SUPPRESSION OF HARD-BOTTOM MARINE MULTIPLES WITH THE WAVE EQUATION

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Conventional techniques for multiple suppression on deep water data discriminate against multiples on the basis of velocity. This approach is quite successful in attenuating water bottom multiples because the water velocity is both well known and quite distinct from primary velocities. It fails, however, to suppress pegleg multiples whose velocities seldom satisfy either of these conditions. The removal of these multiples requires a "wave-predictive" approach.

In any area with a hard-bottom multiple problem the free surface, sea bottom, and sedimentary reflectors are of order 1, c, and c^2 respectively. The multiples which cause interpreters the most trouble are the visible ones (of order c^2 and c^3). Our goal will be to eliminate all sea bottom multiples and all pegleg multiples that have undergone only one subsurface bounce. If this goal is attained then all multiples to at least order c^3 will have been suppressed.

Our general approach will be to use the wave equation to generate seafloor reverberation models from the observed surface data and then subtract these models from the data in some statistically optimal sense. Confining our attention to this subset of surface multiples has several advantages over schemes which attempt to remove all surface multiples. The most important one is that we need make no more than two subtractions from our original dataset to obtain a multiple free section. One of these subtractions corresponds to the removal of shot-associated reverberations - the other to removal of geophone reverberations. A minimal number of subtractions will ensure that the degradation of signal to noise remains tolerable. This is in contrast with algorithms (e.g., FGDP-p.257) which call for a subtraction of downgoing waves from upcoming waves at all depth levels in the section.

A second advantage in dealing with this class of multiples is that multiple models can be generated without having to downward continue the surface data past the seafloor. All wavefield extrapolation can be confined to the water layer where the velocity is constant and well known. Continuation of the data to any significant depth past the seafloor would result only in the prediction of multiples of $O(c^4)$. Such attempts are sure to be frustrated by poor knowledge of the velocity and attenuation structure of the sedimentary section. Too, the absence of transmission and absorption effects in the water layer makes true amplitude processing feasible.

Heuristic Development of the Multiple Dereverberation Operator

The classical 1-D approach to deconvolving a water-confined reverberation spike train uses the Backus "3-point" operator (Backus,'59). The assumption is that a reverberation given by

$$R(z) = \sum_{i=0}^{\infty}(-cz^n)^i = \frac{1}{1+cz^n} \tag{1}$$

filters the primary seismogram twice prior to observation – once as the seismic energy passes through the sea-bottom to the deeper strata and a second time on return to the surface. The reflection coefficient at the sea-bottom is c and n is the two-way traveltime of the water layer in time fiducials. The three-point dereverberation operator is thus

$$(1+cz^n)^2 = 1+2cz^n+c^2z^{2n} \tag{2}$$

This analysis ignores the fact that both water depth and seafloor reflectivity are distinct functions of shot and geophone locations (figure 1).

To motivate this discussion we'll write equation (2) as:

$$(1+cz^n)^2 = (1+cz^n)(1+cz^n) = (1+z^{n/2}cz^{n/2})(1+z^{n/2}cz^{n/2}) \tag{3}$$

We can think of one of these brackets as zeroing the reverberations associated with the shots and the other as a filter cancelling geophone reverberations. In the more general case we are not dealing entirely with vertical plane waves. Plane wave components with non-zero stepouts undergo a smaller delay proportional to the cosine of

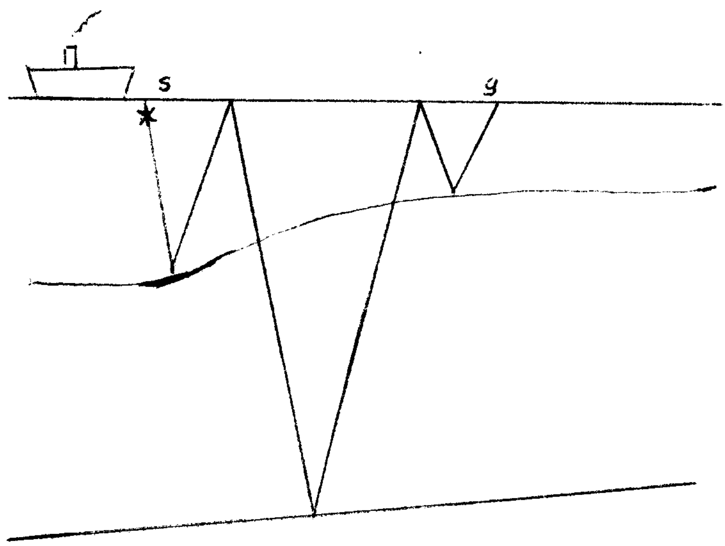

FIG. 1. Water depth and seafloor reflectivity are functions of shot and geophone location. The Backus operator makes no allowance for this.

