2.0 WHY USE FINITE DIFFERENCING?

In the previous chapter we learned how to extrapolate wave fields down into the earth. The process proceeded rather simply being just a multiplication in the frequency domain by $\exp[ik_z(\omega,k_x)z]$. Finite-difference techniques will be seen to be complicated. They will involve new approximations and new pitfalls. Why should we trouble ourselves to learn them?

The situation is analogous to the one encountered in ordinary frequency filtering. Frequency filtering can be done as a product in the frequency domain or a convolution in the time domain. With wave extrapolation we have a product in both the temporal frequency ω-domain and the spatial frequency k_x-domain. The new element is that now we are in two-dimensional (ω,k_x)-space instead of the old one-dimensional ω-space. Our question, "Why bother with finite differences?" is a two-dimensional form of an old question, "After the discovery of the fast Fourier transform, why should anyone bother with time-domain filtering operations?"

Our question will be asked many times and under many circumstances. Later we will also have the axis of offset between the shot and geophone and the axis of midpoints between them. Then again we will have a choice whether to work on these axes with finite differences or to use Fourier transformation on them. Neither is it an all-or-nothing proposition: for each axis separately we may make the choice of Fourier transform or convolution (finite difference).

The answer to the question is many-sided, just as geophysical objectives are many-sided. And most of the criteria for answering the question are already familiar from ordinary filter theory. Those electrical engineers and old-time deconvolution experts who have pushed themselves into wave processing have turned out to be delighted by it. They hadn't realized their knowledge had so many applications!

Lateral Variation

In ordinary linear filter theory, a filter can be made time-variable. This is quite useful in reflection seismology because the frequency content of echoes changes with time. An annoying aspect of time-variable filters is that they cannot be described by a simple product in the frequency domain. So when an application of time-variable filters comes along we either abandon the frequency domain or we go into all kinds of contortions (stretching the time axis, for example) to try to make things appear time-invariant.

All the same circumstances apply when we transfer attention from the time axis t to the horizontal space axis x. Now the factor of major concern is the seismic velocity v. If it is space-variable, say $v(x)$, then the operation of upward and downward extrapolating wave fields can no longer be expressed as a product in the k_x-domain. So when trying to extrapolate waves into the earth we abandon the spatial frequency domain and go to finite differences, or else we go through all kinds of contortions (such as stretching the x-axis) to try to make things appear to be space-invariant.

In two or more dimensions, stretching tends to become more difficult and less satisfactory.

A less compelling circumstance of the same type which suggests finite difference rather than Fourier methods is lateral variation in channel location. If geophones somehow have become unevenly separated so that the Δx between channels is not independent of x, then we have a choice of (1) resampling the data at uniform intervals before Fourier analysis, or (2) processing the data directly with finite differences.

Causal All-Pass Filters

The upward and downward wave-field extrapolation filter $\exp[ik_z(\omega,k_x)z]$ is basically a causal all-pass filter. (Under some circumstances it is anticausal.) It moves energy around without amplification or attenuation. I suppose this is why migration filtering is more fun than minimum-phase filtering. Migration filters gather energy from all over and drop it in a good place, whereas minimum-phase filters hardly move things at all - they just scale some frequencies up and others down. Any filter of the form $\exp[i\varphi(\omega)]$ is an all-pass filter. What are the constraints on the function $\varphi(\omega)$ which make the time-domain representation of $\exp(i\varphi)$ causal? That is a question more easily understood in the time domain.

Causal all-pass filters turn out to have an attractive representation, with Z-transforms as $Z^N \bar{A}(1/Z)/A(Z)$. Those who are familiar with filter theory will realize that the division by $A(Z)$ raises a whole range of new issues: feedback, economy of parameterization, and possible instability. These issues will all arise when we use finite differences to downward extrapolate wave fields. It is a feedback process. The economy of parameterization is attractive. Taking $A(Z) = 1 + a_1 Z + a_2 Z^2$ the two adjustable coefficients are sufficient to select a frequency and a bandwidth for selective delay. Economy of parameterization also implies economy in application. That is nice. It is also nice having causality as an automatic implication of the functional form. On the other hand, the advantages of economy are offset by some dangers. Now we must learn and use some stability theory. $A(Z)$ must be minimum phase.

Being Too Clever in the Frequency Domain

In the frequency domain it is easy to specify sharp cutoff filters, say a perfectly flat passband between 8 Hz and 80 Hz, zero outside. But such filters have problems in the time domain. Such a filter is necessarily non-causal, giving a response before energy enters the filter. Another ugly aspect is that the time response drops off only inversely with t. What happens when we try to look at distant echoes which normally have amplitudes weakened as inverse time squared? What happens is that they get lost in the long filter response of the early echoes.

