REFLECTION SEISMOLOGY LITERATURE IN CHINA

Richard Ottolini

This report covers books and journals published in China about geophysical petroleum exploration. It is intended to complement the report of the 1979 SEG delegation to China (Stan Jones, *Geophysics*, October 1980). This report is derived from information obtained during a six month, non-professional trip by the author to the People's Republic of China in late 1979 and early 1980. Information comes from technical bookstores, the National Library, the State Seismological Bureau Library, the Peking University Library, and personal contacts. Unfortunately, at that time the major petroleum geophysical research institutes seemed unwilling to engage in dialog with the Stanford Exploration Project, so information from these sources was limited. The situation is changing rapidly in China, so that the information in this report may soon be outdated.

Structure of Petroleum Exploration Geophysics in China

This topic is best covered in the above cited *Geophysics* article. To summarize, the major geophysical petroleum exploration efforts are conducted under the Ministry of Petroleum and Ministry of Geology. The Ministry of Petroleum runs the China National Oil and Gas Company plus over a dozen provincial and oilfield centered divisions. In addition a very small amount of reflection seismology is done at the National Academy of Sciences Geophysical Research Institute and the State Seismological Bureau.

Journals

I came across about twenty-five journals about the petroleum industry while in China. Most were classified *neibu* which is short for "for internal circulation only". However, the recent trend is towards removing these circulation restrictions. Four of these journals dealt mostly with reflection seismology theory and practice, while another four contained geological use of reflection seismograms. Two which contain mostly reflection seismology are

(1) *Oil Geophysical Prospecting*
published bimonthly by the Ministry of Petroleum's Geophysical Exploration Bureau-
Editor Lin Yuangen.
This journal can be suscribed from the Guoji Shudian, Box 399, Peking, China.
Subscription price is unknown.

(2) *Geophysical Prospecting for Petroleum*
published quarterly by the Ministry of Geology's Research Institute of Geophysical Prospecting for Petroleum-
Editor Huang Xude.
This journal is available from the Editorial Board, Shiyou Wutan, 29-1 Weigang Street, Nanjing, China. Subscription price is unknown.

Recent tables of contents and two translated articles are attached as appendices.

Articles on reflection seismology have also occasionally appeared in

(3) *Acta Geophysica Sinica*
Published quarterly by the Chinese Geophysics Society-
Editor Chuan Chengyi.

Internationally distributed journals about geology and reflection seismology include

(4) *Acta Petroleum Sinica*
Published quarterly by the Chinese Petroleum Society-
Editor Jiang Qi.

(5) *Oil and Gas Geology*
Published Quarterly by the Ministry of Geology's Petroleum Exploration Bureau-
Editor Guan Shicong.

The last three journals are all available from the Guoji Shudian, Box 399, Peking, China. The subscription prices are unknown.

Books

In a survey conducted in October, 1979 there were slightly over 250 titles about the earth sciences available in Peking bookstores. Fifty one were on geophysics, with only a few about reflection seismology. Nothing was observed on reflection seismology during rare visits to *neibu* bookstores. Books about reflection seismology include:

(1) Fundamentals of Geophysical Data Processing
J.F. Claerbout
Petrochemical Industry Publishers,
16 Heping Liqi District, Peking
1979, 357 pages, Y1.20
(A pirate translation of this classic.)

(2) Dictionary of Exploration Geophysics
Editorial Group
Science Publishers
137 Chaoyang Mennei Street, Peking
1976, 151 pages, Y1.40
(The best translation aid for reflection seismology articles.)

(3) Seismic Prospecting Instruments
B.S. Evenden
Petrochemical Industry Publishers
16 Heping Liqi District, Peking
1978, 237 pages, Y0.80
(Another translation.)

(4) Numerical Techniques in Seismic Exploration (3 Volumes)
Ministry of Petroleum Geophysics Computation Center
Science Publishers
137 Chaoyang Mennei Street, Peking
1974, 1977, 522 pages, Y3.60
(Excellent series covering higher mathematics
and fundamentals of reflection seismology.)

(5) Digital Seismic Instruments
Xian Petroleum Instruments Factory
Petrochemical Publishers
16 Heping Liqi District, Peking
1975, 144 pages, Y0.52

Appendix 1: Selected Recent Contents of "Oil Geophysical Prospecting"

1980 #3:

Retaining Actual Frequency Information, *Weng Wendo*

Analysis of the Seismo-Geological Conditions in the Hydrocarbon Bearing Areas of China, *Wan Youlin Ou Qingxian*

*Migration Velocity Analysis with Sign Bit Statistical Detection, *Shao Youqing*

*Implementation of Migration in the F-K Domain, *Tian Xiaokun Wang Xihuai*

*An Additional Parameter in Claerbout's Equations, *Zhao Zhenfei Zhao Zhenwen*

CDP Stacking on Crooked Lines, *Cheng Jiang*

A Recursion Solution of the Toeplitz Equations, *Peng Yongmeng*

On Filling Zeros in FFTs, *Yin Bajin*

1979 #6:

Computation and Application of Residual Statics, *Xiong Zhu*

The Balance Between Automatic Statics and Automatic Recording- A Discussion, *Chen Xiao*