their dip. Taking our cue from equation (3), we expect a more general dereverberation operator to be of the form:

$$D=(1+L_s cL_s)(1+L_g cL_g) \tag{4}$$

L_s and L_g are the linear operators which extrapolate the wavefield from one datum to another in either shot or geophone coordinates.[1]

The relative order of the L_g and L_s brackets is of no importance since s and g are independent variables. i.e.

$$D = (1+L_g cL_g)(1+L_s cL_s) \tag{5}$$

is equally valid.

[1] To be rigorous we should differentiate between the L_s operator which takes shots from the free surface to the seafloor and the operator which returns the shot wavefield from the seafloor to the free surface. These operators only coincide when the seafloor is flat. There is a similar ambiguity in the definition of L_g. Equations (4) and (5) and all future references to L_s and L_g should be interpreted with this in mind.

Equation (4) is the wave equation dereverberation operator consistent with the philosophy of the 3-point operator. (i.e.- The notion that the only important multiples are those which arise from interactions between the primary reflectors and the seafloor.) It contains two quantities that we will need to approximate - the wave extrapolators and the seafloor reflection coefficients. Almost any approximation to L_s or L_g is an improvement over $z^{n/2}$. The "reflection coefficients" are an empirical set of parameters that will have to account for a number of very complex effects including elastic reflections, intra-bed multiples within the first few hundred meters of sediment and 3-D effects. In this author's opinion, previous attempts to suppress multiples with the wave equation emphasized approximations to L_s and L_g below the seafloor and failed to adequately treat the seafloor reflection. Since the c's are, in general, a function of space, they will not necessarily commute with L_s and L_g (see appendix A). Hopefully, however, they will be relatively local in time and space. Experience has shown that the seafloor region cannot be modelled as a medium in which transmission effects are negligible. Only experimentation can decide how it is best parameterized but a statistical approach is almost certainly in order.

Offset Extension of Backus Operator (Zero-Dip Case)

The first real extension of the Backus operator is to increased offsets. Such an extension has been made by the authors of *WEMUL*[2] (Lerat, Tariel, and Fourmann, '79). This program generates a multiple model for each CDP gather by diffracting the surface data through a double water layer. To do this each midpoint gather is transformed to the $\omega - k_h$ domain, phase delayed by the diffraction operator $exp(2i\omega\tau\sqrt{1-(vk_h/\omega)^2})$ and transformed back to $h-t$ space. (τ is the one way vertical watertime at the CDP in question and k_h is the Fourier dual of full offset, h). The resulting model is then subtracted by adaptive least squares from the data either before or after stack to yield a "multiple-free" product.

Extensions beyond WEMUL depend on whether one feels that structural dip or variation in the seafloor reflection coefficients are more important. In this section we will examine the problem of multiple suppression on data with small dips but variable

[2] "WEMUL" - CGG trademark

seafloor reflectivity. This problem deserves consideration from both a theoretical and a practical standpoint. WEMUL, at present, has no concept of an underlying physical model of seafloor reflectivity. The following theory clears up some of the uncertainty on this point. From the practical side, we expect that implementing this (relatively cheap) algorithm will reveal whether variable seafloor reflectivity or dip are to blame for most of our present woes.

Figure (2) explains the basic concepts. Our observed dataset is given as a surface wavefield downgoing in shot and upcoming in geophone coordinates. Our goal (figure 2a) is to obtain a downgoing shot and upcoming receiver wavefield just below the seafloor. This will be done through the process of downward continuation and application of the seafloor boundary condition:

$$\underline{U_g} = \overline{U}_g - \hat{c}_g \overline{D}_g \,|_{z=seafloor} \qquad (6)$$

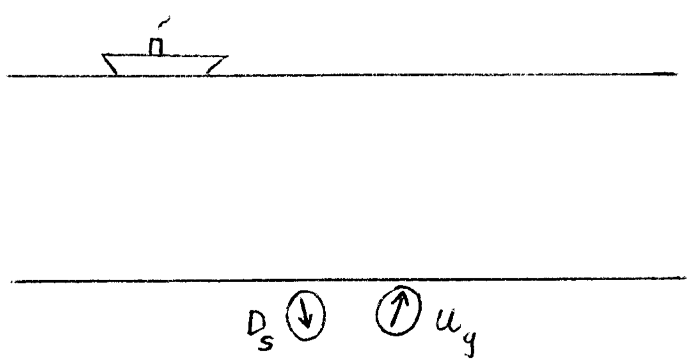

FIG. 2a. Objective wavefield consists of downgoing shot and upcoming receiver both below the seafloor.

