves. The distance from shot to receiver varies from 5.0 to 11.3 km in intervals of .1 km. The numbering along the time axis is purely arbitrary.

giving me the results shown in Figures 2 through 5.

As shown by Yedlin and McMechan, by drawing lines through the points of maximum amplitude of the $p-\omega$ plot (as has been done in Figure 2), it is possible to measure how phase velocity (or slowness) varies with frequency for the various modes. For the purpose of this report, I'm not too interested in the precise nature of the relation between p and ω (thus the lack of meaningful values for p and ω on the axes of the figures). Rather, I am concerned with the uncertainty in this relation. For example, looking at Figure 2, we notice some spreading in the dispersion curve. The width of this spreading roughly corresponds to the uncertainty in our knowledge of p versus ω.

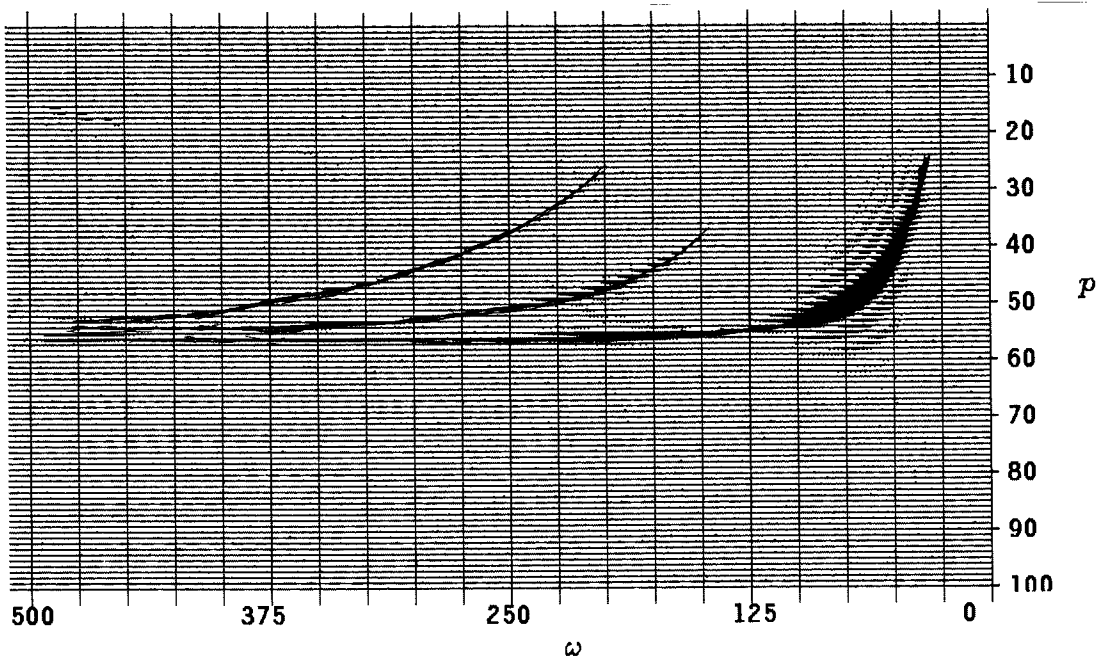

FIG. 2. The entire data set of Figure 1, transformed into $p-\omega$ space. The three dispersion modes are clearly visible. The values along the p and ω axes are purely arbitrary, but they correctly show the location of the origin and the direction of increase of p and ω.