A more common problem arises with the 60 Hz powerline frequency rejection filters found in much recording equipment. Notch filters are easy to construct in the Z-transform domain. You start with a zero on the unit circle at exactly 60 Hz. That kills the noise but it distorts the passband at other frequencies. So then a tiny distance away, outside the unit circle, you place a pole. The separation determines the bandwidth for the notch. The pole has the effect of nearly cancelling the zero if the pair are seen from a distance. So you have an ideal flat spectrum away from the absorption zone. You record some data with this. Late echoes are weaker than early ones, so on your plotting machine you let the gain increase with time. After installing your powerline reject filters you discover that they have *increased* the powerline noise instead of decreasing it. Why? The reason is that you tried to be too clever when you put the pole too close to the circle. The exponential gain effectively moved the unit circle away from your zero towards the pole. You may have ended up with the pole on the circle! Putting the pole further from the zero will give you a much broader notch, less attractive in the frequency domain, but at least the filter will work when the gain varies with time.

So the frequency domain easily leads to pitfalls. Sharp cutoffs are not as attractive as they seem at first because they imply long and possibly unexpected time responses. Don't be too disappointed that this anomalous behavior usually fails to occur with simple finite-difference representations. This can be an advantage.

Zero Padding

When fast Fourier transforms came into use, one of the first applications was convolution. If a filter has more than about 50 coefficients, then it may be faster to apply it by multiplication in the frequency domain. The result will be identical to convolution provided that care has been taken to pad the ends of the data and the filter with enough zeroes. They make invisible the periodic behavior of the discrete Fourier transform. For filtering time functions whose length is typically about one thousand, this is a small price to pay in added memory compared to the time saved. Seismic sections are commonly thousands of channels long. For migration, zero padding must simultaneously be done on the space axis and the time axis. This gets painful even in a large machine. To add further insult, there are three places where zeroes are required, as indicated below:

data	0
0	0

To make matters worse, the filter of interest is basically an expanding spherical wave. So instead of having an expanding circle we have concentric circles, the separation between them being controlled by the number of points on the frequency axis. So when a circle "wraps around," it has an amplitude diminished only by spherical spreading. See figure 1. Energy density drops off on a sphere of size t in proportion to $1/t$. So amplitude drops as $t^{-1/2}$. But that is for three dimensions and we almost always process in two dimensions, in which case amplitude scales inversely as the 1/4 power of t. That drops off slowly, so wraparound is something to worry about.

Looking Ahead

Some problems of the Fourier domain have just been summarized. The problems of the space domain have yet to be seen. Seismic data processing is a multidimensional task, and the different dimensions are often handled differently. But if you are sure you are content with the Fourier domain then you could skip much of this chapter and the next and jump directly to Chapter 4 to learn about shot-to-geophone offset, stacking, and migration before stack. You could also start on multiple reflections in Chapter 5, but you won't get far there with just Fourier transforms. Multiple reflections are strongly influenced by lateral variation in reflectivity as well as by velocity. I suggest you proceed some distance into both Chapters 2 and 3 before jumping to Chapters 4 and 5.