In equation (6) $\overline{U}_g$ and $\underline{U}_g$ are the upcoming geophone wavefields above and below the seafloor respectively. The seafloor reflectivity filter, $\hat{c}_g$, is estimated by minimizing the power in $\underline{U}_g$ over a time gate where multiple energy is expected to dominate.

In migration, it suffices to consider the operators $\pm(\sqrt{1-S^2}+\sqrt{1-G^2})$. In this problem it is also of interest to consider the operators $\pm(\sqrt{1-S^2}-\sqrt{1-G^2}) = \pm HY$. Physically these operators denote the wavefields of frames (2) and (3) in figure 2(b). They vanish for either 0 dip (our present assumption) or 0 offset angle. They are important only when large water depth or dip become a consideration.

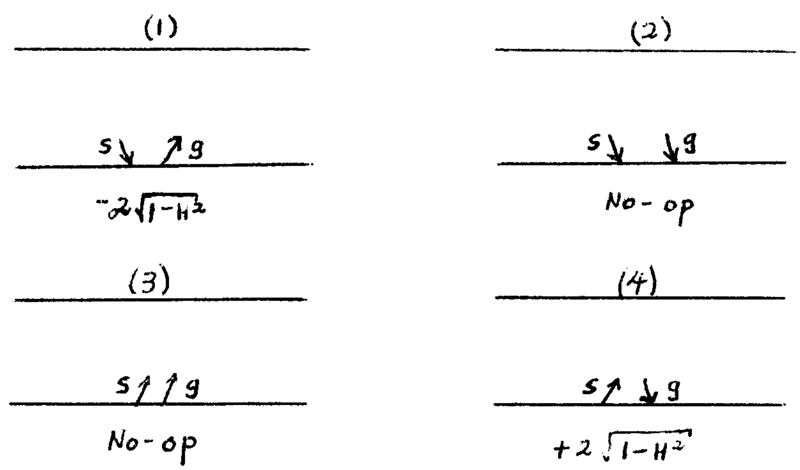

FIG. 2b. The four operators obtained by varying signs in $\pm(\sqrt{1-S^2}\pm\sqrt{1-G^2})$ for 0 dip.

Implementation

It is desirable to apply our wave operators in midpoint - offset (y-h) space. There are two reasons for this. The most important one is that truncation and aliasing artifacts are much less severe in (y-h) than in (s-g) space. A second reason is that operations in (y-h) space mesh better with standard data processing procedures.

The proposed algorithm for 0 dip is explained in figure (2c). The basic steps are:

I) Downward continue the surface data with operators (3) and (4) of figure (2b) to the seafloor and apply the shaper filter (6) to all geophone locations. Ideally, this yields a collection of data consisting of an upcoming shot wavefield just *above* the seafloor and an upcoming geophone wavefield just *below*. We'll denote this situation by $(\overline{U}_s, \underline{U_g})$.

II) Interchange shots and geophones, upcoming and downgoing waves, and reverse time. This is equivalent to a formal statement of reciprocity. Our wavefield is now $(\overline{D}_g, \underline{D_s})$.

III) Upward continue the configuration of (II) to the seafloor. This is merely a conceptual (i.e. no-operation) step. The "x" across the "S" field indicates that this is now a "multiple-free" wavefield.

IV) Apply the boundary condition $U_g = -D_g$ at $z=0$.

V) Downward continue the wavefields from steps 'III' and 'IV'. This is done with operators (3) and (4) of figure (2b). The sign on the $\sqrt{1-H^2}$ operator is positive since time is reversed. A final application of equation (6) gives us our objective wavefield (figure 2a).

In practice the proposed algorithm consists of two 0-dip diffractions of the common midpoint gathers, each followed by a least squares fitting procedure (figure 3). The first fit is done over common geophone or shot gathers. The second fit is done over the remaining coordinate. It may be necessary to allow the seafloor reflectivity to vary with offset. This question, however, is best left to experiment.

