Chapter IV
Wave Theory Derivation of the Lateral Velocity Equations

This chapter develops a velocity estimation theory for two-dimensional media based on the downward continuation of seismic data with the wave equation. Such a theory is advantageous since it provides a mathematical framework for velocity estimation in diffracting or dipping earth models. The first two sections discuss a general seismic imaging principle for stratified media and characterize it mathematically in what is known as the double-square-root equation. The conventional processing sequence of velocity estimation, stacking, and migration can then be represented by some part of or an approximation to the double square root equation. In the following section the conventional hyperbolic travel time equation is derived for a laterally invariant media from the double square root equation. In the final section, the double square root equation is rewritten for laterally varying media and the effects of the lateral velocity changes on the different order terms of the double square root equation are examined. The zeroth order term (i.e. small dip) leads to a traveltime equation which is identical to the traveltime equation developed in the Chapter II [equation (2.4)].

4.1. Seismic imaging principles

Imaging the earth with surface-recorded seismic reflection data requires two basic ingredients, a downward-continuation operator and an imaging principle. The imaging principle is needed to know when to stop the downward-continuation process. There are three main imaging principles of use in reflection seismology which apply to the following experiments: 1) one surface source and many receivers, 2) many surface sources and many receivers, and 3) a source distribution at depth and many surface receivers. The typical reflection seismology experiment falls into the second category and its imaging principle is based on the downward continuation of sources and receivers. This is: ***a reflector exists where upgoing energy is received after zero traveltime from the***

source to the receiver. This implies, of course, that the source and receiver have zero offset and are located at the reflector. Theoretically, the entire reflecting structure can be mapped by placing a source-receiver pair at every point in the subsurface and observing the upgoing energy arriving at t=0.

For the case of a two-dimensional medium and letting the shot and geophone locations be (s,z_s) and (g,z_g), the above imaging principle can be described as a mapping of the wavefield P from

$$P(s,z_s=0,g,z_g=0,t) \quad \text{to} \quad P(s=g,z_s=z_g,t=0).$$

With the following definitions of midpoint and half-offset,

$$y = \frac{(g+s)}{2} \quad \text{and} \quad h = \frac{(g-s)}{2}, \tag{4.1}$$

the imaging principle can be written as the following mapping

$$P(y,z_s=0,h,z_g=0,t) \quad \text{to} \quad P(y,z_s=z_g,h=0,t=0)$$

Figure 4.1 shows the location of the seismic traces in shot-geophone space for a typical marine experiment and also defines the nomenclature for gathers and sections used throughout the remainder of this thesis. We are considering a two-dimensional experiment, so the coordinates s and g both refer to locations along the traverse line. The y-axis represents the zero-offset (h=0) section. The imaging principle tells us that any energy lying along this axis at zero traveltime must be from a reflector at the source-receiver depth.

In order to use the imaging principle it is necessary to find the wavefield for the sources and receivers at non-zero depth. To help do this we have the full wave equations in two space-dimensions

