SPATIAL INTERPOLATION OF STEEP DIPS

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Abstract

A common misconception is that, to avoid spatial aliasing, we must spatially sample our seismic data at a wavenumber (spatial frequency) twice the highest wavenumber contained in our data. Since high wavenumbers correspond to steeply dipping events, we tend to think that steep dips imply spatial aliasing. This paper attacks this misconception by showing (1) that the *range*, not the absolute magnitude, of dips present in our data determines whether or not the data is spatially aliased; and (2) that this range of dips need only be restricted *locally*, that the range may change from sample to sample without danger of aliasing.

Aliasing implies an inability to accurately reconstruct a continuous wavefield from the discrete data obtained by sampling; i.e., it implies an inability to interpolate correctly. Actually, correct interpolation requires only knowledge of the limited range of dips present at any given sample and of how that range changes from sample to sample. This knowledge is often obtainable with seismic data, and an example is provided to illustrate an interpolation method based on this knowledge.

Spatial aliasing and interpolation

"Spatial aliasing" is one of our favorite scapegoats in seismic data processing. When some processing tool produces strange artifacts, we often (and sometimes rightfully) attribute them to spatial aliasing (SA). SA is a particularly handy excuse since we usually are not expected to do anything about it except perhaps to ask the field crew to lay down more geophones in more carefully designed arrays next time. But are we giving up too easily?

We expect to find SA whenever we have data with steep dips such as the tails of reflection hyperbolas found in common-shot gathers or steep reflections or diffractions in constant-offset sections. Steep dips cause trouble because, for a given frequency, steeply dipping events have higher wavenumbers than flatter events which, if exceeding the Nyquist wavenumber, may be ambiguously superimposed on lower wavenumbers.[1] (Frequency and wavenumber denote the Fourier dual variables to time and space, respectively.) So how steep is steep?

We can be certain that an event is not SA if the following inequality is satisfied:

$$|\rho| \leq \frac{f_{Nyq}}{f_{\max}} \tag{1}$$

where ρ is the dip of the event in samples/trace, f_{Nyq} is the Nyquist frequency, and $f_{\max}$ is the maximum frequency contained in the dipping event.

Suppose for now that $f_{\max} = f_{Nyq}$. Then inequality (1) states that dips between $\pm$ 1 sample/trace will not be SA. If we want to spatially interpolate such data, we can apply some tapered version of the ideal sinc function interpolator to each time-slice of our data. That is, the interpolated values for a particular time will depend only on known values from neighboring traces *at that time.*

Now suppose our data has dips given by:

$$0 < \rho < 2 \tag{2}$$

We tend to think of events with $|\rho| > 1$ as being SA. Lots of 2-D processing tools (e.g., dip-filters, finite-difference migration) which work for $-1 \leq \rho \leq 1$ do not work when $|\rho| > 1$, or they at least produce artifacts. But this data is not actually SA. SA means (or should mean) that we have high wavenumbers *superimposed* on low wavenumbers. If our data were truly SA we could not reconstruct the continuous wavefield from the sampled values; i.e., we could not spatially interpolate the data correctly. But if we *know* that our data satisfies inequality (2) then we can interpolate correctly. A mid-point interpolation scheme might be as follows:

[1] For the reader unfamiliar with the aliasing phenomena in two-dimensional data, Clement (1973) provides a thorough discussion of this subject.

1) apply linear moveout of -1 sample/trace [the data then satisfies inequality (1)]
2) apply a spatial, midpoint interpolator to each time-slice to obtain the interpolated traces
3) apply linear moveout of +1/2 sample/trace

If, for example, our original data contained an event with a dip of 2 samples/trace, that same event would have a dip of 1 sample/trace after spatial interpolation as outlined above; and our 2-D processes will now function correctly (or we at least will not be able to blame "SA" anymore). Another viewpoint is that we have doubled the Nyquist wavenumber by halving the spatial sampling interval, so the high wavenumbers are now less than Nyquist.

Note that correct interpolation was possible because the range of dips was so restricted as to lie between ± 1 *after* linear moveout was applied. This observation suggests a revision of the condition given by inequality (1). We can be certain that our data is not SA if the following inequality is satisfied:

$$\rho_{\max} - \rho_{\min} \leq 2\frac{f_{Nyq}}{f_{\max}} \tag{3}$$

If this inequality is satisfied, we can apply our interpolation procedure (with the linear moveout corrections adjusted according to the range of dips present) as many times as necessary until we obtain the condition given by inequality (1).