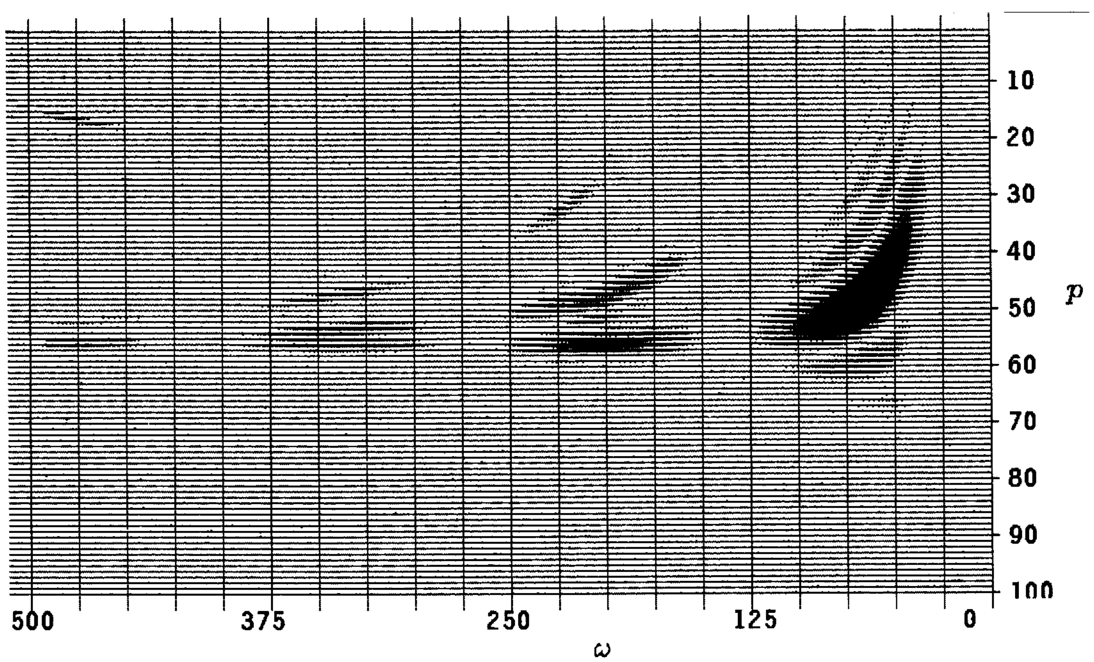

FIG. 3. The near half (5.0 to 11.3 km) of the data set shown in Figure 1, transformed into $p-\omega$ space. Note that the dispersion curves in this Figure are broader than the corresponding curves in Figure 2. Again, the numbers along the p and ω axes are purely arbitrary.