Proposed Method for Non-Zero Dip

We have just outlined a strategy for multiple suppression which requires small dips but accurately accounts for variation in seafloor reflectivity. When dip or water depth is large, we must return to our original form for the dereverberation operator (equations 4 and 5). It is again desirable to work in (y-h) space. In order to implement L_s and L_g in this space we have to think about representations for the differential operators $(1-(Y \pm H)^2)^{1/2}$. An appropriate expansion is

$$(1-(Y \pm H)^2)^{1/2} = \sqrt{1-H^2}(1 + \frac{HY}{1-H^2} - \frac{Y^2}{2(1-H^2)^2} + \ldots) = \sqrt{1-H^2} + \frac{HY}{\sqrt{1-H^2}} - \cdots \qquad (7)$$

Using a stationary phase approximation (Yilmaz, '79), we can approximate H in the second term of the final expression by $\hat{H} = \frac{2h}{vt}$. In migration, the double square root

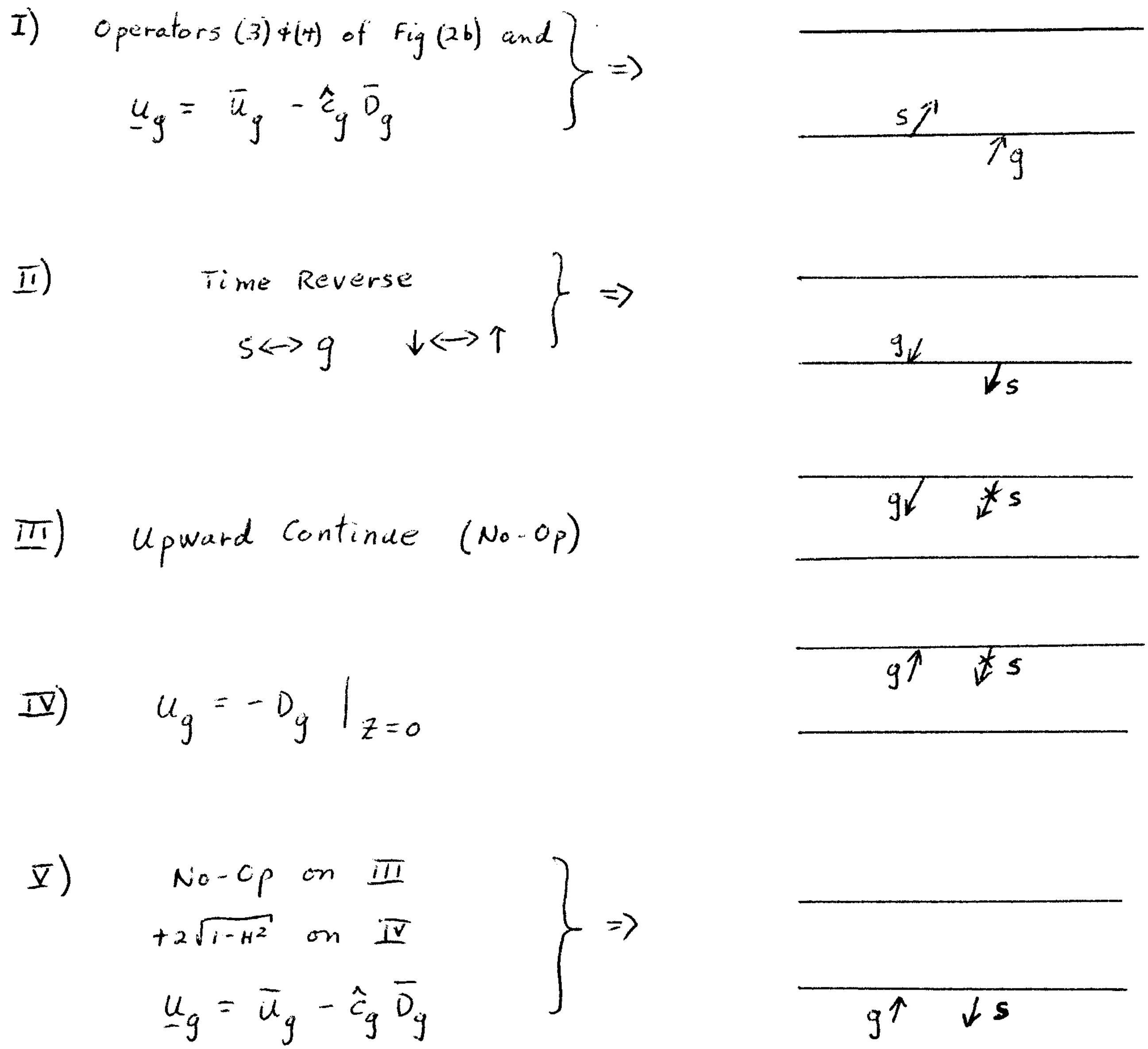

FIG. (2c). Conceptual steps in the 0-dip algorithm.