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*translations available or in progess

Numerical Computing Methods, *Wang Kulin*

A Computation Method for Continuing Seismic Wavefields, *Gu Lanming*

The Use of the Karhunen Transformation in Seismic Data Processing (translation), *Hemon*

Chapter 11 of Fundamentals of Geophysical Data Processing, *Claerbout*

1979 #5:

Crooked Line Processing Methods, *Xiong Zhu*

Determination of Several Parameters in Crooked Line Multiple Coverage, *Huang Shuiming*

Limitations of Multiple Coverage in Crooked Survey Lines, *Deng Dongrun*

Sum Weighted Velocity Spectra, *Qin Zheng*

The Effect of Time Shifts of W Transform Amplitudes, *Zhang Zhenping*

The Wave Equation and Modeling (translation), *Hemon*

Chapter 10 of Fundamentals of Geophysical Data Processing (translation), *Claerbout*

Chapter 8 of Introduction to the 1724 Seismic Processing System (translation)

(Contents of 10 other issues are also available.)

Appendix 2: Selected Recent Contents of "Geophysical Prospecting for Petroleum"

1980 #3:

(Special issue on Carbonate Exploration)

Views on Carbonate Geophysical Exploration, *Gu Gongxu*

Guiding Principles and Methods for Central Problems in Carbonate Areas of Southern China, *Ou Qingxian*

The Central Problems of Geophysical Exploration for Carbonates in Southern China, *Zhang Guanghua*

Seismic Experiment for Obtaining Reflections from a Carbonate Reservoir in the Jin Chang Region of Southern Jiangsu Province, *Zhou Jikang   Hu Guolian   Chen Guanghua*

Seismic Field Work in Mountainous Limestone Regions, *Tang Wenbang*

Using Geophysical Methods to Study Carbonates (translation), *J. Delplanche   D.Michon*

Why Static Corrections Didn't Work In a Mountainous Region, *Cao Jinsheng*

Seismic Work in Hilly Land with Outcropping Carbonates, Central District, Hunan Province, *Chen Xianggun   Xiao Xuezheng*

The Possiblity of Using Surface Sources in Dry Mountainous Regions, *Huang Yongming*

Overseas Methods of Carbonate Exploration

Computer Processing and Interpretation of Carbonate Reservoir Well Logs, *Xiao Cixuan   Liang Gancai*

Number Theory Transformations and Seismic Signal Filters, *He Zaitian*

Application of the Iterative Frequency Domain Method for Gravity Anomalies, *Li Guozhi Chen Zhonghua*

1979 #2:

The Residual Static Correction of Autostatics- The Method of a Model Trace Formed by Multichannel Aligned Stacking, *Nie Xunbi He Yuchun*

*Some Problems Concerning the Application of Wave Equation Stack Migration, *Wang Zhaohua Xu Boxun Xu Yun Zhao Jingxuan*

*Wave Equation Migration for Steep Dips, *Wu Lu Ma Yanru He Jia*

Analytic Continuation of Gravity and Magnetic Fields in the Hefei Basin and their Significance to Deep Structure Studies, *Li Xiuxin Liu Deliang*

The Application of Data Compression Techniques to Seismic Prospecting, *Wang Tianwei*

(Contents of four other issues are also available.)

Appendix 3:
Wave Equation Migration for Steep Dips

Wu Lu   Man Yanru   He Jia

Translated from "Geophysics for Petroleum Prospecting"
1979, #2, pages 67- 72 by Richard Ottolini

Abstract

This article introduces a migration method suitable for layers dipping to 40 degrees which takes only forty percent the computation time of Claerbout's method. At the end of the article is an actual data processing example.

(1)

Wave equation migration is an important research topic. Reports and theses on wave equation migration make up a comparatively large part of some related academic meetings and journals. This on one hand goes to show the importance of this method; yet on the other hand that some fundamental migration problems remain in seismic data processing.

The many migration methods may be summarized into three kinds- Kirchoff summation, Claerbout finite difference, frequency domain migration[1]. A rather large part of research and application is on Claerbout's finite difference method. Claerbout personally asserts that his method is effective only to angles of 15 degrees. This is to say that it can only be applied for the migration of small angles. When the angles are large there are serious dispersion and phase errors with poor results. This article introduces a migration method suitable for dipping layers up to 40 degrees. Dispersion phenomenon is less than in Claerbout's method. Thus we can use a rather large migrated time interval ($\Delta\tau$) and increase computation efficiency.

(2)

Our theory is only for two dimensional wave equation problems.