$$P_{gg} + P_{z_g z_g} = \frac{1}{v^2(g,z_g)} P_{tt} \tag{4.2a}$$

and

3.0
1-5-82

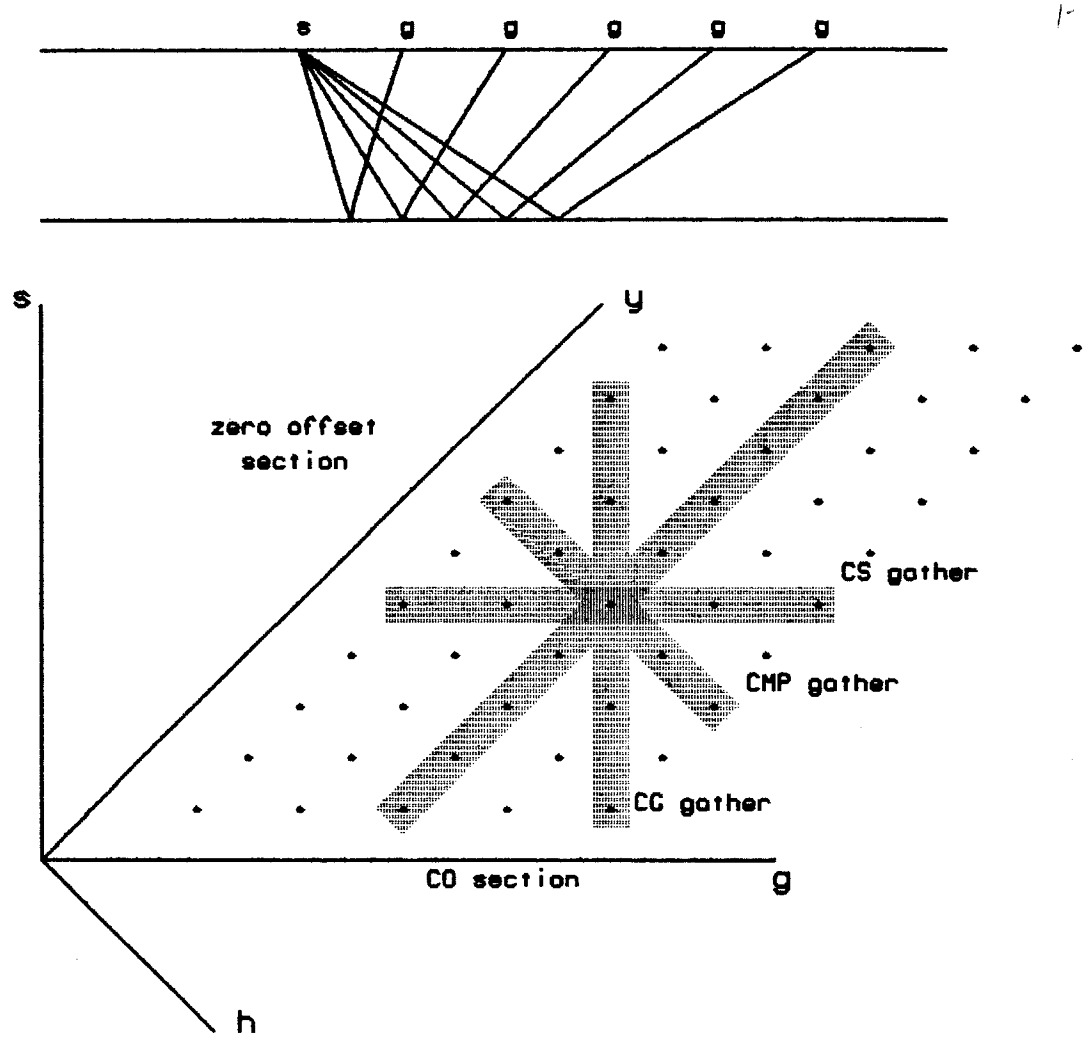

Figure 4.1. Location of seismic traces in shot-geophone (s,g) space for a typical marine experiment. Each dot in the grid represents a seismic trace for a particular shot-receiver location. The time axis can be considered to be into the page. The figure also shows the location of the midpoint [y = (g + s)/2] and half-offset [h = (g - s)/2] axes. The stippled regions define common-shot (CS), common-geophone (CG), and common-midpoint (CMP) gathers and a common-offset (CO) section. The figure at the top depicts some raypaths off a flat reflector for the stippled common-shot gather. The focused earth image is found on the zero-offset section at t=0 for every z-level.

$$P_{ss} + P_{z_s z_s} = \frac{1}{v^2(s,z_s)} P_{tt} \tag{4.2b}$$

To obtain the imaged section, first downward continue the geophones to some Δz using some form of equation (4.1a) on the common-shot gathers. Invoking reciprocity to justify the downward continuation of the sources, we next use an appropriate form of equation (4.2b) to downward continue the shots to Δz using the common-geophone gathers. At each z-level the energy arriving at zero travel time along the zero-offset axis is picked off to form the migrated section. The process is then continued down to the next z-level and so forth. Neglecting any error induced by approximations to the wave equation and if the correct medium velocity is used in the downward continuation, then theoretically *all* of the *primary* energy will migrate to the zero-offset, zero-traveltime plane.