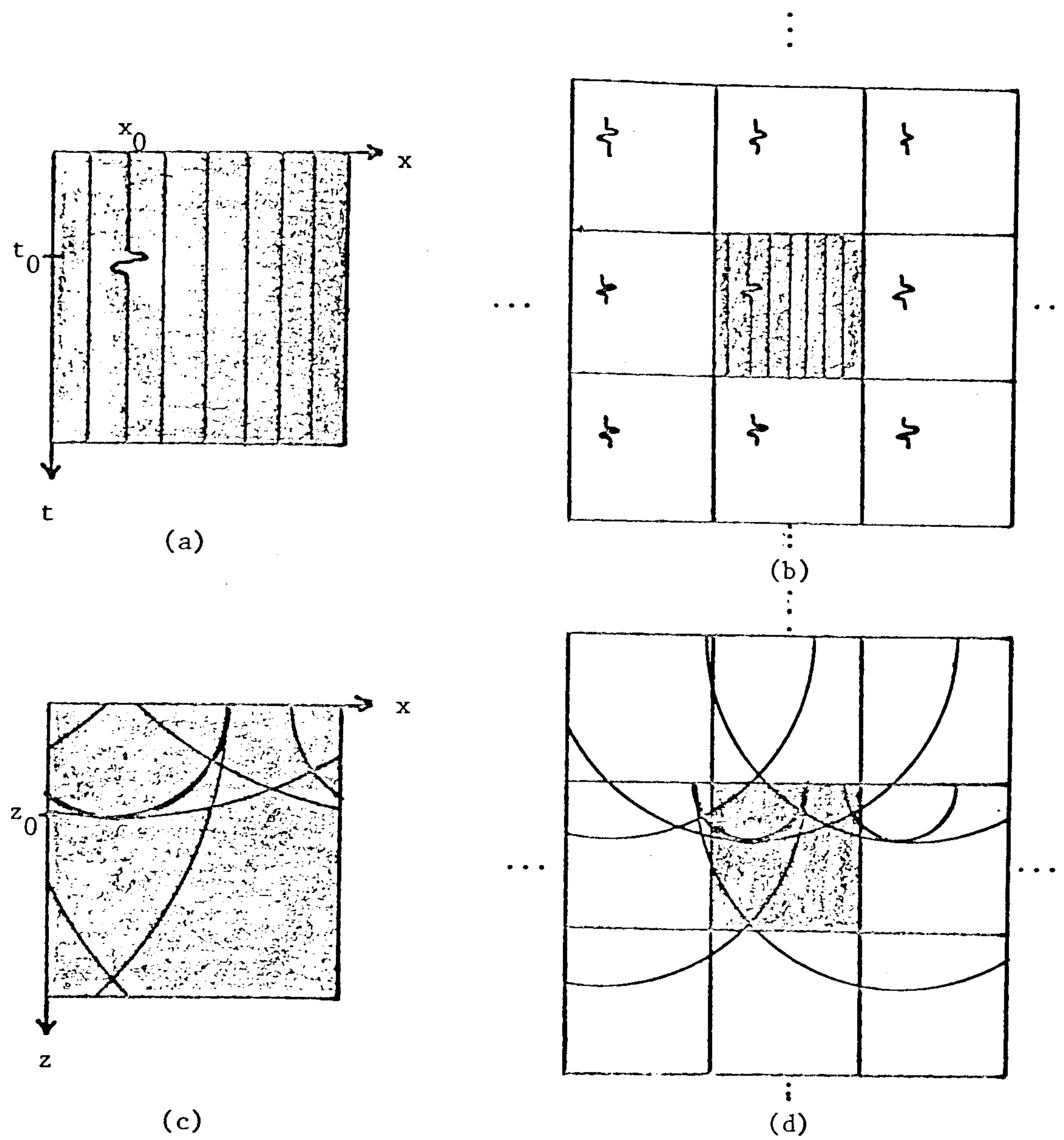

FIG. 1.. (Lynn) Inherent periodicity implied by the use of discrete Fourier transforms. (a) The desired input, $p(t,x)$, which here is $\delta(t-t_0)\delta(x-x_0)w(t)$. (b) The input implied by the periodic boundary conditions of the DFT. The dots imply a continuation *ad infinitum*. (c) The migrated section. The correct structure is shown by the heavy line. (d) The origin of the spurious structures in (c). (Note: later versions of these notes will show a clipped, synthetic example.)

2.6 RETARDED COORDINATES

To examine running horses it may be best to jump on a horse. Likewise, to examine moving waves, it may be better to move along with them. So to describe waves moving downward into the earth we might abandon (x,z) coordinates in favor of moving (x,z') coordinates where $z'=z+tv$.

An alternative to the moving coordinate system is to define *retarded coordinates* (x,z,t') where $t'=t-z/v$. The classical example of retarded coordinates is solar time. Time seems to stand still on an airplane which moves westward at the speed of the sun.

The migration process resembles simulation of wave propagation in either a moving coordinate frame or a retarded coordinate frame.

Retarded coordinates are much more popular in geophysics than moving coordinates. The reason has to do with space variation in material velocity $v(x,z)$. It is not clear which velocity the frame should choose, or whether it should try to move at different velocities in different places. Retarded time, on the other hand, is easily defined with reference to traveltime along some particular family of rays. We will see that a more mathematical reason for preferring retarded coordinates to moving coordinates is to prevent the velocity from being a function of time as well as space. Fourier transformation is a popular means of solving the wave equation, but it loses much of its utility when the coefficients are non-constant. This fact alone can explain why in solid earth geophysics, retarded coordinates are universally preferred to moving coordinates.

Definition of Independent Variables

The definition of retarded coordinates is one of convenience. Commonly the retardation is based on hypothetical rays moving straight downward with velocity $\overline{v}(z)$. The definition of these coordinates may have utility even in problems in which the earth velocity varies laterally, say $v(x,z)$, even though there may be no rays going exactly straight down. In principle, any coordinate system may be used to describe any circumstance. But the utility of the retarded coordinate system generally declines as the family of rays defining it departs more and more from the actual rays.

Despite the rather simple case at hand it is worthwhile to be somewhat formal and precise. Define the retarded coordinate system (t',x',z') in terms of ordinary Cartesian coordinates (t,x,z) by the system of equations

$$t' = t'(t,x,z) = t - \int_0^z \frac{dz}{\overline{v}(z)} \tag{1a}$$

$$x' = x'(t,x,z) = x \tag{1b}$$

$$z' = z'(t,x,z) = z \tag{1c}$$

The purpose of the integral is to accumulate the traveltime from the surface to depth z. The reasons why we bother to define (x',z') when it is just set equal to (x,z) are first, to avoid confusion during partial differentiation and second, to prepare readers for later work where there is a more general family of rays.

