WAVE EQUATION STACKING

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Abstract

A wave equation method of common midpoint stacking is readily derived from the double square root equation. This paper responds to one of the major presumed objections to wave equation stacking- the problem of truncated and aliased offset spreads- by demonstrating through logic and synthetic examples that this objection is illusionary.

Wave Equation Stacking (WES)

Kirchoff migration and conventional CDP stacking are analogous operations- both sum over hyperbolic trajectories predicted by ray tracing equations. An equivalent analogy exists for wave equation stacking and migration methods. The starting point is the double square root equation (1) which encompasses the combined operations of common midpoint stacking and migration of zero offset sections. (Clayton, SEP 14, Yilmaz, SEP 16)

$$P_z = -i\frac{\omega}{v}\left\{\left[1 - \left[Y + H\right]^2\right]^{1/2} + \left[1 - \left[Y - H\right]^2\right]^{1/2}\right\} P \tag{1}$$

Y and H are Fourier wavenumber ratios related to the reflector dip angle α and short-reflector-geophone raypath angle 2β (from Clayton, SEP 14)

$$Y = \sin\alpha \cos\beta \tag{2a}$$

$$H = \sin\beta \cos\alpha \tag{2b}$$

An expression for migrating zero offset sections is derived by assuming that the angle β has been reduced to zero by common midpoint stacking. H then becomes zero giving

$$P_z = -2i\frac{\omega}{v}\left[1 - Y^2\right]^{1/2} P \tag{3a}$$

Likewise, an expression for stacking common midpoint gathers is derived by assuming that the dip angle α is much less than the offset angle β on a common midpoint gather. Y becomes zero giving

$$P_z = -2i\frac{\omega}{v}\left[1 - H^2\right]^{1/2} P \tag{3b}$$

Thus both stacking and migration, whether formulated as raytracing or wave equation operations, are analogous to one another. However, in the case of migration, both raytracing (Kirchoff) and wave equation (Claerbout, Stolt) methods are used, while in the case of stacking, only raytracing methods (CDP stacking) is used. Five possible explanations for the lack of the use of wave equation methods follow.

(1) This method is not widely known.

(2) The additional expense of WES is not worth it. This objection is basically unanswerable until there is substantial experience with this method. WES may reduce such problems as interpolation error and moveout stretch present in conventional stacking methods. Since stacking is universally applied to seismic data, even a slight improvement may be worth the added expense. Also expense is a moot question with the continuing trend towards more powerful computers.

(3) WES cannot handle dipping events. There are at least two ways to manage dipping reflectors with WES. One is to apply a prestack dip correction along the lines of Yilmaz (SEP #16). The other is to modify the WES operator of equation (3b). Clayton (SEP #14) suggests scaling H by a factor of $\sec\alpha$ in order to handle dips.

(4) Velocity analysis cannot be done with WES. Velocity is analyzed in the same way as conventional stacking. Gathers are stacked for a range of trial velocities. The best velocity structure is when events stack in strongest.

(5) WES stacking precludes statistical stacking methods such as medium stacking. The solution to this problem is *wave equation moveout,* such as discussed by Thorson and Yedlin elsewhere in this SEP report. In fact, wave equation moveout just happens to be a special case of the next objection carried to its extreme- single trace spreads.

(6) Short cables, missing inner and outer traces, and dip aliasing would prohibit WES methods because

(a) These factors make it too hard to compute the Fourier wavenumbers (in the frequency domain) or the second derivatives (in the time domain) necessary for wave equation methods.

(b) These factors leave spurious "smile" arcs on processed gathers.

First, answer part a. One must recognize that there are two kinds of truncation and aliasing problems in wave equation processing. The first kind is inherent in the way the data is collected. The second kind is due to a choice of grid size during wave equation processing. The fallacy in the reasoning behind part a is in confusing the second kind for the first kind. Because ray tracing methods *work* this demonstrates that the limitations in the way the data is collected need not affect the way the data is processed. To prevent the problems of the second kind, a sufficient number of zero traces should be padded about the non-zero traces in order to have a satisfactory grid size for the WES operator. The examples at the end of this report demonstrate that this indeed does work.