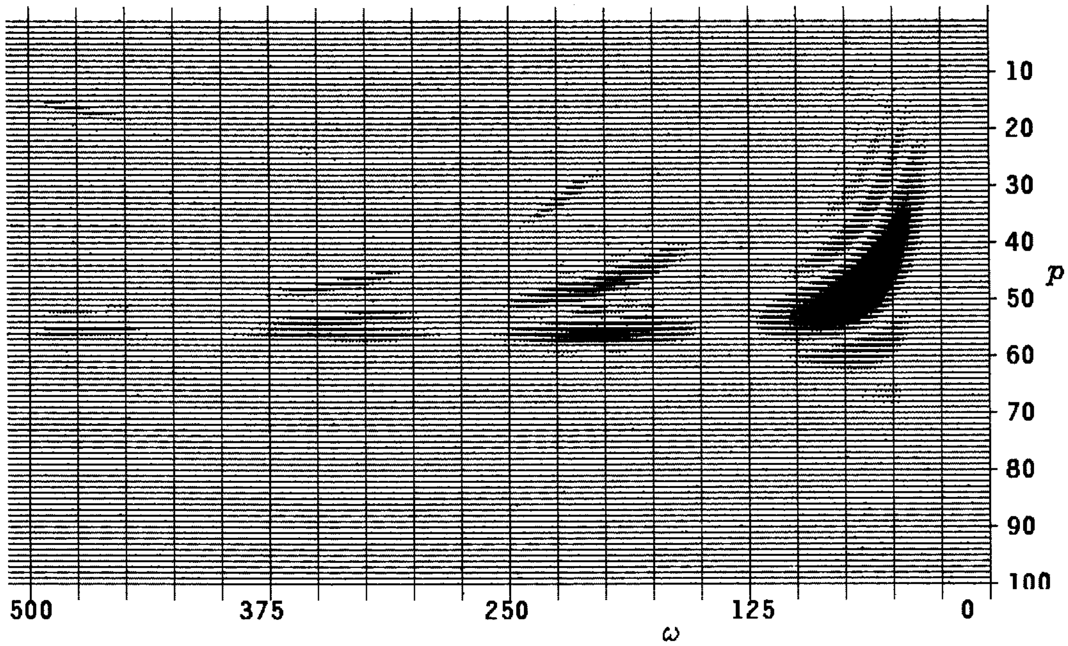

FIG. 4. The far half (11.4 to 17.7 km) of the data set shown in Figure 1, transformed into $p-\omega$ space. Notice that the dispersion curves in this Figure are virtually identical to those in Figure 3. This result suggests that the determining factor in the broadening of dispersion curves is the distance between the near and the far receiver.

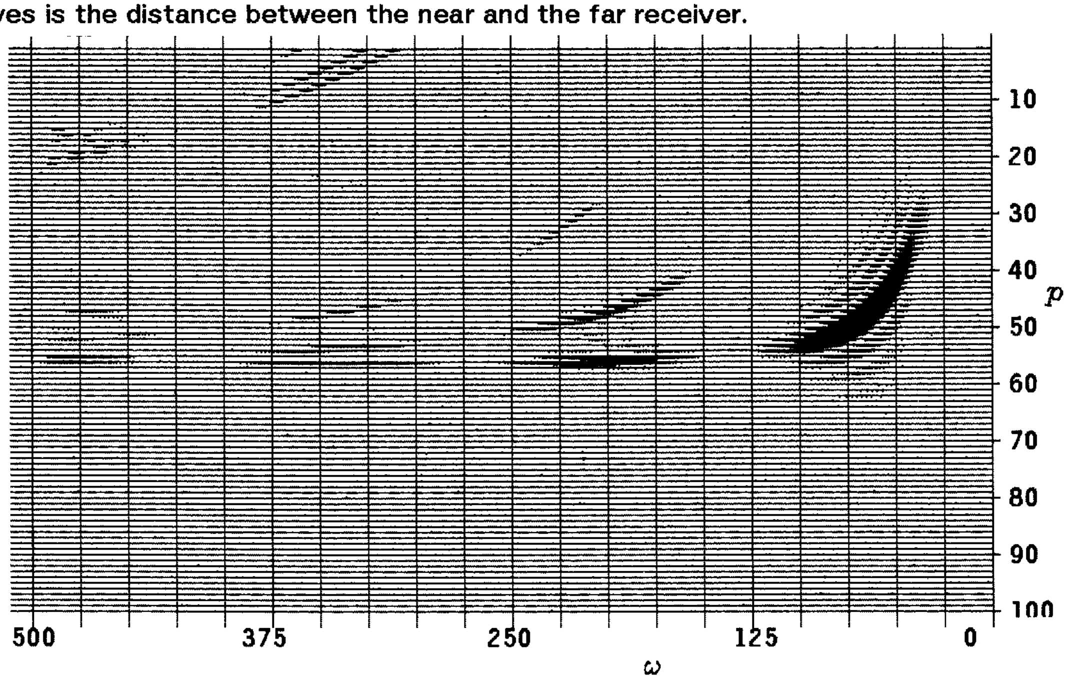

FIG. 5. Every other trace (5.0 to 17.6 km, intervals of .2 km) of the data set shown in Figure 1, transformed into $p-\omega$ space. Notice that these curves are identical to those in Figure 2, with the exception of some artifacts in the upper left part of this Figure, caused by spatial aliasing.

