Inversion of Refracted Free-Surface Multiples By Wavefield Continuation

*Robert W. Clayton and George A. McMechan**

Summary

The inversion scheme for refraction data presented SEP-24 by Clayton and McMechan is easily extendable to free-surface multiples. The extension is based on the property that the first-order free-surface multiple has twice the τ of the corresponding primary, for a given ray parameter. It is implemented in the inversion algorithm by simply doubling the frequency. The analysis of the multiples allows an independent check on the inversion using primaries, and on the assumption of lateral homogenity.

Introduction

In SEP-24, Clayton and McMechan, presented a method for directly inverting refraction profiles that are well-sampled spatially. The method consists of transforming the entire recorded wavefield into the slowness-depth domain where the velocity profile can be picked directly.

The first step in the method is a slant stack of the recorded wavefield (Schultz and Claerbout, 1978; Chapman, 1978; McMechan and Ottolini, 1980; and Chapman, 1980). For arrivals beyond the critical angle, slant stacking unravels the triplications of the travel time curve into a single monotonic curve (the τ-curve). The slant stack transformation is linear and invertable.

The next step is a downward continuation to convert the slant stack in (p,τ)-space into the slowness plane in (p,z)-space. The slowness plane is the plane that contains the turning points of the rays. The velocity-depth curve is then picked from this plane. The last step is iterative because it is necessary to specify the velocity for the continuation. Convergence is determined when the picked velocity curve is the same as the velocity model input to the downward continuation. In the examples given in SEP-24, the convergence was rapid and completely stable.

The method outlined above was directed towards primary events only. The free surface refracted multiples form a false image in the slowness plane that falls below the true velocity-depth curve. However, with a very minor change in the inversion algorithm, the multiples can also be made to form the correct velocity-depth image. In this case the primaries form a false image above the true velocity curve.

Theory

The downward continuation in the inversion scheme is implement with the equation [equation (9) from SEP-24, with a slight change in notation]

$$s(p,z) = \int S(\omega,p)\, e^{-i\omega\Phi(p,z)} d\omega \tag{1}$$

where

$$\Phi(p,z) = 2\int_0^z \left| v^{-2}(z) - p^2 \right|^{1/2} dz$$

In this equation s(p,z) is the slowness plane, p is the ray parameter, S(ω,p) is the Fourier transform of the slant stack of the recorded wavefield, and $v(z)$ is the velocity-depth function.

The phase rotation Φ, that is used in the continuation, may be related to τ by

$$\Phi(p,z) = \tau(p) \tag{2}$$

where it is understood that z refers to the turning point of the ray.

In a laterally homogeneous earth [which equation (1) assumes] the first-order multiples have twice the τ of the corresponding primaries, for a given ray parameter. Hence, the appropriate phase rotation to image the multiples is

$$\omega\, \Phi'(\omega,p,z) = 2\omega\, \tau(p) = 2\omega\, \Phi(p,z) \tag{3}$$

Thus, the phase rotation for multiples can be implemented in equation (1) by simply doubling the frequency. This will cause the primary arrivals to form a false image above the true image. For second-order multiples the frequency would be tripled in equation (1), and so forth.

One further point that needs to be discussed, is the phase shift to be applied to the multiple image. For primaries, a frequency-independent phase shift of $-5\pi/4$ to