The two dimensional wave equation after Claerbout's time domain upgoing wave coordinate transformation is

$$\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Z^2} = - \frac{2}{V} \frac{\partial^2 U}{\partial Z \partial t} \tag{1}$$

where v is constant velocity. We take the t partial derivative of equation (1)

$$\frac{\partial^3 U}{\partial X^2 \partial t} + \frac{\partial^3 U}{\partial Z^2 \partial t} = - \frac{2}{V} \frac{\partial^3 U}{\partial Z \partial t^2} \tag{2}$$

Again taking the z partial derivative of equation (1) gives

$$\frac{\partial^3 U}{\partial X^2 \partial z} + \frac{\partial^3 U}{\partial Z^3} = - \frac{2}{V} \frac{\partial^3 U}{\partial Z^2 \partial t} \tag{3}$$

Eliminating the second order z partial derivative term and omitting the third order z partial derivative results as

$$\frac{2}{V} \frac{\partial^3 U}{\partial X^2 \partial t} - \frac{\partial 3 U}{\partial X^2 \partial z} + \left[\frac{2}{V}\right]^2 \frac{\partial^3 U}{\partial Z \partial t^2} = 0 \tag{4}$$

We now make a difference approximation to equation 4 using a five point differencing scheme. Use $\frac{T}{\Delta X^2}$p to represent $\frac{\partial}{\partial X^2}$, T = (-1,2,-1); in the t-z plane use $Z = n\Delta z$ $(n = 1,2,3 \cdots N)$ $t = j\Delta t$ $(j = 1,2,3 \cdots J)$ to obtain

$$U_j^{n+1} = U_{j+2}^n - \frac{I_n - \alpha T}{I_n + \alpha T} \left[U_{j+2}^{n+1} - U_j^n\right] + \frac{2(I_n - \beta T)}{I_n + \alpha T} \left[U_{j+1}^{n+1} - U_{j+1}^n\right] \tag{5}$$

where

$$\alpha = \frac{V^2 \Delta t \Delta \tau}{32 \Delta X^2} \tag{6}$$

$$\beta = \frac{V^2 \Delta t^2}{32 \Delta X^2} \tag{7}$$

It must be pointed out that according to Claerbout's imaging principle, the velocities during migration in equations (6) and (7) are one half of the velocity values in equations (1) to (5), $\Delta\tau = \frac{2\Delta Z}{V}$. We can assume the z direction velocity variation is the τ direction variation provided that the velocity is constant within intervals.

In equation (5), I_n is the identity matrix as large as U. T is the tridiagonal matrix. Based upon the properties of characteristic vectors in linear algebra it can be proven that T is positive definite. An equivalent formula can be obtained for equation (5).

$$U_j^{n+1} = U_{j+2}^n - \frac{1 - \alpha T}{1 + \alpha T}\left[U_{j+2}^{n+1} - U_j^n\right] + \frac{2(1 - \beta T)}{1 + \alpha T}\left[U_{j+1}^{n+1} - U_{j+1}^n\right] \qquad (8)$$

Nonetheless, in equation (8) T is a characteristic value and U is a scalar.

Then we use the recurrence relation $T = \frac{D}{1 - \gamma D}$, $\gamma = \frac{1}{6}$ and Z transforms in the x direction to obtain

$$U_j^{n+1} = U_{j+2}^n - \frac{(\alpha + \gamma) + [1 - 2(\alpha + \gamma)]Z + (\alpha + \gamma)Z^2}{-(\alpha - \gamma) + [1 + 2(\alpha - \gamma)]Z - (\alpha - \gamma)Z^2}\left[U_{j+2}^{n+1} - U_j^n\right]$$

$$+ 2\frac{(\beta + \gamma) + [1 - 2(\beta + \gamma)]Z + (\alpha + \gamma)Z^2}{(\alpha - \gamma) + [1 - 2(\alpha - \gamma)]Z + (\alpha - \gamma)Z^2}\left[U_{j+1}^{n+1} - U_{j+1}^n\right] \qquad (9)$$

If the third right hand term in equation (5) is set to zero, we have the same formula as Claerbout's method. Conditions for setting the third term to zero are: (1) $U_{j+1}^{n+1} = U_{j+1}^n$; (2) if not equal then $U_{j+1}^{n+1} - U_{j+1}^n$ is a solution to the homogeneous linear equation $(I - \beta T)X$. There are already many articles discussing initial and boundary conditions that will not be repeated here.

(3)

Modeling experiments were conducted in order to research the effectiveness of various methods. It can be said that the method of this article is an improvements over Claerbout's basic method. Of course it is most convincing to conduct comparisons with the modeling research results of already published articles on Claerbout's method. Ray calculation methods were used to obtain synthetic sections similar to those of reference [3] and equivalent to CDP stacks. The geophone distance is 30 meters and sampling interval is 4 milliseconds.

First Claerbout's method was used in a DJS-11 computer to construct two testable models. The results are comparable to those in reference [3]. Afterwards the method of this article was used to construct numerical models.

Figure 1a is a simple, single reflector with a dip of 12 degrees. The migrated result using Claerbout's method with a continuation interval Δτ of 64 milliseconds is shown in figure 1b. The basic waveform is unaltered and there aren't any phase errors. Just the arrival is lengthened by a factor of $\frac{1}{\cos}\vartheta$ where ϑ is the dip angle (3). Carefully observe that figure 1b still has arc phenomenon such as where the arrows show, but none of the methods eliminate this.

Figure 2a is a four layer structure. The dip angles of the first and fourth layers are the same- 22 degrees. the dip angles of the second and third layers are the same- 12 degrees. After Claerbout's migration method, besides the characteristics of figure 1b, the waveforms of the 22 degree dipping layers change. The period increases and the event bifurcates. The left 15 traces disappear on the fourth layer. These are all created by the influence of dispersion. More serious effects result from even steeper dip angles.