Definition of Dependent Variables

There are two kinds of *dependent* variables, those which characterize the medium and those which characterize the waves. We characterize the medium by its velocity v and its reflectivity c. To characterize the waves we use U for upcoming wave, D for downgoing wave, P for pressure, and Q for a modulated form of pressure. Let us say $P(t,x,z)$ is the mathematical function to find pressure given (t,x,z), and $P'(t',x',z')$ is the mathematical function given (t',x',z'). The statement that the two mathematical functions P and P' both refer to the same physical variable is this

$$P(t,x,z) = P'[t'(t,x,z),x'(t,x,z),z'(t,x,z)] \tag{2}$$

$$P(t,x,z) = P'(t',x',z')$$

Obviously we also have analogous expressions for the other dependent variables. One advantage of solid-earth geophysics over other branches of geophysics is that the specifications of the medium, i.e. v and c, are not dependent on time. Now we can state this more precisely, that the advantage of retarded coordinates compared to moving coordinates is that v' and c' do not depend on t'.

The Chain Rule and the High Frequency Limit

The familiar partial differential equations of physics come to us in (t,x,z)-space. To use them in (t',x',z')-space we need to learn to convert the partial derivatives. This is done with the chain rule for partial differentiation. For example, differentiating (2) with respect to z we get

$$\frac{\partial P}{\partial z} = \frac{\partial P'}{\partial t'}\frac{\partial t'}{\partial z} + \frac{\partial P'}{\partial x'}\frac{\partial x'}{\partial z} + \frac{\partial P'}{\partial z'}\frac{\partial z'}{\partial z} \tag{3a}$$

Using (1) to evaluate the coordinate derivatives we get

$$\frac{\partial P}{\partial z} = -\frac{1}{\bar{v}}\frac{\partial P'}{\partial t'} + \frac{\partial P'}{\partial z'} \tag{3b}$$

There is nothing special about the variable P in (3). We could as well write

$$\frac{\partial}{\partial z} = -\frac{1}{\bar{v}'}\frac{\partial}{\partial t'} + \frac{\partial}{\partial z'} \tag{4}$$

where the left side is for operation on functions which depend on (t,x,z) and the right side is for functions of (t',x',z'). Differentiating twice we get

$$\frac{\partial^2}{\partial z^2} = \left[\frac{-1}{\bar{v}'}\frac{\partial}{\partial t'} + \frac{\partial}{\partial z'}\right]\left[\frac{-1}{\bar{v}'}\frac{\partial}{\partial t'} + \frac{\partial}{\partial z'}\right] \tag{5}$$

Using the fact that the velocity is always time-independent we get

$$\frac{\partial^2}{\partial z^2} = \frac{1}{(\bar{v}')^2}\frac{\partial^2}{\partial t'^2} - \frac{2}{\bar{v}'}\frac{\partial^2}{\partial t'\partial z'} + \frac{\partial^2}{\partial z'^2} - \left[\frac{\partial}{\partial z'}\frac{1}{\bar{v}'}\right]\frac{\partial}{\partial t'} \tag{6}$$

Except for the rightmost term with the square brackets it could be said that "squaring" the operator (4) gives the second derivative. This last term is almost always neglected in data processing. The reason is that its effect is very similar to that of other first derivative terms with material gradients for coefficients. The effect of such terms, described along with the derivation of the single square root equation, is to cause amplitudes to be more carefully computed. If this term is to be included, then it would seem that all such terms should be included, from the beginning.

Fourier Transforms in Retarded Coordinates

Given a pressure field $P(t,x,z)$ we may Fourier transform it with respect to any or all of its independent variables (t,x,z). Likewise if the pressure field is specified in retarded coordinates we may Fourier transform with respect to (t',x',z'). As we conventionally refer to the Fourier dual of (t,x,z) as (ω,k_x,k_z) it seems appropriate to refer to the dual of (t',x',z') as (ω',k_x',k_z'). Now the question is, "How are (ω',k_x',k_z') related to the familiar (ω,k_x,k_z)?" The answer is contained in the chain rule for partial differentiation. Any expression like (4)

$$\frac{\partial}{\partial z} = -\frac{1}{\overline{v}'}\frac{\partial}{\partial t'} + \frac{\partial}{\partial z'} \tag{4}$$

upon Fourier transformation says

$$ik_z = -\frac{-i\omega'}{\overline{v}'} + ik_z' \tag{7}$$

Computing all the other derivatives, we have the transformation

$$\omega = \omega' \tag{8a}$$

$$k_x = k_x' \tag{8b}$$

$$k_z = k_z' + \frac{\omega'}{\overline{v}} \tag{8c}$$

Recall the dispersion relation for the scalar wave equation

$$\frac{\omega^2}{v^2} = k_x^2 + k_z^2 \tag{9}$$

Performing the substitutions from (8) into (9) we have the expression of the scalar wave equation in retarded time, namely

$$\left[\frac{\omega'}{v}\right]^2 = (k_x')^2 + \left[k_z' + \frac{\omega'}{\overline{v}}\right]^2 \tag{10}$$

These two dispersion relations are plotted in figure 1 for the retardation velocity chosen equal to the medium velocity.