In answering part b, it is true that there are spurious "smile" arcs at non-zero offsets on a WES processed gather. However, only the result at zero offset is desired in the stack. As shown in figure 5b, even though there are spurious arcs at non-zero offsets in the WES processed, aliased gather, these arcs constructively sum into the correct place at zero offset, provided the correct stacking velocity is used.

Synthetic Examples

Various truncated and aliased versions of a common midpoint gather were wave equation stacked at a correct and incorrect velocity using a Stolt algorithm. The Stolt algorithm was also used to generate the original gathers. Figures 1a-5a display the unstacked gathers, 1b-5b the correctly stacked gathers, and 1c-5c the incorrectly stacked gathers. Figure 6 assembles and compares the zero offset trace from each of the ten stacks. It demonstrates how remarkably well WES works even in the case of seriously truncated and aliased gathers.

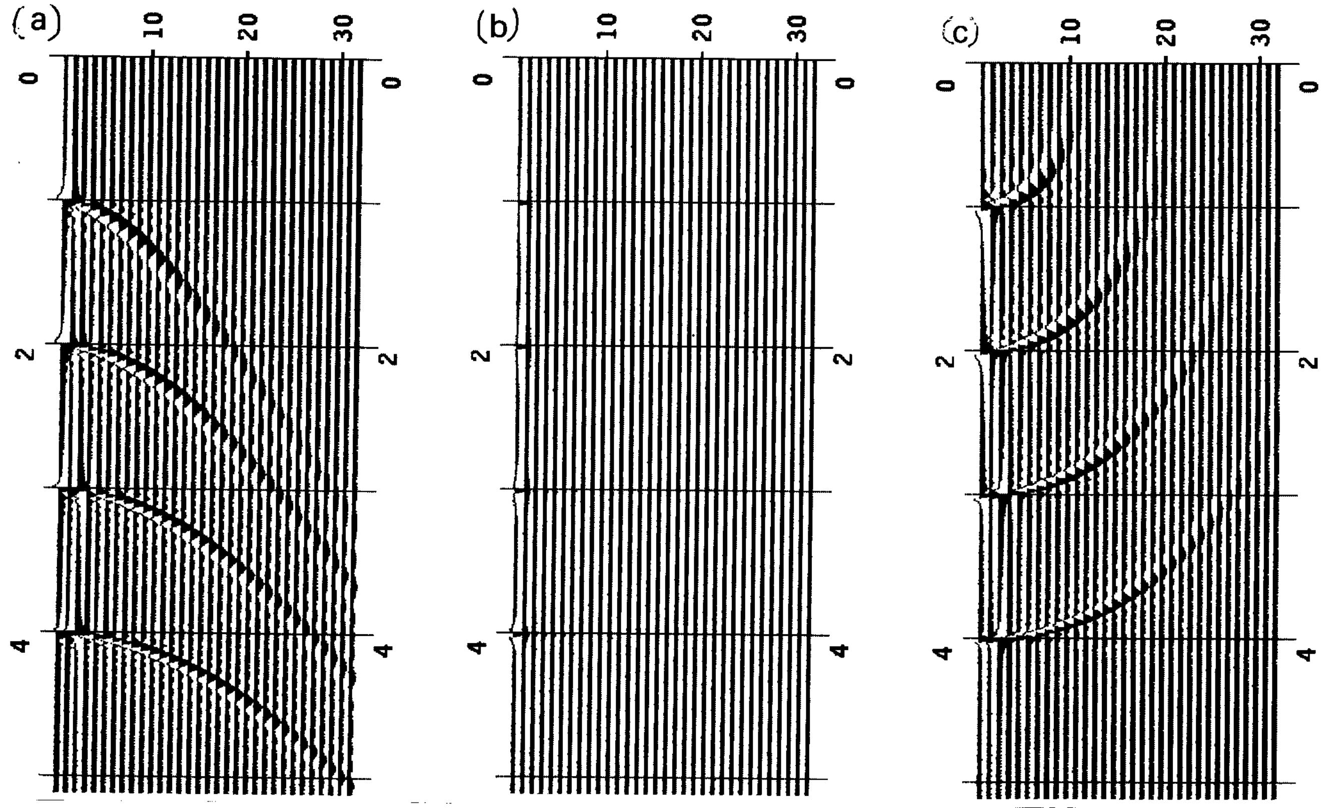

FIG. 1. Wave equation stacking of a complete gather. Part a is the dataset generated by a Stolt algorithm (dt=.005sec, dh=62.5m, v=2500m/s). The display is gained a factor of ten above normal. Part b is the Stolt wave equation stack of part a with the correct velocity. Hence, it is also the earth image. Though the stack is only the zero offset, other offsets are shown. Part c is a wave equation stack at an erroneous velocity of 3500 m/s. The events at zero offset are dspread out away from the correct time value.