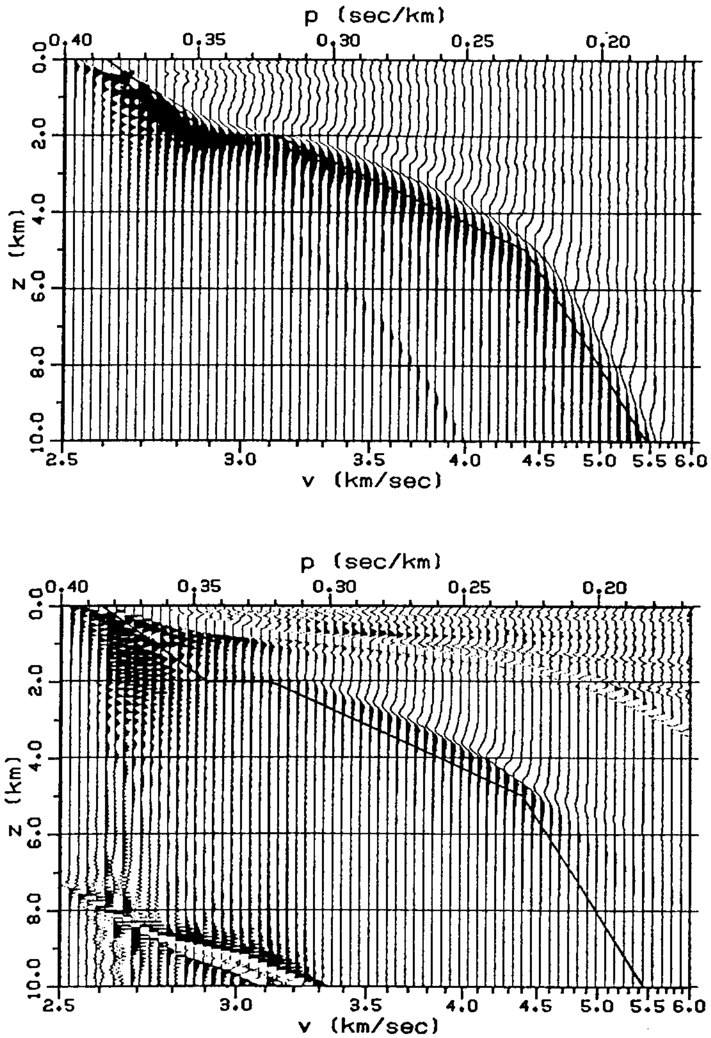

FIG. 1. A synthetic example of free-surface multiple inversion. The upper panel shows the inversion based on primary arrivals. The solid curve shows the velocity model used to generate the synthetics. The multiples form a false image below the true curve. In the lower panel, the inversion with the multiples is shown, and in this case the primaries form a false image above the true curve. The multiple image does not span the same depth range as the primary inversion because the slant stack was truncated in τ.

the slowness plane before the velocity curve was picked. This accounted for the "reflection coefficient" of refracted waves $-i \, sgn\,\omega$ (a $\pi/2$ phase shift), and a $\pi/4$ phase shift that occurs in converting a line source to a point source. Chapman (1980) has pointed out that the correct phase shifts are frequency dependent, but we will assume that the static phase shifts are sufficient for bandlimited data. For the first multiples we have applied a phase shift of $-7\pi/4$ which accounts for the caustic and for the free-surface reflection coefficient.