4.2. *The double-square-root equation*

Equations [4.2(a,b)] cannot be used in their exact form to downward continue the sources and receivers since they are second order in z and by the nature of the experiment there exists only one boundary condition in that dimension. The full wave equation, however, can be split into two parts, one for upgoing waves and one for downgoing waves (Claerbout, 1976). An exact form of the one-way wave equations can be obtained by assuming a laterally invariant velocity and Fourier transforming equations [4.2(a,b)] with respect to s, g, and t to obtain the second-order ordinary differential equations

$$P_{z_g z_g} = -\left[\left(\frac{\omega}{v(z_g)}\right)^2 - k_g^2\right] P$$

$$P_{z_s z_s} = -\left[\left(\frac{\omega}{v(z_s)}\right)^2 - k_s^2\right] P$$

where k_s, k_g, and ω are the frequency-domain counterparts of s, g, and t respectively and $P = P(k_s, z_s, k_g, z_g, \omega)$. For constant velocity media, these equations have solutions

$$P(\ldots, z_g,) = P(\ldots, 0,) \exp\left\{ -\frac{i\omega}{v}\left[1 - \left(\frac{vk_g}{\omega}\right)^2\right]^{\frac{1}{2}} z_g \right\} \qquad (4.3a)$$

$$P(,z_s,..) = P(,0,..)\exp\left\{-\frac{i\omega}{v}\left[1-\left(\frac{vk_s}{\omega}\right)^2\right]^{\frac{1}{2}} z_s\right\} \qquad (4.3b)$$

Thus, to move both the sources and receivers to depth z, we first apply equation (4.3a) to the surface common-shot gathers. Then, applying equation (4.3b) to this result yields

$$P(k_s,z,k_g,z,\omega) = P(k_s,0,k_g,0,\omega)\exp\left\{-\frac{i\omega}{v}\left[\left(1-G^2\right)^{\frac{1}{2}}+\left(1-S^2\right)^{\frac{1}{2}}\right] z\right\} \qquad (4.4)$$

where

$$G \equiv \frac{vk_g}{\omega} \text{ and } S \equiv \frac{vk_s}{\omega} .$$

G and S can be interpreted physically as the sine of the raypath angle with respect to the vertical at the geophone and shot location respectively (Figure 4.2). Equation (4.4) is known as the double-square-root equation for constant velocity media. For a stratified velocity structure, the double-square-root equation can be written as

$$\frac{dP}{dz} = -\frac{i\omega}{v(z)}\left[\left(1-G^2\right)^{\frac{1}{2}}+\left(1-S^2\right)^{\frac{1}{2}}\right] P \qquad (4.5)$$

In section 4.4 a more general form for laterally varying media is derived. Equations (4.4) and (4.5) are the exact downward-continuation equations and provide a basis on which to compare various approximations to the one-way wave equations.

Since velocity estimation and migration are typically done in midpoint-offset space it is helpful to rewrite equation (4.5) in (y,h)-space defined by equation (4.1). Letting P' be the wavefield in the y,h coordinate system we have