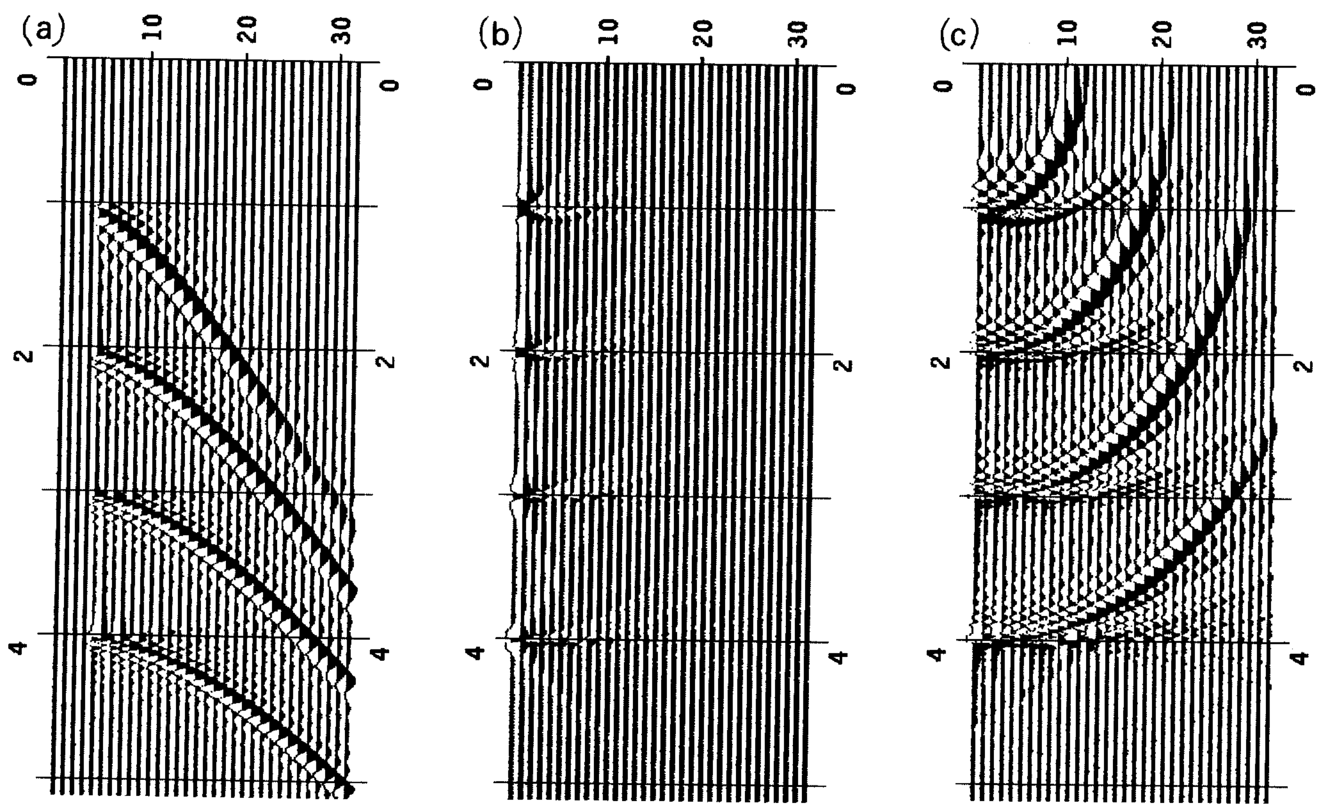

FIG. 2. Part a is the same model as in figure 1a, except with the four inner traces missing. Parts b and c have been processed in the same way as in figure 1. Zero traces replace the missing traces in the actual processing. Observe that the events "stack" at the correct time values for the correct stacking velocity in part b and not so for the incorrect velocity used in part c.