It must be pointed out here that when using Claerbout's method to do migration processing, the ends of events will be altered. If the dip angles of layers are small, arrivals will be stretched out after migration. If the dip angle of a layer reaches a certain angle (we modeled 22 degrees) the ends of the arrivals on migrated sections will be shortened. This undoubtably gives interpreters difficulties.

Figure 3 is a three layer model. The first and third layers have 12 degree dip angles. The second layer is 22 degrees. Using the method of this article with continuation intervals of 100 milliseconds, the migrated section appears in figure 3b. The waveforms do not change, there is no arc phenomena, and the arrivals are neither stretched nor contracted. However, the whole waveforms have moved about 100 milliseconds. The displacements at the endpoints have very small arcs. In actual data processing there is not such an effect because the numerical value of the arcs is several orders of magnitude less than that of the input.

The layer dip angles of figures 4a and 5a are 30 and 40 degrees. After our migration method, except for a weak arrival, the character is the same as figure 3b.

It can also be seen from figures 3b, 4b, and 5b that the dip angles of the layers respective arrivals change. Of course, as you can see, our method compared to Claerbout's has smaller changes.

In reference [3] the long discussion of dispersion and phase shift in Claerbout's method concludes that a continuation interval of only 20 milliseconds can be used. Our experimental results demonstrate that intervals up to 100 milliseconds can be used. In this a large amount of machine time can be saved.

The following are field data processing examples all using a continuation interval of 100 milliseconds.

Figure 6a is a continental six fold CDP stack section from analog recorders. This district has sandy shale layers underlaid with limestone. Due to complicated basal fracturing, the diffraction arcs on the CDP record are strong and long. After

Claearbout's migration the diffractions aren't removed very well. After using our migration, as shown in figure 6b, strong diffraction arcs are removed very well and the section is very clean. There isn't any semi-circle phenomenon.

Figure 7 is a marine sixfold CDP stack section using analog recording. The arrow points to a diffraction arc. As shown in figure 7b our migration method cleanly removes the diffraction arc. In the middle part of figure 7a are the ends of a layer labeled A and B which are not altered after migration. The signal to noise ratio is rather high.

Figure 8 is a continental six fold CDP stack also using analog recording. Complicated fracturing assumes the appearance of mutually crossing diffractions. After out migration method diffractions are removed and fault points can be clearly seen. See figure 8b.

The support and assistance of comrades Zeng Degao, Wang Haisi, and Xu Boxun is greatly appreciated.

References

(1) R. H. Stolt, 1978, Migration by Fourier Transform, Geophysics, v43, p23-48

(2) J. F. Claerbout, Fundamentals of Geophysical Data Processing: New York, McGraw Hill Book Company

(3) D. Loewenthal, L. Lu, R. Robenson, and J. Sherwood, The Wave Equation Applied to Migration, Geophysical Prospecting, v24, p380-399

(4) U. M. Fourmann, Seismic Processing with Wave Equation Migration, CGG Technical Series, #503, 77.05