An easier way

Suppose that our midpoint interpolator is a symmetric, eight-point operator which in some way approximates the ideal sinc interpolator. Then, rather than applying a linear moveout correction to our data traces, we can instead apply linear moveout to the interpolator. We apply this "tilted" interpolator to eight neighboring traces to obtain a single interpolated trace. The actual amount of tilt we apply to the interpolator depends on the range of dips we want to interpolate. Figure 1 illustrates the application of three possible interpolators and gives the range of dips for which each is valid.

Note that for $0 < \rho < 2$ the interpolated samples occur halfway between the original samples *in time* (as well as in space); we, therefore, require an additional temporal, midpoint interpolation. This second interpolation is in fact necessary whenever the interpolator is tilted by an odd number of samples/trace.

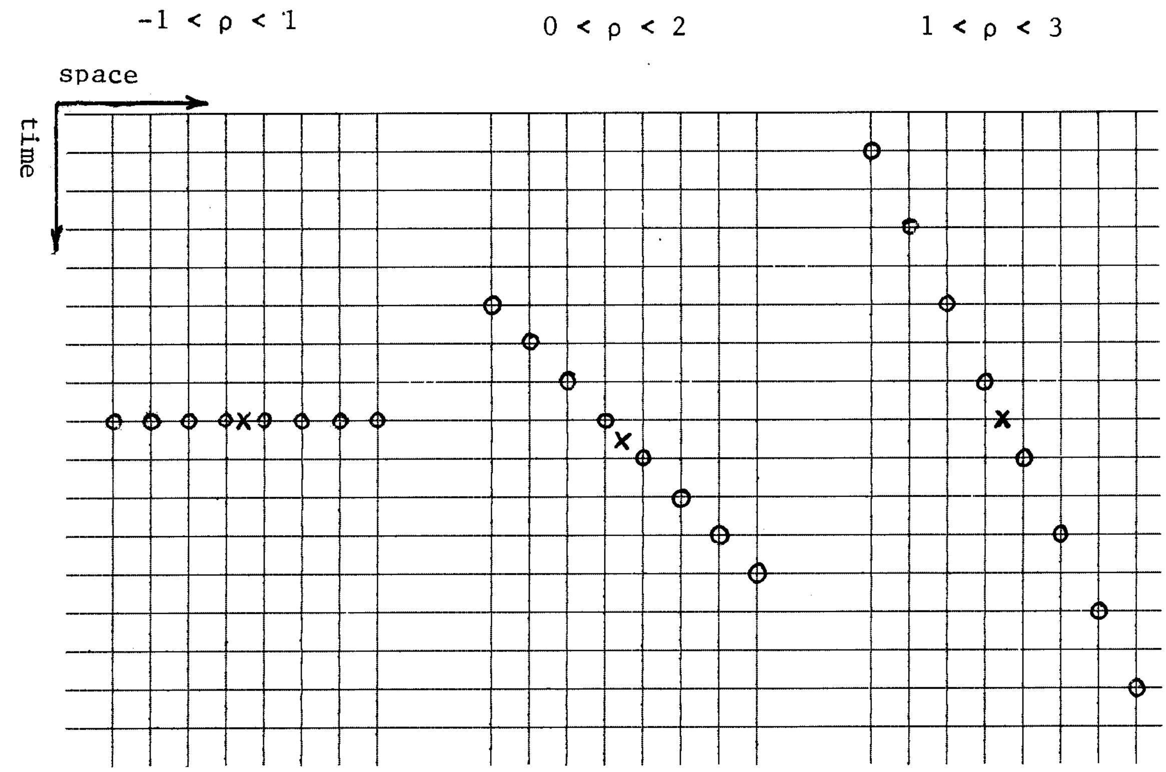

FIG. 1. Three different midpoint interpolators. "x" denotes an interpolated sample which is a linear combination of the circled samples.

Time and space-variable interpolation

If you look at the 2-D Fourier transform of a common-shot gather with steep reflection tails at far offsets, you are likely to discover that the data is truly SA, that high wavenumbers are indeed *superimposed* on low wavenumbers. In other words, if you measure the dips on such data, you will find that inequality (3) *is not* satisfied. So how do we interpolate (de-alias) this data?