Figure 1 graphically illustrates that retardation can reduce the cost of finite-difference calculations. Consider waves going nearly straight down. On the dispersion curves they are near the top of the circle. The effect of retardation is to shift the circle's top down to the origin. Now consider discretizing the x- and z-axes. This means that there will be a spatial folding frequency on both k_x- and k_z-axes. The

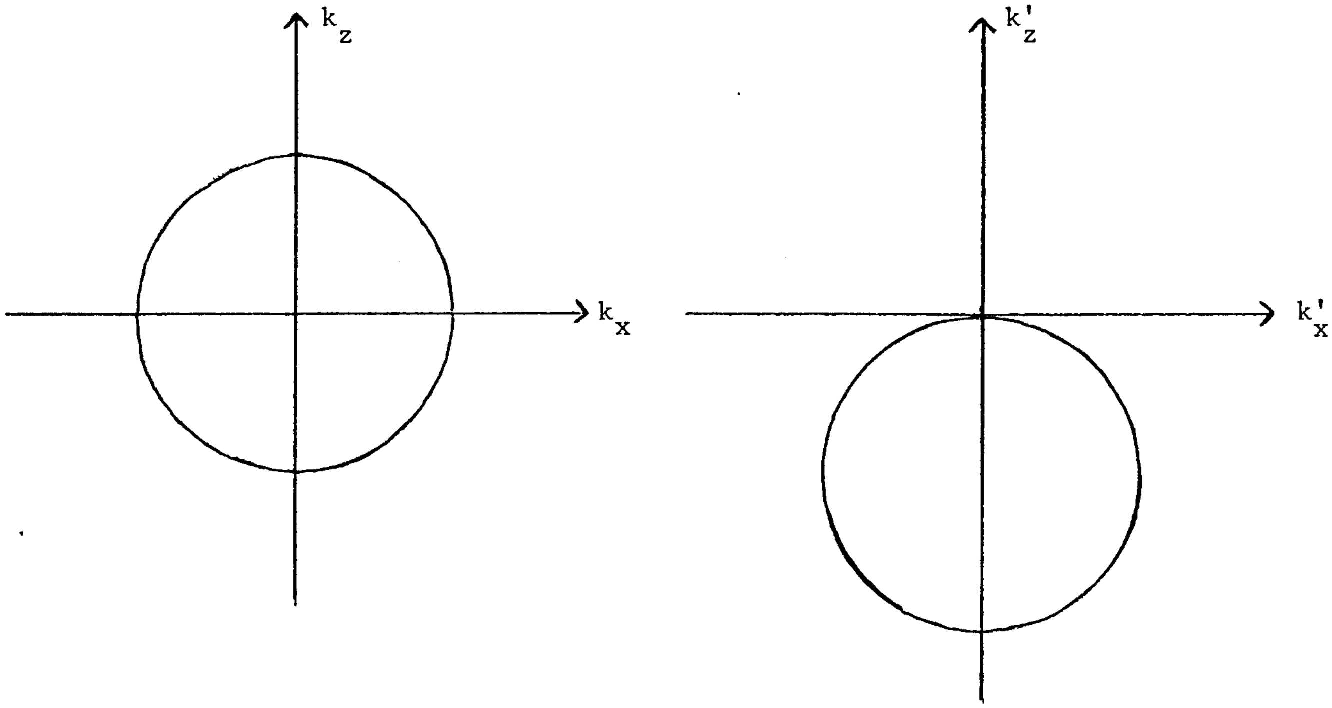

FIG. 1. Dispersion relation of the wave equation in usual coordinates (left) and retarded time coordinates (right).

larger the frequency ω, the larger the circle. Clearly the top of the shifted circle is further from folding. Alternately, Δz may be increased (for the sake of economy) before $k_z \cdot \Delta z$ exceeds the Nyquist frequency.

Interpretation of the Modulated Pressure Variable Q

Earlier we defined a variable Q from the pressure P by the equation

$$P = Q \exp\left[i\omega \int_0^z \frac{dz}{\overline{v}(z)} \right] \tag{11}$$

The right side is a product of two functions of ω. At constant velocity (11) is expressed as

$$P(\omega) = Q(\omega) e^{i\omega z/v} = Q(\omega) e^{i\omega t_0} \tag{12}$$

In the time domain $e^{i\omega t_0}$ becomes a delta function $\delta(t - t_0)$. Thus in the time domain (12) is

$$\begin{aligned} p(t) &= q(t) * \delta(t - z/v) \\ &= q(t - z/v) \\ &= q(t') \end{aligned}$$

This confirms that the definition of a dependent variable Q is equivalent to introducing retarded time t'.

2.7 FINITE DIFFERENCING IN (t,x,z)-SPACE

At the present time much, if not most, production migration work is done in (t,x,z)-space. However, in principle, there seems to be no reason why this work should not be done in (ω,x,z)-space, much as described earlier. It certainly is more confusing to downward continue in the three-dimensional (t,x,z)-space than in the two-dimensional (x,z)-space. The frequency ω enters as a third dimension only when we sum over it to image at $t=0$. Further disadvantages of t-space are the need to learn some stability analysis and the need to consider accuracy as a function of size of Δt. In the next chapter the stability question is completely resolved and the cost of Δt accuracy is determined.