$$P(s,g) = P'(y,h)$$

since it is the same wavefield regardless of coordinate systems. Using

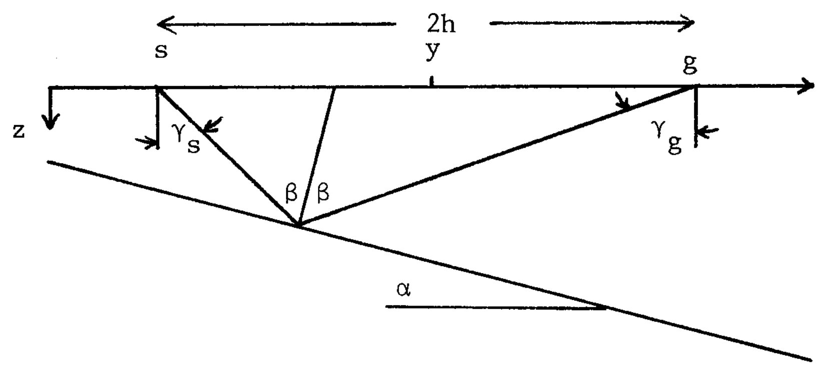

Figure 4.2. Geometry showing a raypath for a shot-receiver distance of $f = 2h$ and reflecting off an interface with dip α. β is referred to as the offset angle. The quantities G and S in equation (4.4) are given by $\sin(\gamma_g)$ and $\sin(\gamma_s)$, respectively. Note that α=0 implies zero dip and β=0 implies zero offset.

the chain rule for partial differentiation gives

$$G = \frac{vk_g}{\omega} = \frac{v\,(k_y+k_h)}{2\omega} = Y + H \tag{4.6a}$$

and

$$S = \frac{vk_s}{\omega} = \frac{v\,(k_y-k_h)}{2\omega} = Y - H \tag{4.6b}$$

where k_y and k_h are the midpoint and offset spatial frequencies respectively, $Y = \frac{vk_y}{2\omega}$, and $H = \frac{vk_h}{2\omega}$. Inserting these expressions into equation (4.5) and dropping the prime from P we obtain the double-square-root equation in midpoint-offset space

$$\frac{dP}{dz} = -\frac{i\omega}{v(z)}\left[\left(1 - (Y+H)^2\right)^{\frac{1}{2}} + \left(1 - (Y-H)^2\right)^{\frac{1}{2}}\right] P \,. \tag{4.7}$$

Y and H are quantities which can be related to the dip and offset angle in a constant-velocity medium. Referring to Figure 4.2, S and G can be expressed in terms of the dip angle, α, and the offset angle, β, as

$$G = \sin \gamma_g = \sin(\beta+\alpha)$$

and

$$S = \sin \gamma_s = -\sin(\beta-\alpha)$$

Using equations (4.6a,b) yields

$$Y = \sin \alpha \cos \beta \qquad (4.8a)$$

and

$$H = \sin \beta \cos \alpha \qquad (4.8b)$$

Thus, for zero offset ($\beta = 0$), Y is given by the sine of the dip angle and for zero dip (Y=0), H is given by the sine of the offset angle.

Midpoint-offset space is the preferred coordinate system to use in conventional processing. There are inherent difficulties with this coordinate system, however, since unless either Y or H equals zero, the effects of the midpoint and offset derivatives implied by Y and H cannot be uncoupled from equation (4.7). This means that common-offset sections theoretically cannot be migrated independently of one another unless H is equal to zero. Doherty and Claerbout (1976) recognized this fact and first put the wavefield into a normal moveout corrected coordinate system to get H as small as possible. However, unless Y equals zero, the NMO correction is not independent of offset.

One possible means of dealing with this problem is to approximate equation (4.7) by expanding the square roots around Y or H equal to some constant. Since wide offsets (large H) are of more importance than large dips it is advantageous to expand both of the square roots about zero dip. Performing a Taylor-series expansion and retaining terms to second order gives

$$\frac{dP}{dz} = -\frac{i\omega}{v(z)}\left[2(1 - H^2)^{\frac{1}{2}} - \frac{Y^2}{(1 - H^2)^{3/2}}\right] P \qquad (4.9)$$

Equation (4.9) is like a 15^0 approximation (in dip) (Claerbout, 1976) to the double-square-root equation. As will be seen in the next section, the first term within the brackets governs conventional normal moveout correction and stack. The second term is an offset dependent migration term, so we have not completely succeeded in decoupling the midpoint and offset dimensions (nor can we ever). We can, however, use the offset dependence to our advantage in preprocessing the data before velocity estimation and stack, as will be seen in section 4.4. One final point to make about equation (4.9) is that it is independent of the first-order dip (Y^1) term. This will not be the case for laterally varying media.

4.3. Velocity estimation in laterally invariant media from the double-square-root equation

Conventional velocity estimation is based on a zero dip assumption and the traveltime formula

$$t^2 = t_0^2 + \left(\frac{2h}{v}\right)^2$$

where t_0 is the zero-offset two-way traveltime, h is half-offset, and v is the RMS velocity to the interface.