operator is used and the first order terms in Y are self-cancelling. Here we neglect the term in Y^2. The term in Y represents a shift in midpoint coordinates whose sign depends on whether we are continuing shots or geophones. Its magnitude depends simply on the depth of the seafloor and the ratio of offset to time. It will be more convenient to apply the shift after the $\sqrt{1-H^2}$ operator in order to minimize truncation effects. This is permissible since the operators in (7) are entirely commutative.

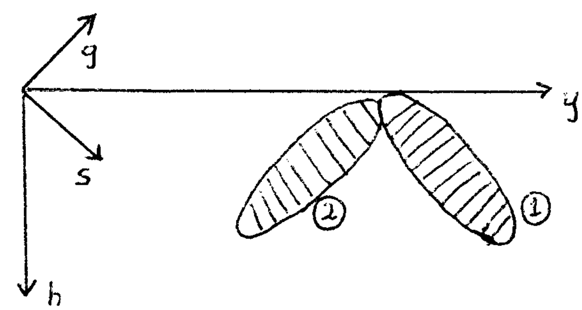

FIG. 3. The correlation between upcoming and downgoing wavefields at the seafloor is removed from the upcomng wave for each common geophone location (1). Residual correlation common to shot locations is then removed (2).

Summary

The Backus operator provides a useful base for developing a general strategy for hard water bottom multiple suppression. This paper has presented the form that a general water layer dereverberation operator must take. The generalization is necessary to account for

(1) non zero offsets

(2) variable depth and reflectivity of the seafloor, and

(3) the effects of structural dip.

For the limit of zero dip but variable seafloor we argued that an approach - algorithmically reminiscent of WEMUL - but with a very different physical interpretation shows promise. This algorithm is now being implemented. For the third level of complexity a stationary phase approach is advocated to split shot and geophone peglegs.

ACKNOWLEDGMENTS

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APPENDIX

(A) Once More (with rigour)

The main thing that will emerge from a rigorous derivation of (4) is a clear statement of why c does not necessarily commute with L_s or L_g. Consider how one would predict "geophone" multiples from a common shot gather. Figure (A1) depicts the basic situation. The total pressure field, P, in the water is the sum of an upcoming wave, U, and a downgoing wave, D. Boundary conditions are that $U=-D$ at $z=0$ and $U=cD$ at the seafloor, $z=z_f(y)$. Decompose U and D into a sequence of reverberatory components U_i and D_i with $U = \sum_{i=0}^{\infty} U_i$ and $D = \sum_{i=0}^{\infty} D_i$.