Since the earth is time-invariant we might be inclined to suspect that we can always Fourier transform the time axis and that there is really no need to learn any time-domain techniques. This belief might be justified for migration of primary reflections on stacked sections. It might be true that the ω-domain is more advantageous than the t-domain, and the t-domain is a "historical relic." So the casual reader may safely skip this section. [Ironically, I did all my early work in (ω,x,z)-space, but I recall no one who saw it as practical until I learned to migrate in (t,x,z)-space!]

The serious reader should not so hastily abandon the time domain. Looking forward to pre-stack partial migration, the Yilmaz approach must be implemented in the time domain. Looking further forward to the section, "Slanted-Ray, Multiple Reflections," we see the first of several processes for suppression of multiple reflections which seem to demand the time domain. Predictive multiple suppression seems to be a non-linear process, a fact which works against frequency space. Recall also that even the simple, time-honored process of time-variable filtering is ill-suited to the frequency domain. It is only prudent to keep an open mind on the question of time domain versus frequency domain.

There are three different planes which slice through (t,x,z)-space, and it is worth having a look at each of them before we enter the full volume. First, we have already looked in considerable detail at wave extrapolation in (x,z)-space for fixed ω. Next we will look at migration in (z,t)-space for fixed k_x. Besides another look at the migration process, this also offers some insights into velocity determination, insights we didn't get in ω-space. The third slice we will look at is (t,x)-space. There we will solve a dip-filtering problem as a simple prototype of migration. Dip filtering is a process of long standing interest in geophysics. It is particularly attractive to dip filter in (t,x)-space rather than the usual (ω,k_x)*-space* because it is frequently important to change the filter parameters with time and space.

Migration in (z,t)-Space [1]

The equation for upcoming waves U in retarded coordinates (t',x',z') is

$$\frac{\partial^2 U}{\partial z'\partial t'} = -\frac{v}{2}\frac{\partial^2 U}{\partial x'^2} \tag{1}$$

To Fourier transform the x-axis we only need assume that v is a constant function of x and that the x-dependence of U is the sinusoidal function $exp(ik_x x)$. Thus

$$0 = \left[+ \frac{vk_x^2}{2} - \frac{\partial^2}{\partial z'\partial t'} \right] U \tag{2}$$

Letting $*$ denote convolution in (z,t)-space, this partial differential equation may be written as

$$0 = \left\{ \frac{vk_x^2}{2}\,\frac{1}{4} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} - \frac{1}{\Delta z'\Delta t'} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \right\} * U \tag{3}$$

where we have taken t' downward and z' to the right. The $1/4$ arises from the average of U over four places on the mesh. Since the coefficients are positive, the sum of the two operators always has $|b| \geq |s|$ in the form

$$0 = \begin{bmatrix} s & b \\ b & s \end{bmatrix} * U \tag{4}$$

Next we consider the task of using the operator of (4) to fill in a table of values for U

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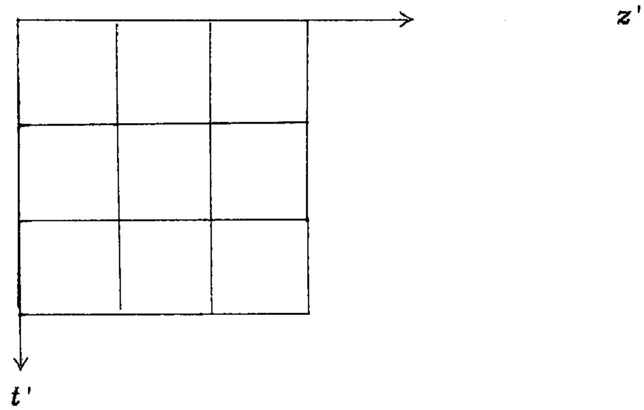
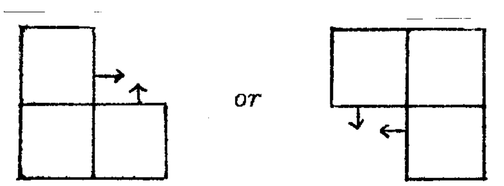
[1] Adapted from SEP-1, p. 73-77.



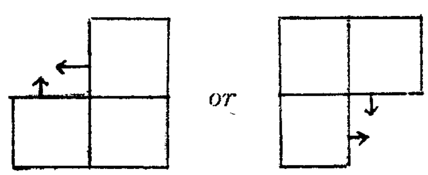
(5)

From equation (4) we see that given the appropriate three values of U a fourth may be determined by either of the two operations



(6a,b)

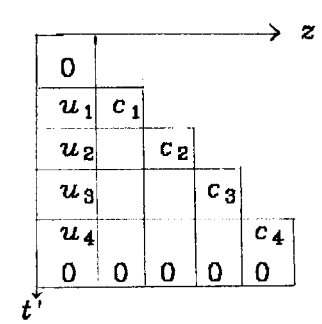
It turns out that because $|b| \geq |s|$ the filling operations implied by



(7)

are unstable. It is obvious that there would be a zero-divide problem if $s=0$, and it is not difficult to do the stability analysis to show that (7) causes exponential growth of small disturbances.