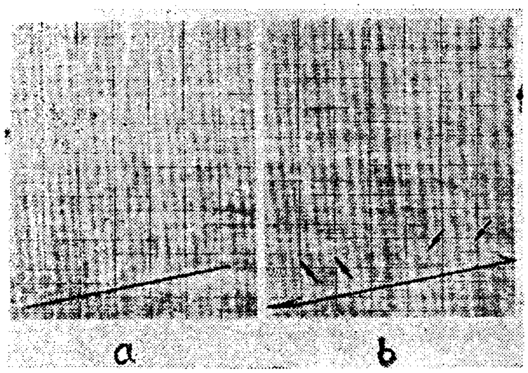

图 1

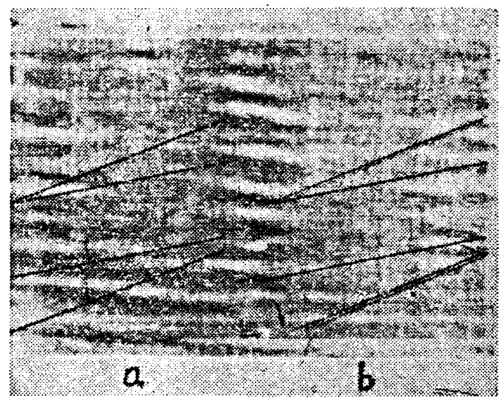

图 2

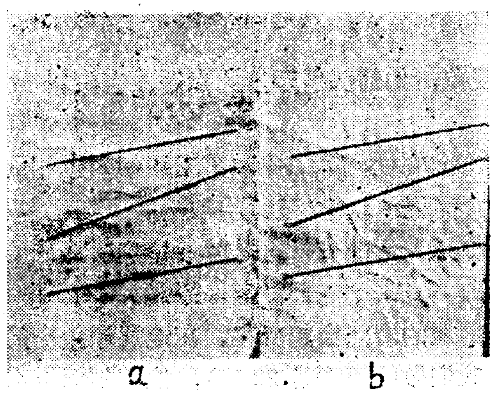

图 3

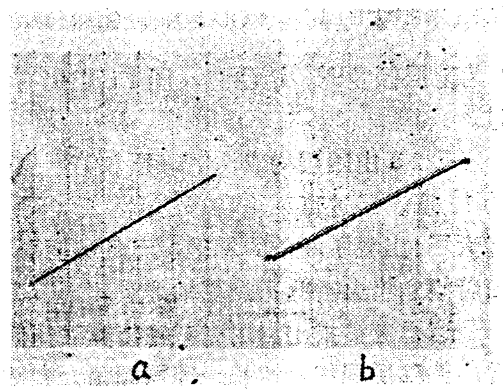

图 4

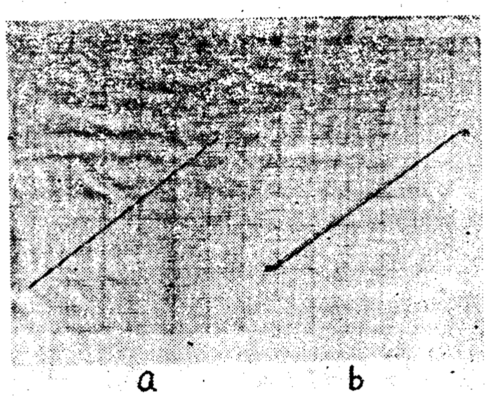

图 5

Selected figures. Figures are not printed very well in the original report, especially those of field data seismograms.

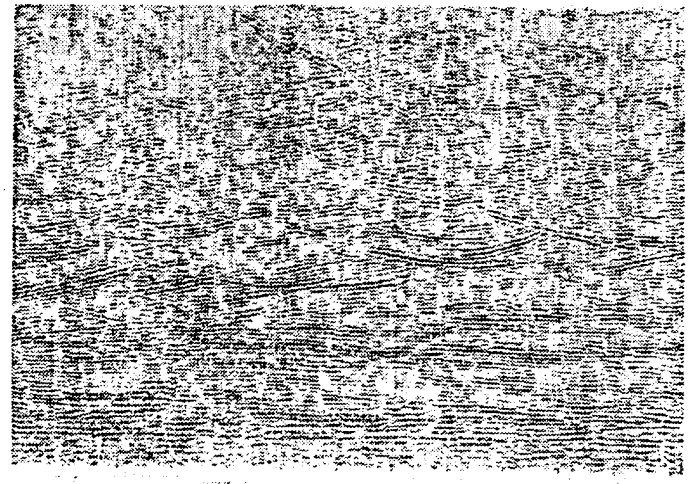

图 8a

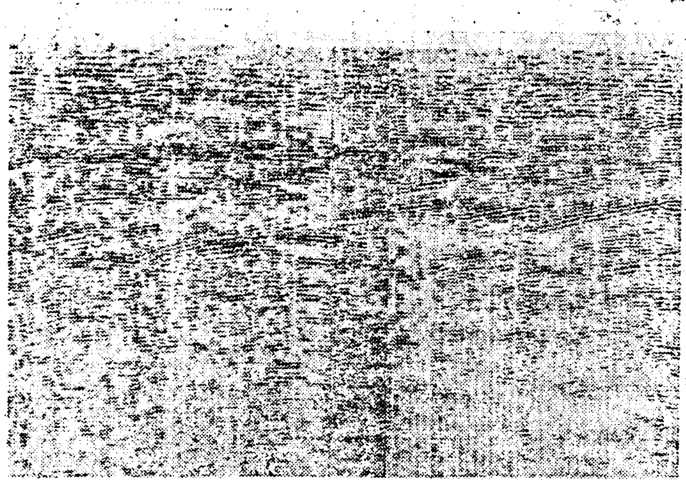

图 8b

Selected figures continued.

Appendix 4:
Some Problems Concerning the Application of Wave Equation Stack Migration

Wang Zhaohua   Xu Yun   Xu Boxun   Zhao Jingxuan
Translated from "Geophysics for Petroleum Prospecting", 1979, #2, pages 58- 66 by Richard Ottolini

Abstract

This article briefly states the realization of finite difference method of wave equation stack migration in a #150 computer, discussing data processing problems including the stability of finite difference schemes, the influence of velocity parameters on migration results, and arc noise phenomenon on seismic sections.

(1) Wave Equation Stack Migration

In the reflection seismic exploration method we research longitudinal wave disturbances formed after a source excitation in a medium distant from the source. Ignoring the sideways reflections of three dimensional energy scattering, the two dimensional longitudinal waves P(X,Z,t) satisfy the scalar wave equation

$$\frac{\partial^2 P}{\partial X^2} + \frac{\partial^2 P}{\partial Z^2} = \frac{1}{C^2}\,\frac{\partial^2 P}{\partial t^2} \tag{1}$$

where C is the propagation velocity of longitudinal waves in the media. The X axis coincides with the survey line, the Z axis is directed vertically downward. Typically, migration is performed after CDP stack. The CDP stack can be viewed as making a co-located shot-receiver time section. For every reflector point of the reflector horizon we can utilize the principle of equal times of upgoing waves and downgoing waves in order to realize migration.