From equation (4.8a), zero dip in a laterally homogeneous media implies Y = 0 which, when substitued into equation (4.7) or (4.9), gives

$$P_z = -\frac{2i\omega}{v}(1 - H^2)^{\frac{1}{2}} P \qquad (4.10)$$

Applying equation (4.10) to a CMP gather will focus the hyperbolic events at zero offset and zero traveltime. To focus the hyperbolae to their tops (i.e. t_0), we need only add a time retardation term of the form (Claerbout, 1976)

$$P_z = -\frac{2i\omega}{v} P$$

to yield

$$P_z = -\frac{2i\omega}{v}\left[(1 - H^2)^{\frac{1}{2}} - 1\right] P. \qquad (4.11)$$

The application of equation (4.11) represents the exact frequency-domain expression of normal moveout and stack. The exact space-time domain expression for equation (4.10) is a differential equation in h, z, and t. Clearly, applying the NMO-correction formula in equation (4.9) and then stacking can be a crude approximation to applying equation (4.11), especially over a large range of offsets.

To see that equation (4.11) does indeed represent the normal moveout correction, an approximate expression can be found in the space-time domain by making a stationary phase approximation to the inverse Fourier transform of the constant-velocity solution of equation (4.10):

$$p(h,z,t) = \iint_{-\infty}^{\infty} \exp\left\{-\frac{2i\omega}{v}\left[1 - \left(\frac{vk_h}{2\omega}\right)^2\right]^{\frac{1}{2}} + \frac{2i\omega}{v}\right\} z \exp\left(-ik_h h + i\omega t\right) d\omega\, dk_h$$

Setting the derivatives of the exponents with respect to k_h and ω equal to zero gives

$$\frac{z}{h} = \frac{2\omega}{vk_h}\left[1 - \left(\frac{vk_h}{2\omega}\right)^2\right]^{\frac{1}{2}} \qquad (4.12a)$$

and

$$\frac{2z}{v(t+t_0)} = \left[1 - \left(\frac{vk_h}{2\omega}\right)^2\right]^{\frac{1}{2}} \qquad (4.12b)$$

where t_0 has been substituted for $2z/v$. Combining equations (4.12a) and

(4.12b) to eliminate ω and k_h yields

$$t = \left[t_o^2 + \left(\frac{2h}{v}\right)^2\right]^{\frac{1}{2}} - t_o \tag{4.13}$$

which is the normal moveout correction time. Consider now the case where the medium velocity is constant but the wavefield is laterally varying, such as a dipping bed below a constant velocity medium. The migration part of equation (4.9)

$$\frac{dP}{dz} = -\frac{i\omega}{v} \frac{Y^2}{(1 - H^2)^{3/2}} P \tag{4.14}$$

is now non-zero and thus presents a dilemma as to how to handle the offset derivatives contained in H in the migration. Clearly, we could do a total imaging by downward continuing the data in (y,h)-space and computing all of the necessary derivatives but this would be very expensive. It would be economically advantageous to decouple the y- and h-coordinates by approximating H with some $\hat{H}$ which did not contain any offset derivatives. Such an approximation, as discussed by Yilmaz (1979), is

$$\frac{vk_h}{2\omega} \simeq \frac{v}{2}\frac{dt}{dh} \equiv \hat{H} .$$

For a constant velocity medium, where $t^2 = 4(z^2+h^2)/v^2$, $dt/dh = 4h/v^2t$ and thus,

$$\hat{H} = \frac{2h}{vt} \tag{4.15}$$

Note that $\hat{H} = 0$ still implies zero offset as with the exact H.

Making this approximation for the second occurrence of H in equation (4.9), gives

$$\frac{dP}{dz} = -\frac{i\omega}{v(z)} \left[2(1 - H^2)^{\frac{1}{2}} - \frac{Y^2}{(1 - \hat{H}^2)^{3/2}}\right] P \tag{4.16}$$

The NMO part of equation (4.16) still contains H and thus applying it first would render the offset dependence of the migration academic as we would now have a stacked section. Hence, prior to velocity estimation and stacking we first perform a partial migration on the common-offset sections to take care of the non-zero-offset part of the migration.