Looking at Figures 3 and 4, we notice that these dispersion curves are identical to each other and wider than the curves in Figure 2. Figure 3 represents the transformed data from the near 64 receivers, while Figure 4 represents the transformed data from the far 64 receivers, and so we may conclude that the uncertainty in our dispersion measurement depends on the distance between the near and far receivers, rather than on the distance from the source. Figure 5 appears nearly identical to Figure 2, although Figure 5 represents data from only every other receiver. Thus again we see that to some extent the uncertainty in our measurements is related to the distance between the near and far receivers, and, as shown by this example, that it is not related to the distance between individual receivers.

In addition, Figure 5 illustrates an effect unrelated to the broadening of the dispersion curve. This new effect is spatial aliasing, and it makes itself apparent in the form of artifacts on the upper edge of Figure 5.

That spatial aliasing is unrelated to broadening can be illustrated by the next two figures. Figure 6 shows a new data set, composed of 8 traces taken at intervals of 5 kilometers, from 5 to 40 km from the source. By transforming this extremely sparse and spatially aliased data set into $p-\omega$ space, we get the dispersion curves shown in Figure 7. Only the top group of curves (representing the lower frequencies) is at all usable. Notice how sharp these low frequency dispersion curves are. We can easily see that one of these curves directly corresponds to the low-frequency dispersion curve in Figure 2, which suggests that such spatially aliased data can give us accurate dispersion measurements, once we know which curve to believe. In Figure 7, then, we have very aliased but very sharp dispersion curves. The sharpness must be due to the fact that the distance from the near to the far receiver is 35 km (the near-to-far distance for the data in Figure 2 was 12.7 km), and the aliasing is due, of course, to the fact that the receivers are 5 km apart.

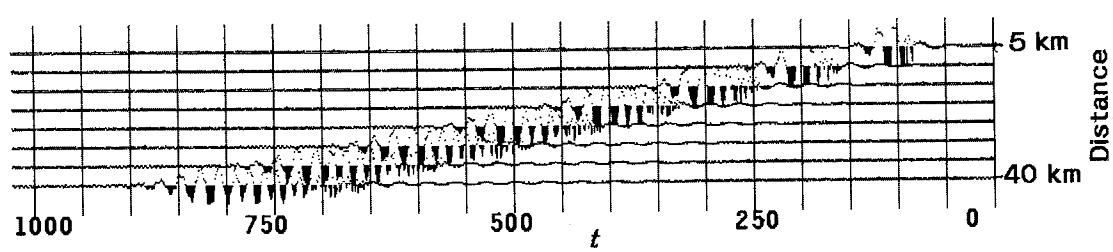

FIG. 6. Synthetic three-mode Love waves. The distance from shot to receiver varies from 5 to 40 km in intervals of 5 km, thus producing an extremely spatially-aliased data set. The numbering along the time axis is purely arbitrary.