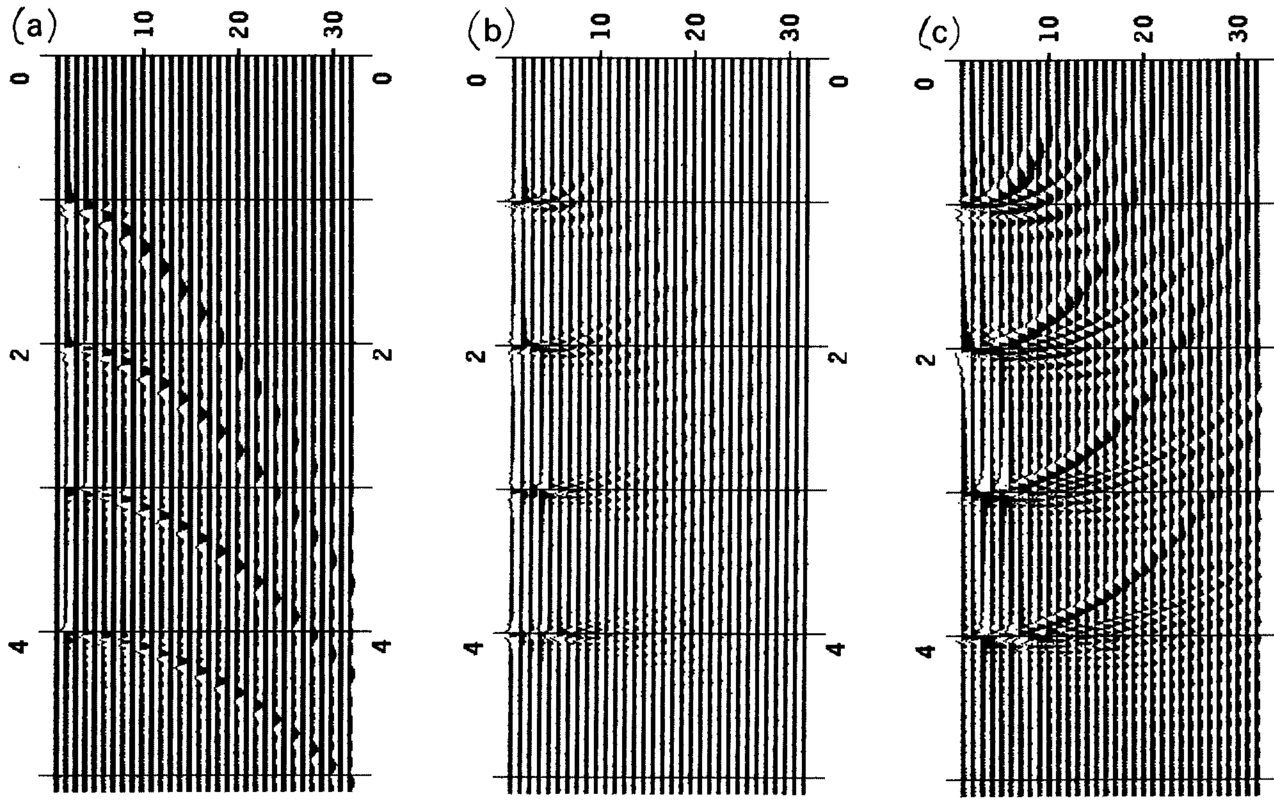

FIG. 3. Part a is the same model as in figure 1a, except slightly aliased with every other trace missing. Parts b and c have been processed the same way as in figure 1. Zero traces replace the missing traces in the actual processing. Events still stack coherently for the correct stacking velocity