The solution comes from noting that we do not expect to find steeply dipping events and flat events *at the same sample* in our gather. In other words, while the data may be terribly aliased in a global sense, locally it may not be aliased at all. Furthermore, we can predict, using a rough guess of a velocity function, the dip of coherent events at each sample in the gather. Now we can see the advantage of using a "local", midpoint interpolator -- we can easily change the tilt of the interpolator as a function of space and time as we must if we hope to interpolate correctly. Fourier transform methods of interpolation, for example, do not permit this variability.

Remember that we are not required to know the dip (or velocity) exactly; each of the interpolators illustrated in Figure 1 has a dip-bandwidth (i.e., range of validity) of 2 samples/trace if the data has frequencies up to the Nyquist frequency. When frequencies up to the Nyquist are not present, as they usually are not, the dip-bandwidth is greater -- $2f_{Nyq}/f_{\max}$ samples/trace. Also, note the considerable overlap in the dip-bandwidths of each interpolator in Figure 1; this overlap is even greater if $f_{\max} < f_{Nyq}$ and, in any case, will permit some error in deciding where to switch interpolators.

The 48-trace, common-shot gather shown in Figure 2 was used to test the tilted interpolation scheme. We first threw away every other trace (the even traces) in Figure 2 to obtain the 24-trace gather shown in Figure 3. Tilted interpolation was applied to this gather; the variable tilt was determined at each sample using a constant velocity of 1500 m/sec (water velocity). Only even, integer tilts were used in order to avoid the additional temporal interpolation required for odd tilts. The interpolated, 48-trace gather is shown in Figure 4a, and it compares quite well with the original gather of Figure 2.

To provide a more quantitative comparison, the gather of Figure 4a was subtracted from that of Figure 2 to obtain the "error" shown in Figure 4b. The error is, of course, non-zero only for the interpolated (even) traces. Trace 28 should not be considered error since the original trace 28 in Figure 2 is obviously ill; the interpolated trace is more feasible.

Figures 5a and 5b show the interpolated gather and error for a constant, no-tilt interpolation. As expected, this purely horizontal interpolation is valid only for small offsets and large traveltimes where the flatter events lie.

We should note that gathers are sometimes interpolated by applying normal moveout (NMO) followed by spatial interpolation followed by inverse NMO. Tilted interpolation is easier (and faster) since (1) no trace shifting or non-linear stretching is necessary, and (2) only a single, midpoint interpolator is required. (NMO and its inverse require temporal interpolation of non-midpoint values.) Tilted interpolation also avoids artifacts which may arise from NMO stretch at far offsets.

Conclusions

Steep dips alone do not imply SA. For if (1) we can make some reasonable estimate of the local dip at each point in our grid of data and (2) we can limit the range of dips present at each point, then we can spatially interpolate correctly and, hence, increase the Nyquist wavenumber of our data. The method outlined in this paper seems particularly suited to the spatial interpolation of gathers where these two conditions are easily satisfied.