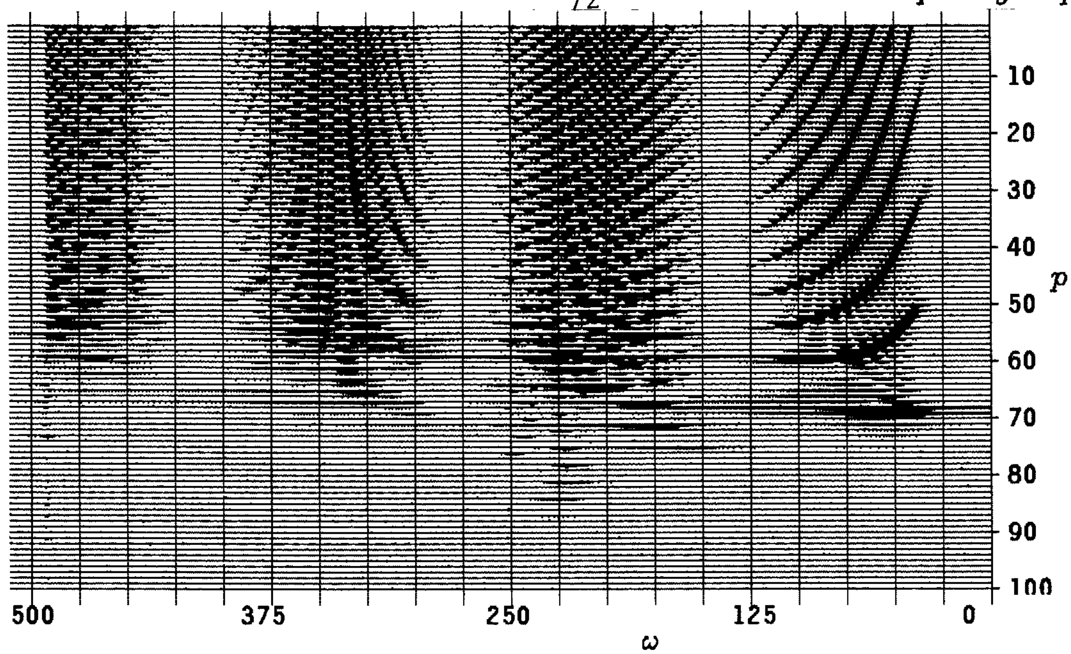

FIG. 7. The spatially aliased data set of Figure 6, transformed into $p-\omega$ space. Notice the sharpness of the low-frequency (right-most) dispersion curves. One of these curves corresponds exactly to the low-frequency dispersion curve in Figure 2. Even in the presence of extreme spatial aliasing it is possible to find a dispersion curve.

We have thus learned two things. First, we have seen that the uncertainty (the width) in dispersion measurements is in some way proportional only to the distance between the near and far receivers. This would not be a very surprising result if we were measuring group velocity by comparing the near and the far trace, but it is not obvious that such a result should hold for a phase velocity measurement involving a large number of traces. The other thing that we have learned is that spatial aliasing manifests itself in the form of extraneous lines on the $p-\omega$ plot. This in itself is not too interesting, but as we saw in the case of Figure 7, even a fairly large degree of aliasing does not destroy the usefulness of the data. When used in conjunction with unaliased data taken over a fairly short spread, aliased data taken over a long spread can be useful (by using both Figure 2 and Figure 7, we can get more precise information than we could get by using either Figure individually).

REFERENCES

McMechan G.A., and Yedlin M.J., 1980, Analysis of dispersive waves by wave-field transformation: SEP-25, pp. 101-113.

Remember, even if you win the rat race -- you're still a rat.

Steele's Plagiarism of Somebody's Philosophy:
Everybody should believe in something -- I believe I'll have another drink.

Condense soup, not books!

Not far from here, by a white sun, behind a green star, lived the Steelypips, illustrious, industrious, and they hadn't a care: no spats in their vats, no rules, no schools, no gloom, no evil influence of the moon, no trouble from matter or antimatter -- for they had a machine, a dream of a machine, with springs and gears and perfect in every respect. And they lived with it, and on it, and under it, and inside it, for it was all they had -- first they saved up all their atoms, then they put them all together, and if one didn't fit, why they chipped at it a bit, and everything was just fine...
-- Stanislaw Lem

Modern man is the missing link between apes and human beings.

Love is a word that is constantly heard,
Hate is a word that is not.
Love, I am told, is more precious than gold.
Love, I have read, is hot.
But hate is the verb that to me is superb,
And Love but a drug on the mart.
Any kiddie in school can love like a fool,
But Hating, my boy, is an Art.
-- Ogden Nash

THEORY
Into love and out again,
Thus I went and thus I go.
Spare your voice, and hold your pen:
Well and bitterly I know
All the songs were ever sung,
All the words were ever said;
Could it be, when I was young,
Someone dropped me on my head?
-- Dorothy Parker

Losing your drivers' license is just God's way of saying "BOOGA, BOOGA!"