Migration and data synthesis may be envisioned in the (z',t')-space on the following table contains the upcoming wave U



(8)

In this table the observed upcoming wave at the earth's surface $z' = 0$ is denoted by u_t. The migrated section is, denoted by c_t, is depicted along the diagonal. The migrated section is shown on a diagonal since the imaging condition of exploding reflectors at time $t=0$ is represented in retarded space as

$$z' = z \tag{9a}$$

$$t' = t + z/v \qquad (+ \text{ for } up) \tag{9b}$$

$$0 = t = t' - z'/v \tag{10}$$

Actually the best focused migration need not fall on the 45-degree line as depicted in (8) but it might be on any line or curve as determined by the earth velocity. Indeed, the concept forms the basis for velocity determination in a later chapter. This concept is not so apparent in frequency-domain migration.

It is a worthwhile exercise to make the zero-dip assumption $(k_x = 0)$ and use the numerical values in the operator of (4) to fill in the elements of the table (8). It will be found that the values of u_t move laterally across the table with no change, predicting as it should, that $c_t = u_t$. Slow change in z suggests that we have oversampled the z-axis. In practice, effort is saved by sampling the z-axis with fewer points than the t-axis.

(t, x)-Space, Recursive Dip Filters [2]

The motivation for this work is to provide simple, causal, recursive dip filters which can be easily applied and made time- and space-variable. Such filters may make possible the observation of important weak events that are obscured by strong events. For example, weak fault diffractions carry velocity information, but they may often be invisible because of the dominating presence of flat layers. For data recorded at late times at only modest offsets, such diffractions could be the only way to measure velocity. This situation applies, for instance, to deep continental soundings.

So-called "pie-slice" filters offer considerable control over the filter response in k/ω dip space. While recursive filters are not controlled as readily, they do meet the same general needs as pie-slice filters and offer the advantages of (1) causality, (2) time- and space-variability, and (3) simple and economic recursive implementation.

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[2] Adapted from SEP 20, p. 235.

Let P denote raw data and Q denote filtered data. When seismic data is quasi-monochromatic, dip filtering can be achieved with spatial frequency filters.

Dip Filters for Monochromatic ($\omega \approx Const$) Data	
Low Pass	High Pass
$Q = \frac{b}{b + k^2} P$	$Q = \frac{k^2}{b + k^2} P$

To apply these filters in the space domain it is necessary only to interpret k^2 as a tridiagonal matrix, call it $\mathbf{T}$, with (-1, 2, -1) on the main diagonal. Specifically, for the low pass filter it is necessary to solve a tridiagonal set of simultaneous equations like

$$(b\mathbf{I} + \mathbf{T})\mathbf{q} = b\,\mathbf{p} \tag{11}$$

in which $\mathbf{q}$ and $\mathbf{p}$ are column vectors whose elements denote different places on the x-axis. We did this while solving the heat flow equation. To make the filter space variable, the parameter b can be taken to depend on x so that $b\mathbf{I}$ becomes a general diagonal matrix. It doesn't matter whether $\mathbf{p}$ and $\mathbf{q}$ are represented in the ω domain or the t domain !

Turning attention from narrow-band data to data with a broader spectrum, we have

Dip Filters for Moderate Bandwidth ($\Delta\omega$) Data	
Low Pass	High Pass
$Q = \frac{b}{b + \frac{k^2}{-i\omega}} P$	$Q = \frac{\frac{k^2}{-i\omega}}{b + \frac{k^2}{-i\omega}} P$

Naturally these filters can be applied to data of any bandwidth. However the filters are appropriately termed "dip filters" only over a modest bandwidth.

To understand these filters we need to look in the (ω,k)-plane and draw contours of constant k^2/ω, i.e. $\omega = \alpha k^2$. Such contours, examples of which are shown in figure 1, are curves of constant attenuation and constant phase shift. Inspecting the low pass filter, we see that there is no phase shift in the flat pass zone, but that there is time differentiation in the attenution zone. Inspecting the high-pass filter we see that there is no phase shift in the flat pass zone but that there is time integration in the attenuating zone.