The difference between the zero-offset migration [equation (4.14) with H = 0] and non-zero-offset migrations [equation (4.14) with $H = \hat{H}$] is given by

$$\frac{dP}{dz} = \frac{-i\omega}{v}\left[1 - \frac{1}{(1-\hat{H}^2)^{3/2}}\right] Y^2 P \tag{4.17}$$

and is called the ***Deviation operator*** by Yilmaz (1979); it is also related to the so called ***Devilish operator*** (Judson, et al., 1978). To understand its effect consider the model shown in Figure 4.3a. The model consists of a flat and dipping interface underlying a constant-velocity medium. At some midpoint, y_0, the zero-offset traveltimes to each interface are identical, but because of the differing dips they will appear with different apparent velocities on the common-midpoint gather (Figure 4.3b). This presents an ambiguity as to which velocity to stack them with as only one event can be enhanced. If equation (4.17) is applied to the common-offset sections first, however, the curvature of the dipping event will increase slightly to match that of the non-dipping event, the subsequent velocity analysis will yield the true media velocity, and the stack will enhance both events.

Equation (4.17) contains a velocity, which implies that a velocity estimation must be done prior to the common-offset migrations. It was shown by Yilmaz (1979) that this operator is not very sensitive to velocity because so little is done in the migration and hence even a gross velocity estimate will suffice to enhance the subsequent velocity estimation and stack.

In summary, the process of normal moveout correction and stacking can be characterized as a space-time approximation to the zero dip term of the double-square-root equation given in (4.11). The conventional velocity estimation formula can be derived from a stationary-phase