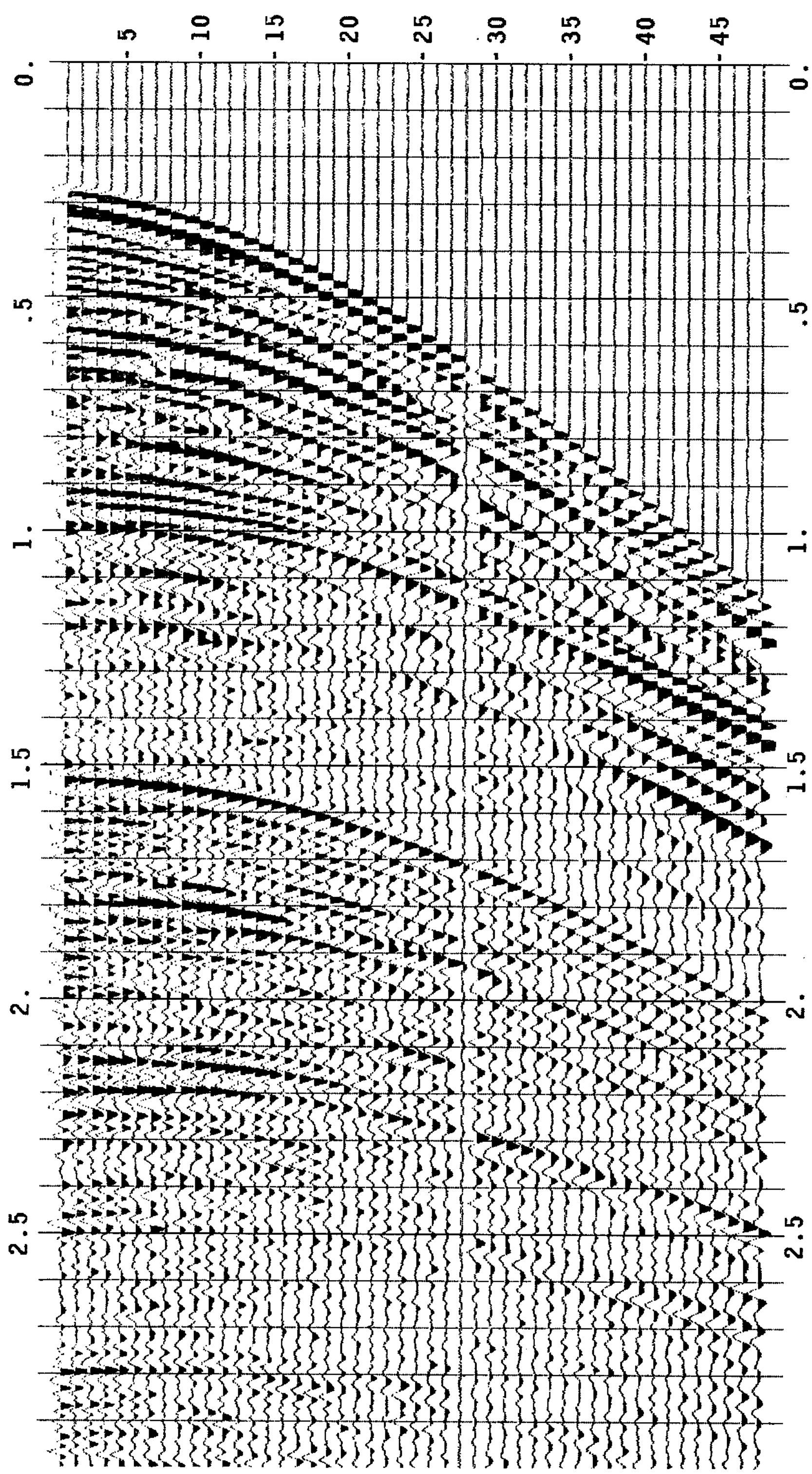

FIG. 2. A three second window of a common-shot gather used to test the tilted interpolation method. (Add one second to get true time.) The data is from the Gulf of Alaska and was provided by the USGS.