In order to separate the upgoing wave equation from equation 1 [1], we introduce the following coordinate transformation

$$X' = X$$

$$Z' = Z$$

$$t' = t + \frac{Z}{C}$$

In the new coordinate system the two dimensional longitudinal waves are defined by

$$P(X,Z,t) = U(X',Z',t')$$

This leads to the two dimensional upgoing wave equation

$$\frac{\partial^2 U}{\partial Z' \partial t'} = - \frac{C}{2} \left[\frac{\partial^2 U}{\partial X'^2} + \frac{\partial^2 U}{\partial Z'^2} \right]$$

Using the parabolic approximation and dropping primes gives

$$\frac{\partial^2 U}{\partial Z \partial t} = - \frac{C}{2} \frac{\partial^2 U}{\partial X^2} \tag{2}$$

Using the Crank-Nicolson scheme obtains the explicit finite difference solution

$$U_j^{n+1} = \frac{1 - \alpha D}{1 + \alpha D} \left[U_{j+1}^{n+1} + U_j^n \right] - U_{j+1}^n \tag{3}$$

where $\alpha = \frac{C^2 \Delta t \, \Delta \tau}{32 \Delta X^2}$, Δt is the sampling interval, $\Delta\tau$ the continuation interval, and ΔX is the same as the geophone spacing. D is the second order difference operator of the following form

$$D = \frac{T}{1 - \frac{T}{6}}$$

where T is the three point convolution factor (-1,2,-1), when inserted into equation 3 gives

$$U_{n+1}^j = \frac{1 - (\alpha+\beta)T}{1 + (\alpha-\beta)T} \left[U_{j+1}^{n+1} + U_j^n \right] - U_{j+1}^n \tag{4}$$

where $\beta = \frac{1}{6}$.

Using Z transforms, equation 4 changes into an operational formula for an electronic computer

$$U_j^{n+1} = \frac{-(\alpha+\beta) - [1 - 2(\alpha+\beta)]Z - (\alpha+\beta)Z^2}{(\alpha-\beta) - [1 + 2(\alpha-\beta)]Z - (\alpha-\beta)Z^2} \left[U_{j+1}^{n+1} + U_j^n \right] - U_{j+1}^n$$

$$= \frac{\left[\frac{\alpha+\beta}{\alpha-\beta}\right]\rho + \left[1 - \frac{2(\alpha+\beta)}{\alpha-\beta}\rho\right]Z + \left[\frac{\alpha+\beta}{\alpha-\beta}\right]\rho Z^2}{(1-\rho Z)\,(Z-\rho)} \left[U_{j+1}^{n+1} + U_j^n\right] - U_{j+1}^n \qquad (5)$$

where

$$\rho = \frac{1 - 2(\alpha-\beta) - \left[1 + 4(\alpha-\beta)\right]^{1/2}}{2(\alpha-\beta)}$$

We have programmed a #150 (DJS-11) computer according to equation 5. The flow chart is in figure 1.

Figure 2 is an example of our using the wave equation stack program to process a seismic exploration section. Part (a) shows a CDP stack section and part (b) shows a wave equation stack migration section. The continuation interval $\Delta\tau$ is 32 ms. At location A there is an obvious curved reflector. Because the radius of curvature of this reflector is shorter than the distance between the reflector and geophone, it appears as a reversed polarity reflection. The geometric form of the geologic structure has been contorted. Distinct curved reflectors are brought out through migration and the original features of the section are restored. Wave equation stack migration has a special use toward the so-called "geometric pitfalls". At location B-B' there is a fault concealed by diffractions. After migration the diffractions disappear and the fault is clear. It makes the interpretation of the fault easy. In other locations, such as C, slanted reflections are automatically repositioned and anticlines contracted. The obvious effectiveness of migration is displayed at locations D, E, F, and G where diffractions disappear, interference is resolved, and faults clarified.

(Translator's note: figures 1 and 2 appear at this place in the report. I shall just translate the captions because the seismic sections are printed too badly in the report to reproduce them here.)

Figure 1: Wave equation stack migration program flow chart.

Figure 2: (a) CDP stack time section (b) Wave equation stack migration time section.

(2) The Finite Difference Stability Problem

The so called stability problem is a boundary value or round off shortcoming. Whether the difference equation recurrence function increases or not, if the the recurrence function does not continuously increase, or if increases are set not to exceed a rather slowly increasing rate, then we can say the finite difference form is stable.

Rewriting equation 4 as

$$[1 - (\alpha+\beta)T]U_{j+1}^{n+1} - [1 + (\alpha-\beta)T]U_j^{n+1}$$

$$= [1 + (\alpha-\beta)T]U_{j+1}^{n} - [1 - (\alpha+\beta)T]U_j^{n} \tag{6}$$

We can can use Z transforms to discuss the stability problem of the difference scheme. Towards wavefields the nth layer of the Z variable has

$$U^n(Z) = U_0^n + U_1^n Z + \cdots \quad \cdots + U_K^n Z^K$$

Connecting formula (6) with Z transforms gives

$$\{[1 - (\alpha+\beta)T] - [1 + (\alpha-\beta)Z\}U^{n+1}(Z)$$

$$= \{[1 + (\alpha-\beta)T] - [1 - (\alpha+\beta)Z\}U^n(Z)$$

$$U^{n+1}(Z) = \frac{[1 + (\alpha-\beta)T] - [1 - (\alpha+\beta)Z}{[1 - (\alpha+\beta)T] - [1 + (\alpha-\beta)Z} U^n(Z) \tag{7}$$