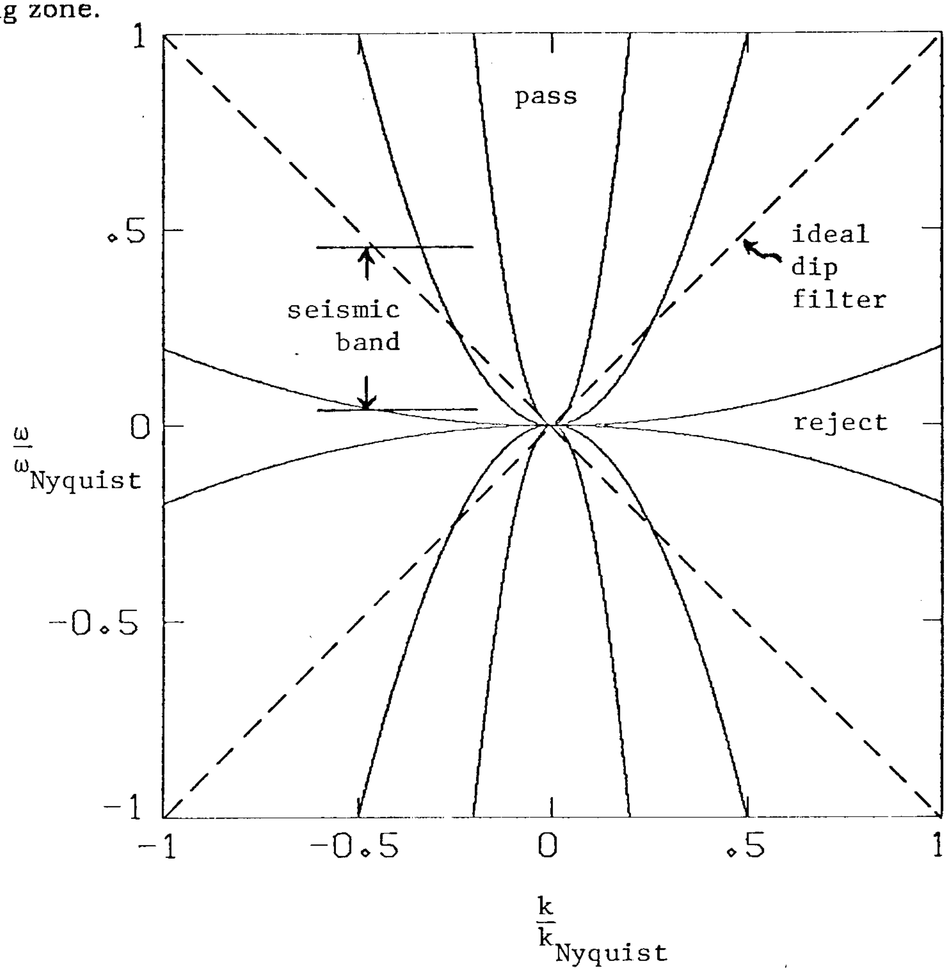

FIG. 1. (Hale) Constant attenuation contours of dip filters. Over the seismic frequency band these parabolas may be satisfactory approximations to the dashed straight line.

Implementation of the moderate bandwidth dip filters is again a straight forward matter. The only trick is to realize that the differentiation implied by $-i\omega$ is performed in a Crank-Nicolson sense. For example, the high pass filter becomes

$$(-i\omega b\mathbf{I} + \mathbf{T})\mathbf{Q} = \mathbf{TP}$$

$$\mathbf{I}\,\frac{b}{\Delta t}\left[\mathbf{q}_{t+1}-\mathbf{q}_t\right] + \frac{1}{2}\mathbf{T}\left[\mathbf{q}_{t+1} + \mathbf{q}_t\right] = \frac{1}{2}\mathbf{T}\left[\mathbf{p}_{t+1} + \mathbf{p}_t\right]$$

$$\left[\frac{b}{\Delta t}\,\mathbf{I} + \frac{1}{2}\mathbf{T}\right]\mathbf{q}_{t+1} = \left[\frac{b}{\Delta t}\,\mathbf{I} - \frac{1}{2}\mathbf{T}\right]\mathbf{q}_t + \frac{1}{2}\mathbf{T}\left[\mathbf{p}_{t+1} + \mathbf{p}_t\right]$$

The last equation is a tri-diagonal system of simultaneous equations for $\mathbf{q}_{t+1}$. As before $\mathbf{q}_t$ is a vector function of x . The equation may be solved recursively for successive

t values

Within reasonable bounds the parameter b which determines the filter cutoff can be chosen to be any function of time and space. A later chapter on stability analysis shows that the recursion is stable. People with a special interest in time series analysis may choose to approach problems of this type by letting the time dependence be represented by Z-transforms. Then differentiation and integration are represented with the bilinear transform $(1-Z)/(1+Z)$.

Another interesting feature of these dip filters is that the low pass and the high pass filters constitute a pair of filters which sum to unity. So nothing is lost if a data set is partitioned into two parts by means of them. The high passed part could be added to the low passed part to recover the original data set.

(t,x,z)-Space

I find that the easiest way to do 15-degree (t,x,z)-space migration is to refer back to the (z,t) space migration but replace k^2 by the tridiagonal matrix $\mathbf{T}$.

The 45 degree migration is a little harder because the operator in the time domain is higher order. When I did this kind of work I found it easiest to use the Z-transform approach where $1/(-i\omega\Delta t)$ is represented by the bilinear transform $(1+Z)/(1-Z)$. There are various approaches to keeping the algebra bearable. One approach is to bring all powers of Z to the numerator and then collect powers of Z. Another approach, called the integrated approach, is to keep $1/(1-Z)$ with some of the terms. Such terms are represented in the computer by buffers which contain the sum from infinite time to time t.