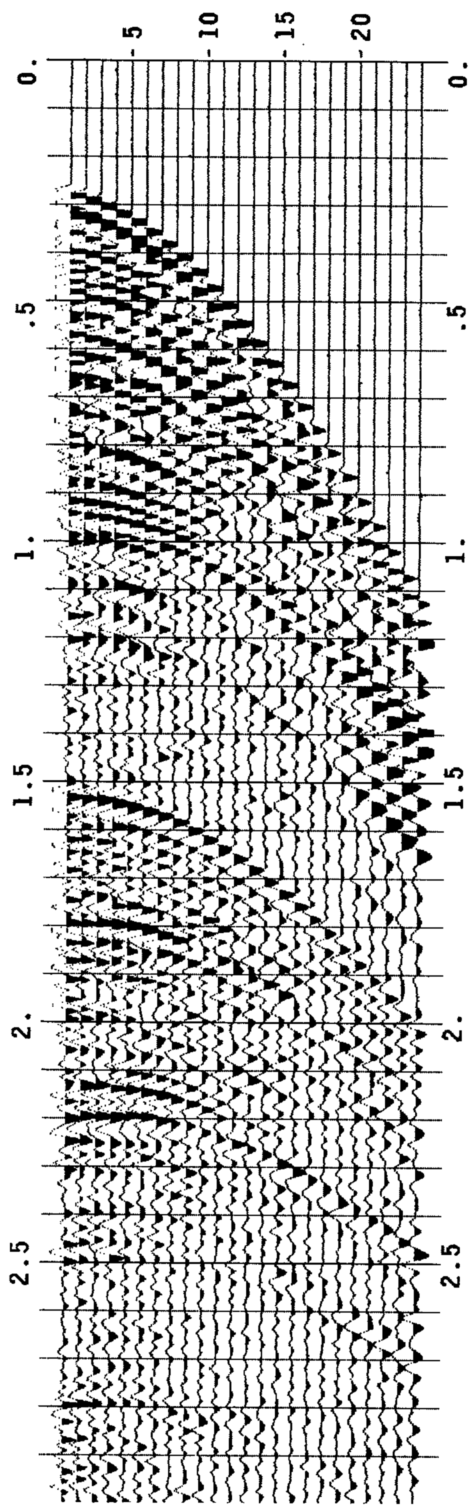

FIG. 3. A spatially subsampled version of the data in Figure 2. The even traces were discarded.

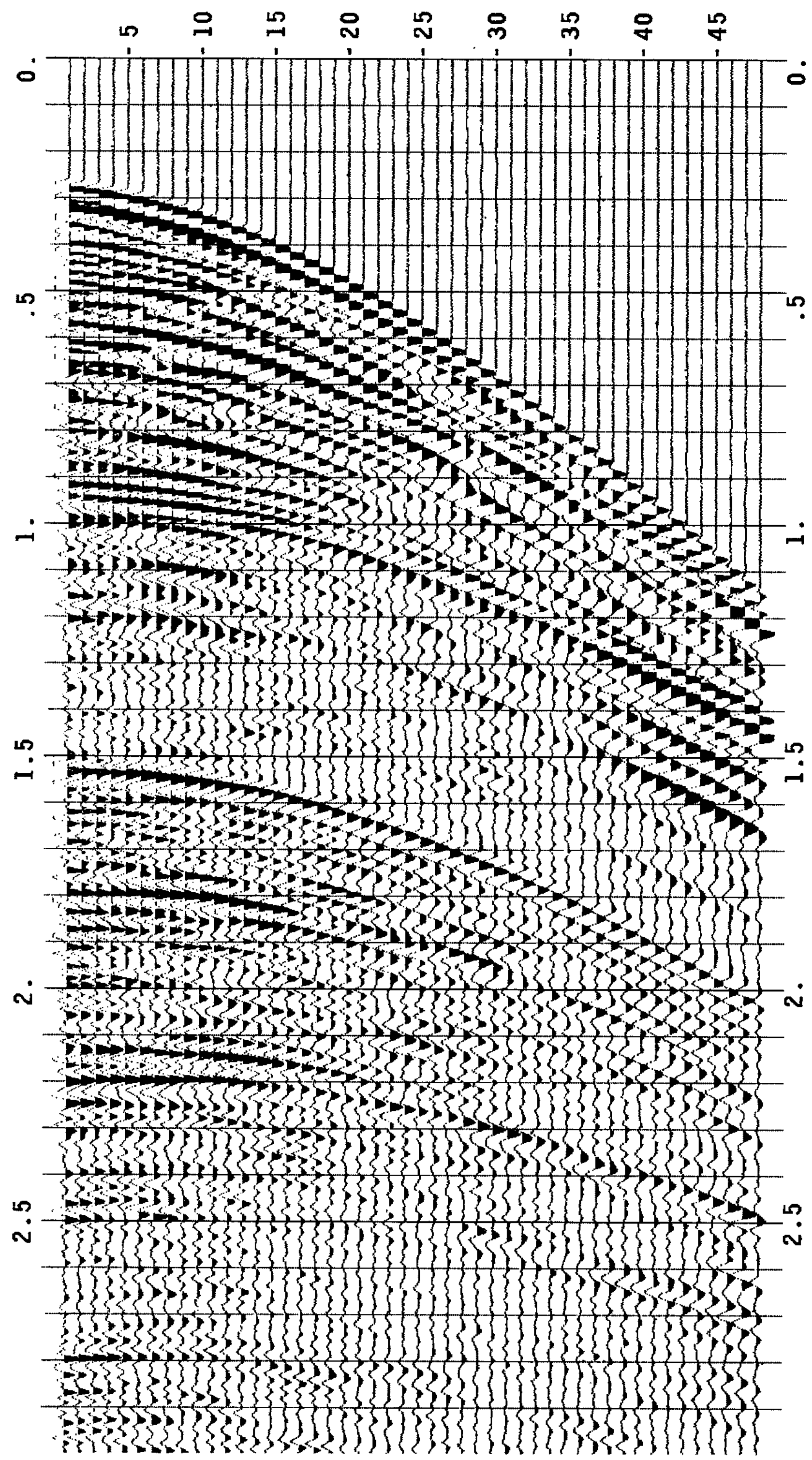

FIG. 4a. The result of applying tilted interpolation to the subsampled gather of Figure 3. The even traces should be compared with the known traces in Figure 2.