and erronously for the incorrect stacking velocity.

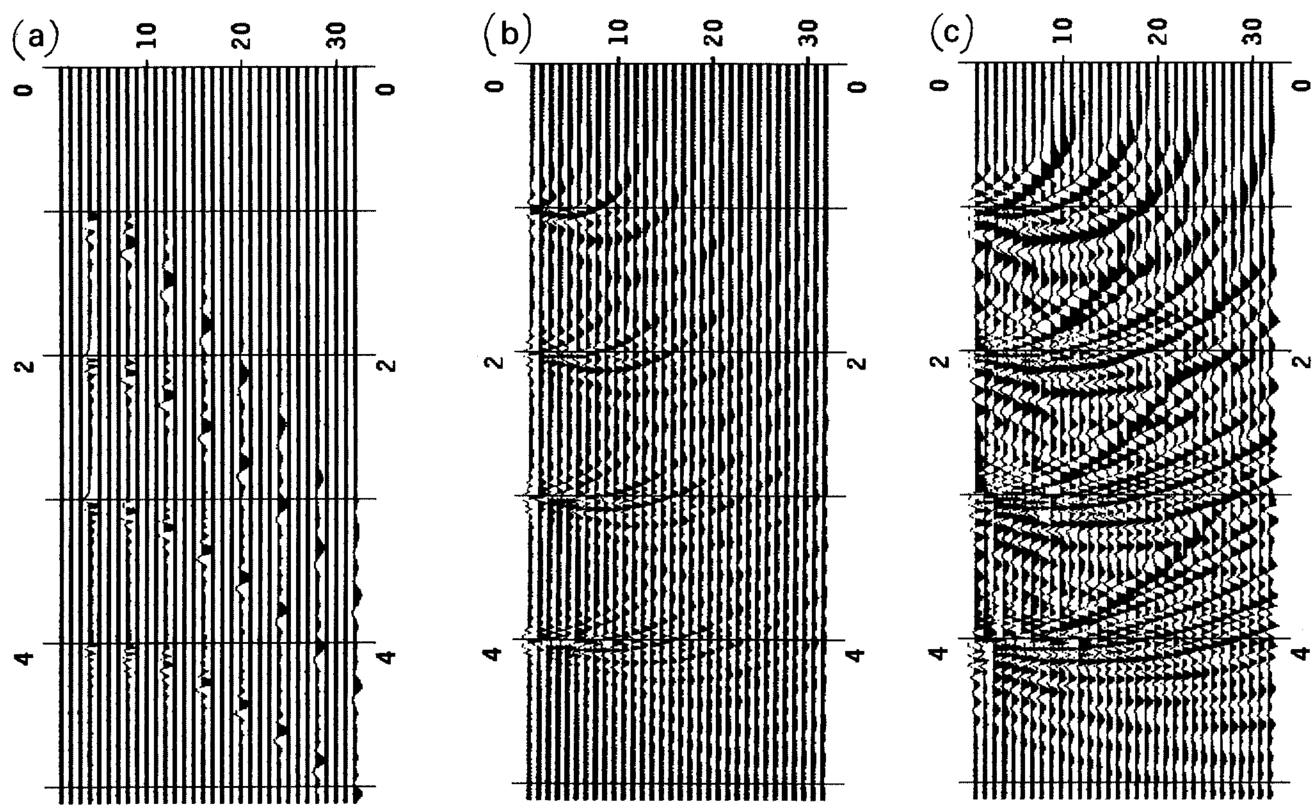

FIG. 4. Part a is the same model as in figure 1a, except strongly aliased with offsets four times that of figure 1a. Parts b and c have been processed the same way as in figure 1. Events still stack well for the correct stacking velocity.