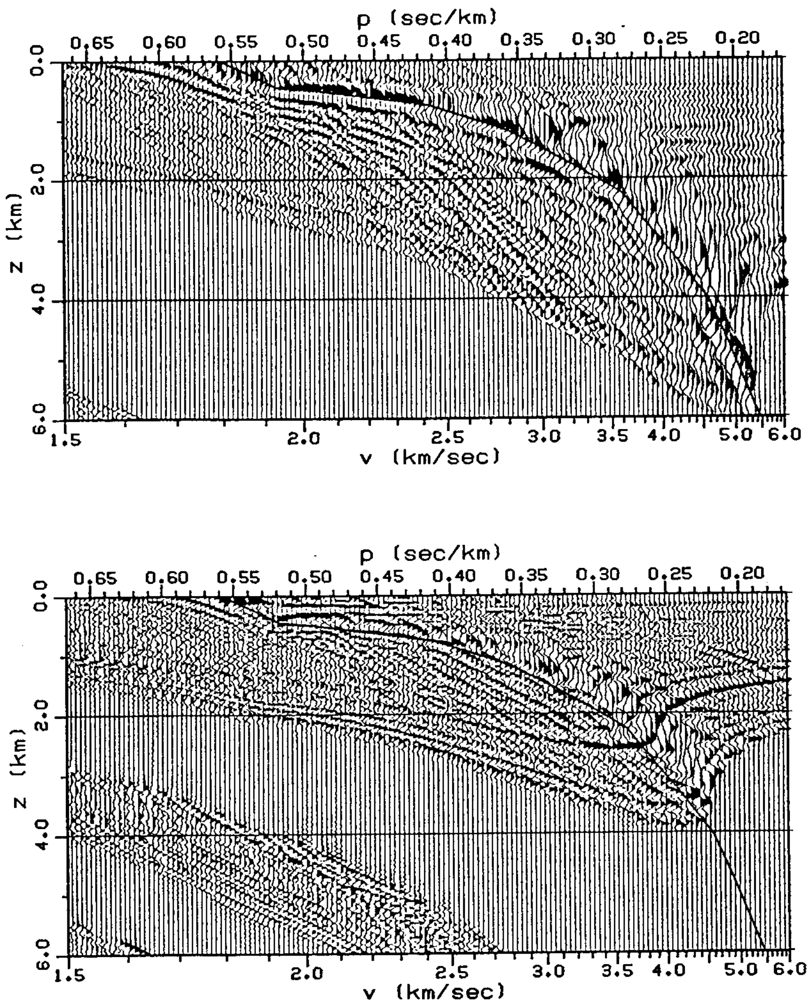

FIG. 2. A real example of free-surface multiple inversion. The data set is a dense refraction profile recorded in the Imperial Valley, California (see SEP-24 for details). The upper panel shows the inversion based on primaries, while the lower panel shows the inversion based on primaries. The solid line in each case is the velocity used in the downward continuation. The multiple inversion conforms the primary inversion in the depth range 0.25-2.0 Km, and also confirms the assumption of lateral homogeneity in that range.

Examples

The imaging of the multiples is illustrated with a synthetic and a real example. The synthetic is taken from SEP-24 (figure 5) In Figure 1a, the image of the primary is shown in the correct position, while the multiple false image of the first multiple appears at a greater depth. In these figures the solid curve shows the input velocity, which in this case is the same as the velocity function used to generate the synthetics.

In figure (1b), the frequency in the continuation algorithm was doubled. Now the multiple forms the true image, while the primary forms a false image at a shallower depth. The "event" in the lower left corner is due to wrap around. The multiple image does not span the full velocity-depth curve, because the slant stack is truncated in τ.

Figure (2) contains a real example from the Imperial Valley in California. The final inversion of the primary data, which is taken from SEP-24 is shown in Figure (2a). Figure (2b) shows the inversion for the first multiple. It confirms the inversion in the region from 0.25 Km to 2 Km. The fact that it also produces the same velocity-depth curve confirms the assumption of lateral homogenity in that range.

Conclusions

Refracted free-surface multiples can be inverted by the same procedure that inverts the primaries. This is useful for confirming both the primary-arrival inversion, and the assumption of lateral homogenity.

Acknowledgements

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REFERENCES

Chapman, C.H., 1978, A new method for computing synthetic seismograms: Geophys. J., v. 54, p. 481-518.

Chapman, C.H., 1980, Generalized Radon transform and slant stacks: a preprint submitted to Geophys. J. R. astr. Soc.

Clayton, R.W., and McMechan G.A., 1980, Inversion of refraction data by wavefield migration: submitted to Geophysics.

McMechan, G.A., and Ottolini, R., 1980, Direct observation of a p-τ curve in a slant stacked wavefield: SSA Bull., v. 70, p. 775-789.

Schultz, P.S., and Claerbout, J.F., 1978, Velocity estimation and downward continuation by wave front synthesis: Geophysics, v. 43, p. 691-714.