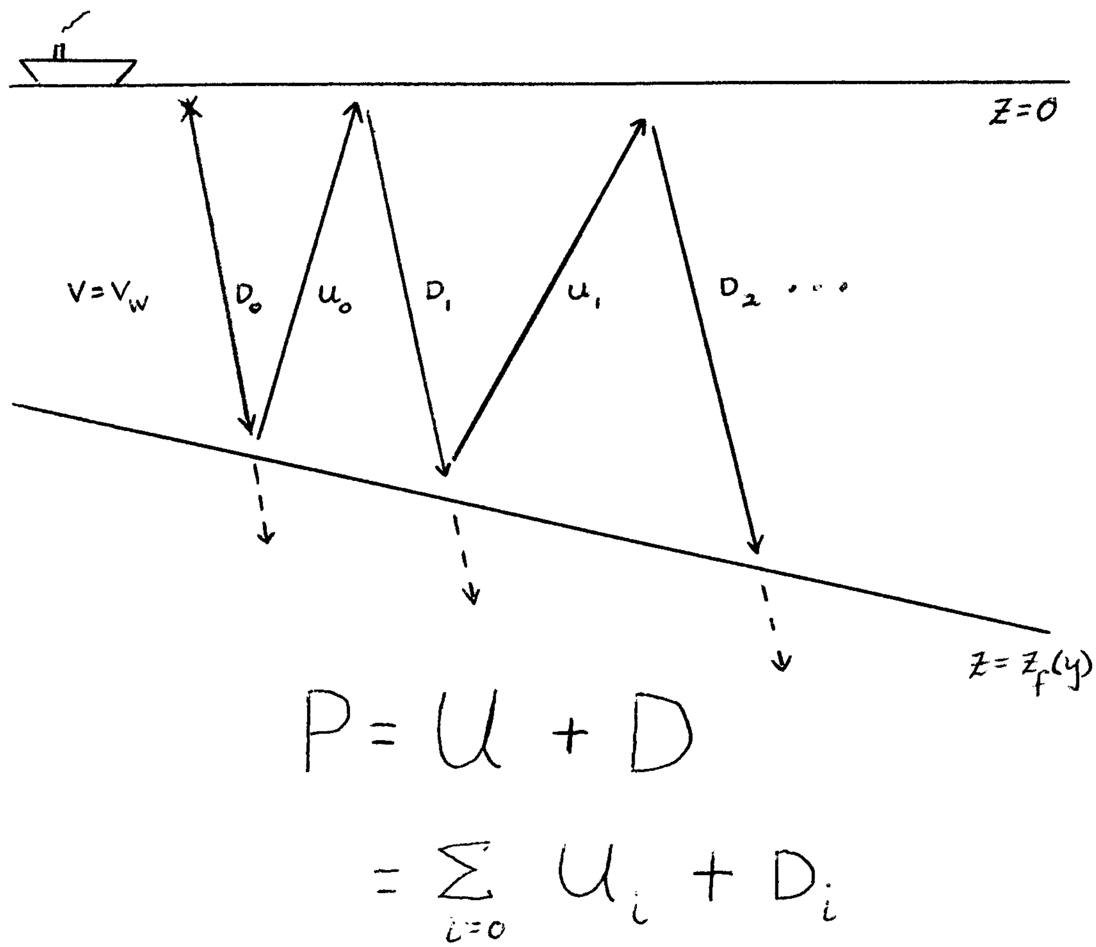

FIG. (A1). Decomposition of upcoming and downgoing pressure field in water layer.

Using the relations:

$$U_{i-1}|_{z=0}=-D_i|_{z=0} \qquad D_i|_{z=z_f}=L_g D_i|_{z=0} \tag{A1}$$

$$U_i|_{z=z_f}=cD_i|_{z=z_f} \qquad U_i|_{z=0}=L_g U_i|_{z=z_f} \tag{A2}$$

gives

$$U_i = -L_g cL_g U_{i-1} \text{ and } U|_{z=0} = \sum_{i=1}^{\infty}(-L_g cL_g)^i U_0|_{z=0} = \frac{U_0|_{z=0}}{(1+L_g cL_g)} \tag{A3}$$

Thus the wavefield, $U_0|_{z=0} = (1+L_g cL_g)U|_{z=0}$ is the total upcoming wavefield at the surface stripped of geophone reverberations. To suppress the shot reverberations we appeal to symmetry and reobtain the dereverberation operator of equation (4).

(B) A Scattering Theory Interpretation

We assume that the Born expansion (see Clayton - SEP 24, '80) for the observed data, D, can be approximated by:

$$D=\sum_{i=0}\sum_{j=0}(G_0\hat{V}G_0\overline{V})^i GVG(\overline{V}G_0\hat{V}G_0)^j \tag{B1}$$

The term "GVG" in the centre of the expansion denotes the primary observations at the sea surface. $\overline{V}=-1$ is the free surface potential. G_0 is the constant velocity Green's function for propagation in water. The "0" subscript emphasizes that - unlike G - this is a *known* Green's function. $\hat{V}$ is a potential which is assumed to have support only in the vicinity of the seafloor. It is estimated from the data by solving the problem

$$\min_{\hat{V}}||DG_0^{-1}-D\overline{V}G_0\hat{V}||^2 \tag{B2}$$

Now define

$$D_1=(DG_0^{-1}-D\overline{V}G_0\hat{V})G_0 \tag{B3}$$

Note that if $\hat{V}$ has been correctly estimated, then

$$D_1=\sum_i\sum_j(G_0\hat{V}G_0\overline{V})^i GVG \tag{B4}$$

We now find

$$D_2=G_0(G_0^{-1}D_1-\hat{V}G_0\overline{V}D_1)=(1-G_0\hat{V}G_0\overline{V})D_1 \tag{B5}$$

$$=GVG \ .$$