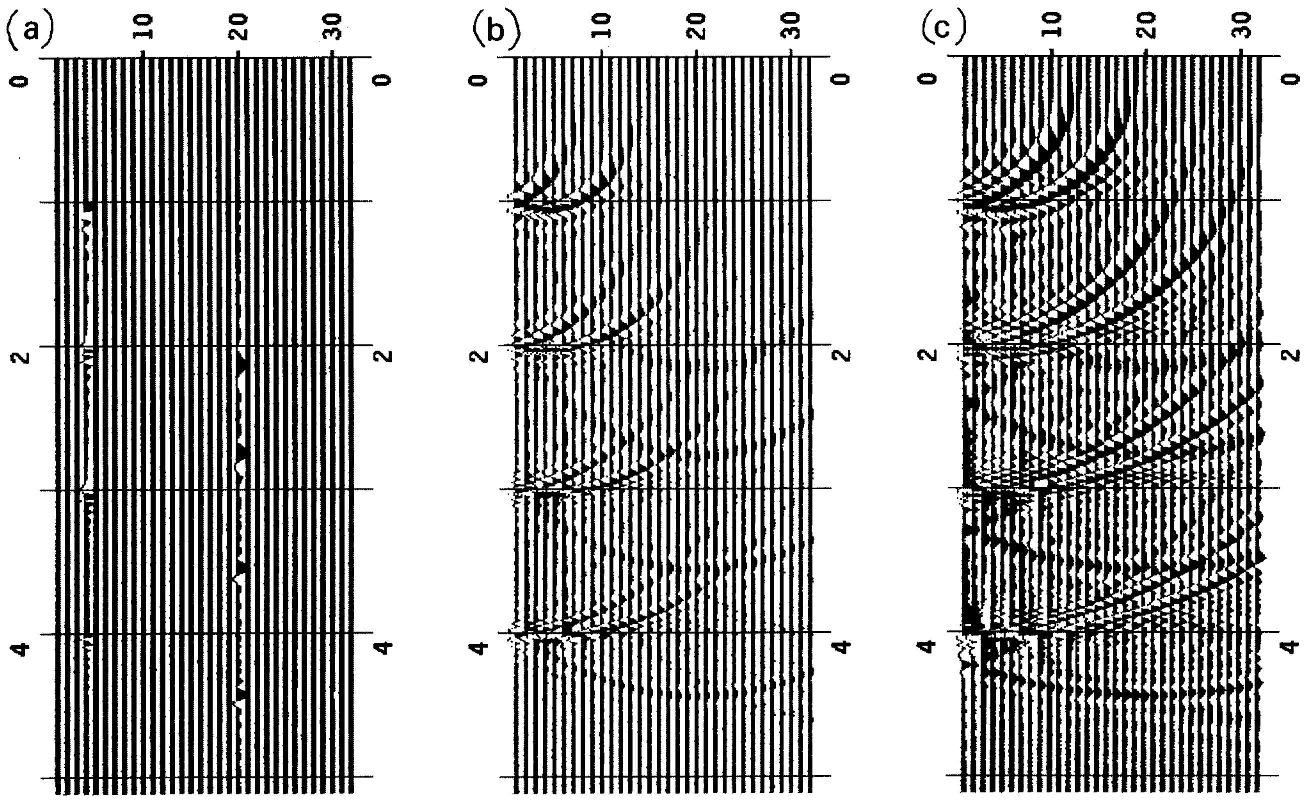

FIG. 5. Dataset with only a pair of widely separated traces. Stacked in the same way as for figures 1. The events for the correct stacking velocity in part b still coherently sum together at the correct time values. This is virtually a demonstration of *wave equation moveout*.