Equation seven explains why computing the (n+1)th layer from the nth layer is a filter process [1]. This filter is

$$\frac{[1 + (\alpha-\beta)T] - [1 - (\alpha+\beta)Z]}{[1 - (\alpha+\beta)T] - [1 + (\alpha-\beta)Z]}$$

Proof of the stability of the finite difference scheme in an arbitrary interval $0 \leq \omega \geq 2\pi$ has

$$\left| \frac{[1 + (\alpha-\beta)T] - [1 - (\alpha+\beta)Z]}{[1 - (\alpha+\beta)T] - [1 + (\alpha-\beta)Z]} \right| \leq 1$$

Setting $Z = e^{-i\omega}$ and substituting into equation 8 gives

$$\left| \frac{[1 + (\alpha-\beta)T] - [1 - (\alpha+\beta)]e^{-i\omega}}{[1 - (\alpha+\beta)T] - [1 + (\alpha-\beta)]e^{-i\omega}} \right|$$

$$= \left| \frac{[1 + (\alpha-\beta)T]e^{\frac{i\omega}{2}} - [1 - (\alpha+\beta)T]e^{-\frac{i\omega}{2}}}{[1 - (\alpha+\beta)T]e^{\frac{i\omega}{2}} - [1 + (\alpha-\beta)T]e^{-\frac{i\omega}{2}}} \right|$$

$$= \left| \frac{[1 + (\alpha-\beta)T]\left[\cos\frac{\omega}{2} + i\sin\frac{\omega}{2}\right] - [1 - (\alpha+\beta)T]\left[\cos\frac{\omega}{2} - i\sin\frac{\omega}{2}\right]}{[1 - (\alpha+\beta)T]\left[\cos\frac{\omega}{2} + i\sin\frac{\omega}{2}\right] - [1 + (\alpha-\beta)T]\left[\cos\frac{\omega}{2} - i\sin\frac{\omega}{2}\right]} \right|$$

$$= \left| \frac{\alpha T\cos\frac{\omega}{2} + i(1-\beta T)\sin\frac{\omega}{2}}{-\alpha T\cos\frac{\omega}{2} + i(1-\beta T)\sin\frac{\omega}{2}} \right| = 1$$

Thus, since this finite difference scheme is stable, why do data values happen to increase during the recurrence process resulting in computational overflow? From equation 5, the computation of U_j^{n+1} is divided into five steps- a summation, a convolution, two recurrences, and a difference. The factors during convolution are

$$\left[\frac{\alpha+\beta}{\alpha-\beta}\rho,\ \frac{1-2(\alpha+\beta)}{\alpha-\beta}\rho,\ \frac{\alpha+\beta}{\alpha-\beta}\rho\right]$$

When $\alpha = \beta$, then computing the $\frac{\alpha+\beta}{\alpha-\beta}$ result is a zero divide condition. How is this resolved? In practical data processing we select a suitable $\Delta\tau$ having the condition that the α value does not exceed the β value. Another method is that when $\alpha=\beta$, the following formula is used.

$$U_j^{n+1} = \frac{-(\alpha+\beta) - [1-2(\alpha+\beta)]Z - (\alpha+\beta)Z^2}{(\alpha-\beta) - [1+2(\alpha-\beta)]Z - (\alpha-\beta)Z^2}\left[U_{j+1}^{n+1} + U_j^n\right] - U_{j+1}^n$$

$$= \frac{2\alpha + (1-2\alpha)Z + 2\alpha Z^2}{Z}\left[U_{j+1}^{n+1} + U_j^n\right] - U_{j+1}^n \qquad (9)$$

This equation is divided into four computational steps- a summation, a convolution, a shift, and a difference.

Some references state [2], when a $\alpha=\beta$ condition is detected in a program. This does not create an artificial instability. We consider the occurence of $\alpha = \beta$ which has a close relation with the media propagation velocity C, sampling interval Δt, half geophone distance Δx, and the continuation interval $\Delta\tau$. In actual data processing, procedures will be often encountered not according to rational creation, nor according to artificial creation. Therefore, one can only rely on the selection of suitable parameters as offered in the solution method of this article. When $\alpha=\beta$, we cannot solve the problem simply by changing the α value, because it will give an inaccurate velocity*, confusing migration results. We consider that according to the calculation in equation 5, $\beta = \frac{1}{6}$ is obtained by Simpson's formula. It cannot be arbitrarily chosen. We have discovered in actual computation, changing the β value will cause computer overflow.

* Because $\alpha = \frac{c^2\Delta t\,\Delta\tau}{32\Delta x^2}$, where the sampling interval Δt, continuation interval $\Delta\tau$ and survey interval $2\Delta x$ all are constants, therefore, altering the α value is equivalent to changing the velocity value C.