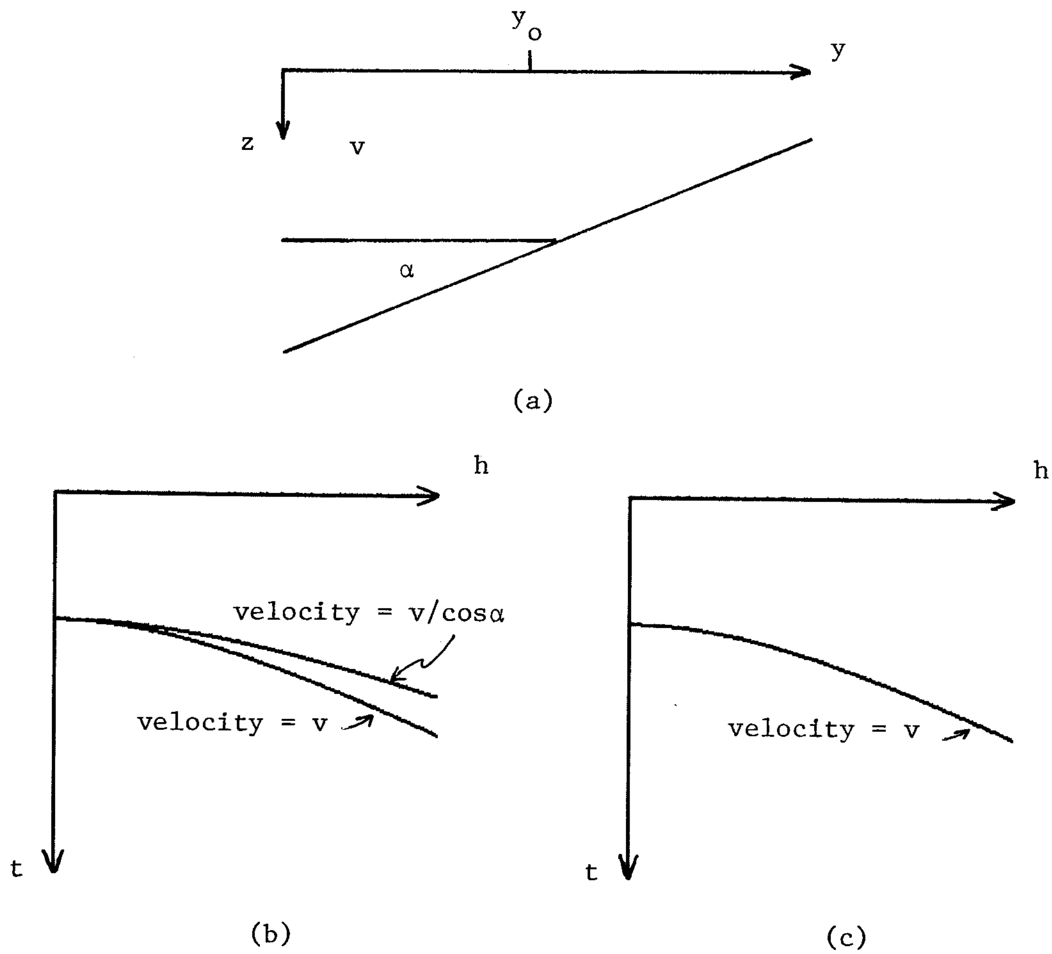

Figure 4.3. Effect of partially migrating common-offset sections prior to stack. a) Model which consists of two intersecting interfaces, on flat and one dipping. At midpoint y_o the zero offset arrival time is identical to each event. b) Common-midpoint gather at y_o. Because the events have different dips they appear to have different apparent velocities (see Levin, 1971). c) Common-midpoint gather at y_o after applying equation (4.17).

approximation to the inverse Fourier transform of the stacking operator. If the wavefield is laterally invariant (fixed h, variable y) the velocity estimation and stack can be improved by first performing a partial migration on the common-offset sections using equation (4.17).

4.4. *Lateral velocity estimation from the double-square-root equation*

In this section the double-square-root equation is generalized to laterally varying media and a lateral velocity traveltime equation is derived. Moreover, we will again consider the effects of the higher-order dip terms and how they can be incorporated into the velocity estimation procedure.

For the laterally varying case where the velocity is a function of the shot and geophone coordinates, the double-square-root equation can be written in operational form as

$$\frac{dP}{dz} = -i\omega \left\{ \left[\frac{1}{v_g^2} + \left(\frac{\partial_g}{\omega} \right)^2 \right]^{\frac{1}{2}} + \left[\frac{1}{v_s^2} + \left(\frac{\partial_s}{\omega} \right)^2 \right]^{\frac{1}{2}} \right\} P \qquad (4.18)$$

where $v_g = v(g,z)$, $v_s = v(s,z)$, $\partial_g = \partial/\partial_g$, and $\partial_s = \partial/\partial_s$. Or, expressing ∂_g and ∂_s in terms of ∂_y and ∂_h:

$$P_z = -i\omega \left\{ \left[M_g + (Y+H)^2 \right]^{\frac{1}{2}} + \left[M_s + (Y-H)^2 \right]^{\frac{1}{2}} \right\} P \qquad (4.19)$$

where Y and H are now given by

$$Y \equiv \frac{\partial_y}{2\omega} \qquad H \equiv \frac{\partial_h}{2\omega}$$

and

$$M_g \equiv \frac{1}{v_g^2} \qquad M_s \equiv \frac{1}{v_s^2}$$

Again, equation (4.19) is not easily implemented in y,h-space because of the coupling of the midpoint and offset derivatives within each square root. Equation (4.19) is put in a more useful form by expanding the square roots around $Y = 0$, giving

$$\frac{-1}{i\omega} P_z = \left\{ \left(M_g + H^2\right)^{\frac{1}{2}} + \left(M_s + H^2\right)^{\frac{1}{2}} \right\} P + \left[\frac{H}{\left(M_g + H^2\right)^{\frac{1}{2}}} - \frac{H}{\left(M_s + H^2\right)^{\frac{1}{2}}} \right] YP +$$

$$\left[\frac{M_g}{\left(M_g + H^2\right)^{3/2}} + \frac{M_s}{\left(M_s + H^2\right)^{3/2}} \right] \frac{Y^2}{2} P \tag{4.20}$$

Equation (4.20) can be considered as the lateral velocity 15^0 approximation to the double-square-root equation. Note that Y = 0 no longer implies zero dip in depth because of the lateral velocity changes. Instead it implies $P_y^t = 0$, which means that the time integral of the wavefield is constant in the midpoint direction. Comparing equation (4.20) with its stratified velocity analog [equation (4.9)] shows that the former has an extra term depending on Y^1.