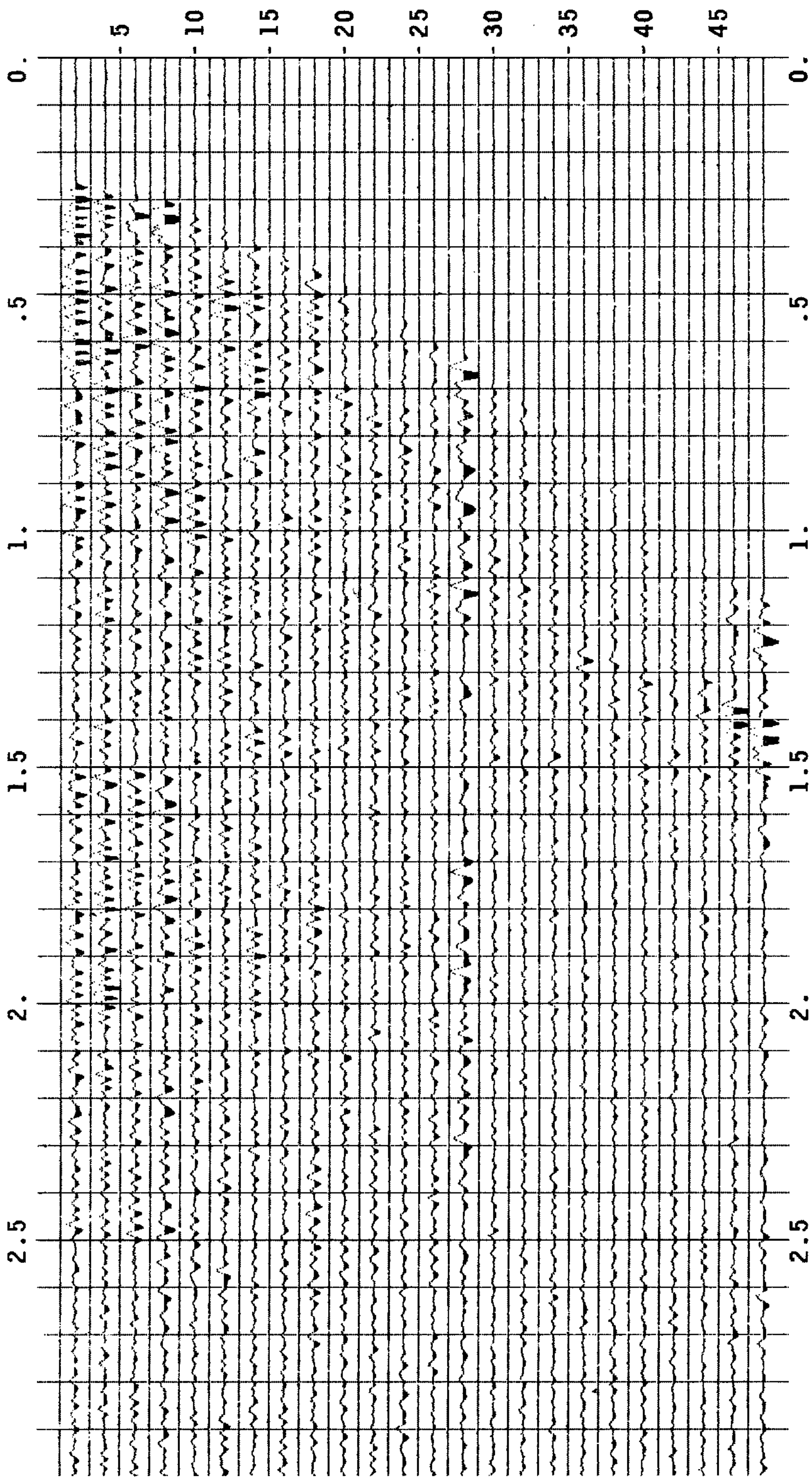

FIG. 4b. The interpolation "error", the difference between Figures 2 and 4a.