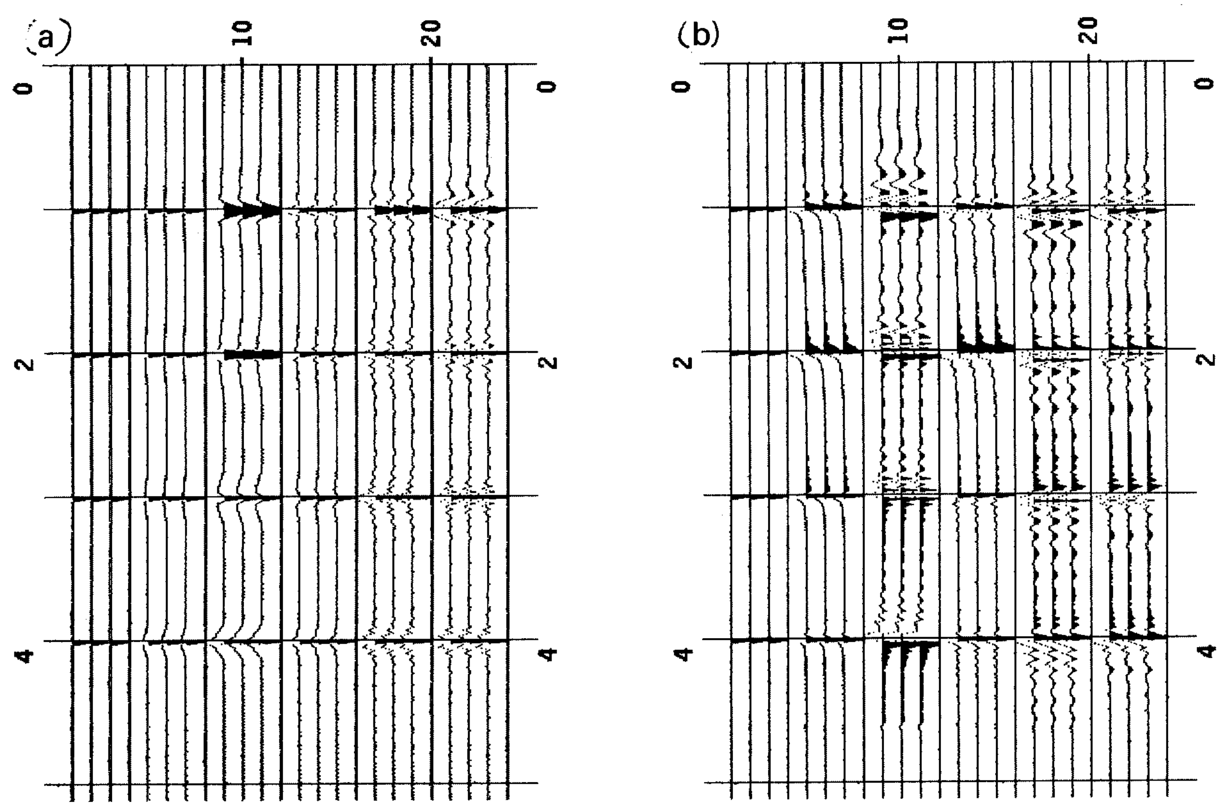

FIG. 6. Collected stacks (zero offset traces) of the earth model and figures 1 to 5 (left to right) for the correct stacking velocity (part a) and incorrect stacking velocity (part b). Traces are repeated three times each. The stacks of part a always come in at the correct time values, though the signal declines with the decreasing number of contributing traces. The stacks of part b are less coherent and often appear at the wrong time values.