By Simpson's formula

$$\int_a^b f(X)dX \sim \frac{b-a}{6}\left\{f(a) + 4f\left[\frac{a+b}{2}\right] + f(b)\right\}$$

know

$$\int_{X_{n-1/2}}^{X_{n+1/2}} f dX = \frac{X_{n+1/2} - X_{n-1/2}}{6}\{f(X_{n-1/2}) + 4f(X_n) + f(X_{n+1/2})\}$$

$$= \frac{\Delta X}{6}\{6f(X_n) - [-f(X_{n-1/2}) + 2f(X_n) - f(X_{n+1/2})]\}$$

$$= \frac{\Delta X}{6}\{6f(X_n) - Tf(X_n)\}$$

$$= \left[1 - \frac{T}{6}\right] f_n \Delta X$$

T represents the second order difference operator which uses the 3 point convolution factor (-1,2,-1). By the centered differencing method we know

$$\int_{X_{n-1/2}}^{X_{n+1/2}} \frac{\partial^2 U}{\partial X^2} dX = \int_{X_{n-1/2}}^{X_{n+1/2}} f dX$$

because

$$\frac{\partial^2 U}{\partial X^2} = \frac{U(X+\Delta X) - 2U(X) + U(X - \Delta X)}{\Delta X^2}$$

therefore

$$\int_{X_{n-1/2}}^{X_{n+1/2}} \frac{\partial^2 U}{\partial X^2} dX = -\frac{T}{\Delta X^2} U_n \Delta X$$

$$f_n\left[1 - \frac{T}{6}\right] = \frac{T}{\Delta X^2} U_n$$

$$f_n \sim \frac{1}{\Delta X^2}\left[\frac{T}{1 - \frac{T}{6}}\right] U_n = -\frac{1}{\Delta X^2} D U_n$$

where

$$D = \frac{T}{1 - \frac{T}{6}}$$

(3) The Influence of Velocity Parameters on Migration Results

The velocity parameter is the most important parameter directly influencing migration. It mainly resides in the α $\left[as\ \frac{C^2 \Delta t\, \Delta \tau}{32 \Delta X^2}\right]$ value. Downward continuation uses interval velocity. If the velocity data are average velocities, then the Dix equation must be used to convert the average velocity into interval velocity.

$$C_i = \left[\frac{C_i^2 t_i^2 - C_{i-1}^2 t_{i+1}^2}{t_i - t_{i-1}}\right]^2$$

We use the velocities of CDP stacking converted into interval velocities. We furthermore say the velocity is constant within the continuation interval.

Parts (a), (b), and (c) in figure 3 are examples of using different velocities to conduct migration corresponding to the CDP stack section of figure 3(d). The velocities in (a) have been increased about .5% over the velocities in (b). The velocities in (c) have been lowered about .5% from the velocities in (b). From these figures one can see the recorded features are different. For example, at location A (about 1.5 seconds) there are obvious changes in the frequencies and position of the wave group. At location B (about 2.4 seconds) the changes are small. Thus it can be said that the velocity sensitivity weakens following an increase in depth.

Figure 3 appear here. Captions:

(a) Wave equation stack migration time section. $\Delta\tau = 16ms$. Velocities increased .5%.

(b) Wave equation stack migration time section. $\Delta\tau = 16ms$.

(c) Wave equation stack migration time section. $\Delta\tau = 16ms$. Velocities lowered .5%.

(d) Adaptive CDP stack time section.

(4) Arc Interference Phenomena in Seismic Sections

Based upon the record length and continuation interval $\Delta\tau$ size, finite difference wave equation migration takes 8-15 minutes on the average to do a stack section. It can be said that a stack section requires several hundred million multiplication, addition, and subtraction operations. Therefore, to require the high quality of CDP stack sections, mainly observe the following points: (1) high signal to noise; (2) no abnormal traces; (3) no abnormal sample values; and (4) the best filtering of CDP stacks to remove low and high frequency interference. If these requirements are not satisfied, serious arc phenomenon will occur. If these requirements are satisfied, editing and filter processing must be done after migration. Figure 4 is a migrated CDP stack which has not been edited for abnormal traces or sample values. Certain interferences occur hiding valid reflections. Actual processing demonstrates for certain abnormal sample values, the shapes present at location A in figure 4 occur. This explains why wave equation migration and summation methods restore arc patterns in the same way.

Figure 4 caption: Special interference to abnormal sample values.

(5) Summary

The finite difference method of wave equation stack migration has excellent restoration results. Finite difference schemes are stable. In actual data processing, a suitable $\Delta\tau$ must be chosen. Avoid the $\alpha = \beta$ condition. If unavoidable, utilize the calculation of equation (9) from this paper. Altering the velocity parameters has a large effect on shallow reflectors and a small effect on deep reflectors. Arc interference phenomenon is primarily caused by the abnormal sample values of the CDP stack recording method.

This article only discusses the basic problems occuring in practical data processing. The intrinsic characteristics of wave equation stacked migration such as boundary influences, patterns of factor changes, wavefield characteristics, and arc interference characteristics, wil be discussed in future publications.

Footnotes:

(1) J.F. Claerbout: Fundamentals of Geophysical Data Processing, Chapter 10, Section 4
(2) Ma Zaitian: Seismic Models and Migration of Seismic Sections, Petroleum Geophysical Prospecting 1978.4
(3) Dong Minyu: Concerning Wave Equation Migration, Petroleum Geophysical Prospecting, 1978, #4