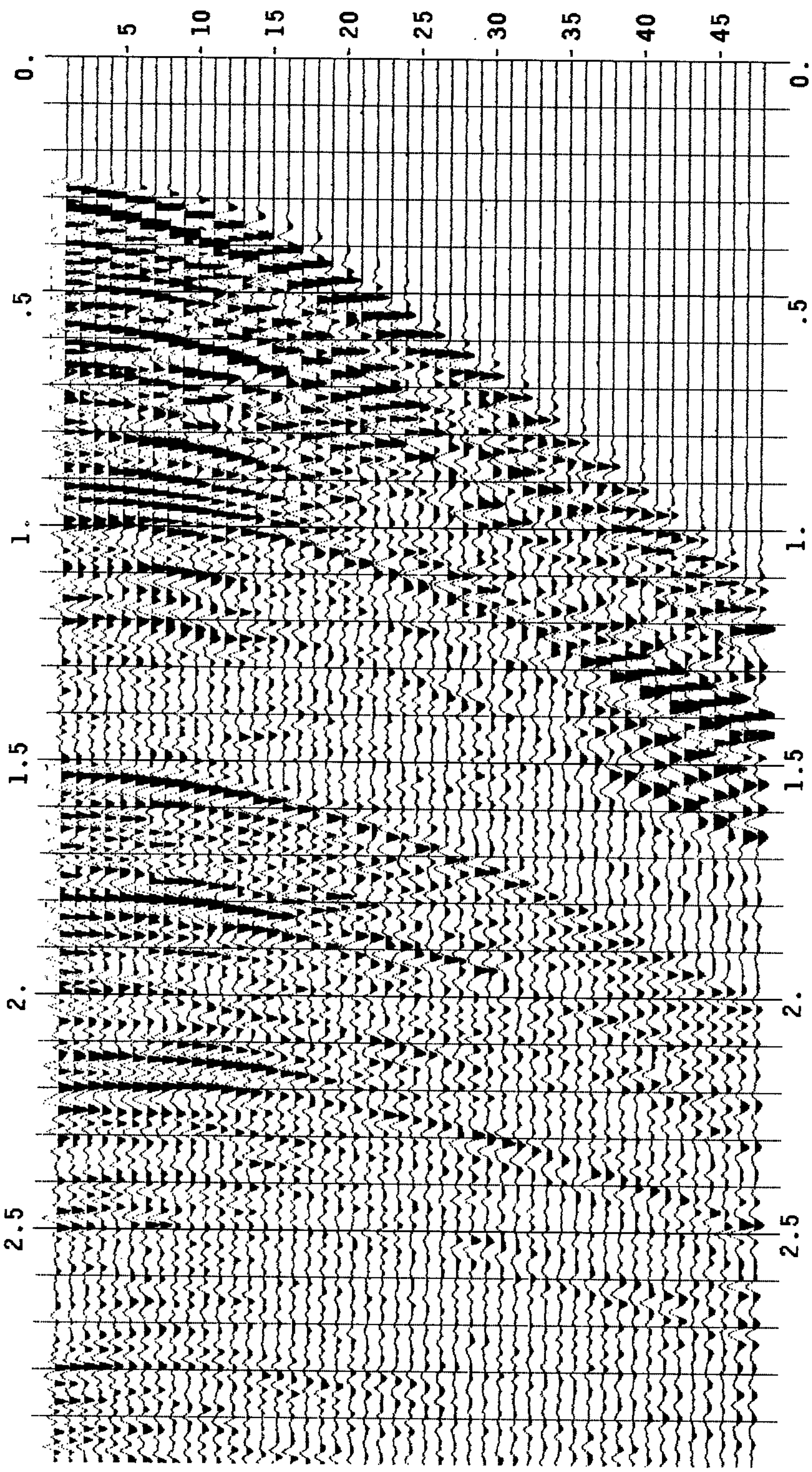

FIG. 5a. The result of applying horizontal, no-tilt interpolation to the subsampled gather of Figure 3. Compare with the known traces in Figure 2.

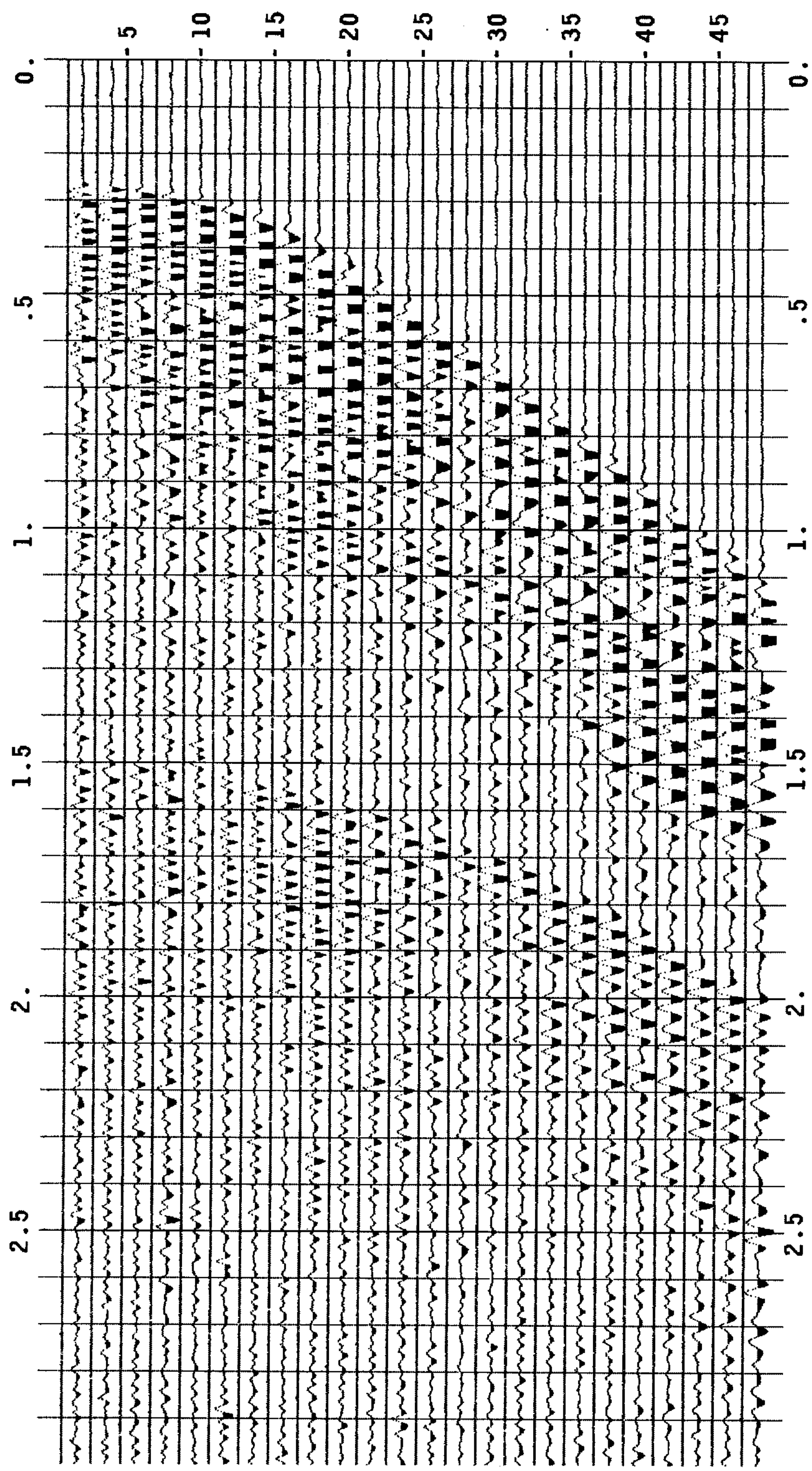

FIG. 5b. The error for no-tilt interpolation. Compare with Figure 4b.

ACKNOWLEDGMENTS

Thanks go to Bob Stolt for constructive insights and criticism and to Ken Larner of Western Geophysical Co. for providing the coefficients of the midpoint interpolator used in this work.

REFERENCE

Clement, W.G., 1973, Basic principles of two-dimensional digital filtering: Geophysical Prospecting, v. 21, p. 125-145.