Equation (4.20) will be considered in three parts, corresponding to the terms depending on Y^0, Y^1, and Y^2. The following mathematics is somewhat cumbersome and so a brief description of the final results is helpful. First, neglecting the Y^1 and Y^2 terms will lead to a lateral velocity analog of normal moveout correction and stacking and a corresponding traveltime formula for velocity estimation from surface data. The most important term in the traveltime formula representing the lateral velocity variation will depend on the second lateral derivative of M. The inclusion of the Y^1 term will help account for the fact that even though a reflecting interface may be flat in z, it will appear as a dipping bed on a time section if the media velocity varies laterally. This term depends upon the first derivative of M. Lastly, the migration term, Y^2, will enter in a way similar to the laterally invariant case.

Consider first the Y^0 terms in equation (4.20). Recalling how this term in equation (4.9) led to the conventional velocity estimation equation, we expect that it will also lead to the first-order velocity estimation equation in laterally varying media.

Expanding M_s and M_g in a second-order Taylor series in the midpoint direction, y,

$$M_g \simeq M + hM' + \frac{h^2}{2} M'' \qquad (4.21a)$$

$$M_s \simeq M - hM' + \frac{h^2}{2} M'' \qquad (4.21b)$$

where $M = M(y)$, $M' = \frac{\partial M}{\partial y}$, and $M'' = \frac{\partial M^2}{\partial y^2}$, the first term of equation (4.20) becomes

$$P_z = -i\omega \left[\left(M + hM' + \frac{h^2}{2} M'' + H^2\right)^{\frac{1}{2}} + \left(M - hM' + \frac{h^2}{2} M'' + H^2\right)^{\frac{1}{2}}\right] P \quad (4.22)$$

Letting $R^2 = M+H^2$, equation (4.18) can be rewritten as

$$P_z = -i\omega R \left[\left(1 + \frac{hM' + \frac{h^2}{2} M''}{R^2}\right)^{\frac{1}{2}} + \left(1 + \frac{hM' - \frac{h^2}{2} M''}{R^2}\right)^{\frac{1}{2}}\right] P. \quad (4.23)$$

Now, for $R^2 >> \pm hM' + \frac{h^2}{2} M''$, equation (4.23) can be approximated by

$$P_z \simeq -i\omega R \left[1 + \frac{hM' + \frac{h^2}{2} M''}{2R^2} - \frac{\left(hM' + \frac{h^2}{2} M''\right)^2}{8R^4}\right] P +$$

$$\left[1 + \frac{-hM' + \frac{h^2}{2} M''}{2R^2} - \frac{\left(-hM' + \frac{h^2}{2} M''\right)^2}{8R^4}\right] P$$

or

$$P_z \simeq -i\omega R \left\{2 + \frac{h^2}{2R^2}\left[M'' - \frac{(M')^2}{2R^2}\right]\right\} P \qquad (4.24)$$

Equation (4.24) is the lateral velocity analog of equation (4.10).

As with the zero-dip term of the constant velocity double-square-root equation, equation (4.24) can also be approximated in the space-time domain to give a traveltime equation useful for velocity estimation. Since the method of stationary phase is very cumbersome in this case we will use a different method here, whereby we will first compute the two-way phase traveltime and then convert this to a two-way group traveltime.

To obtain the phase time we first approximate the solution to equation (4.24) as

$$P(z) \;=\; P(0)\ \exp^{-i\omega t_p} ,$$

where

$$t_p \;=\; \int_0^z R\left[2 + \frac{\frac{x^2}{2}M''}{R^2} - \frac{x^2(M')^2}{4R^4}\right] dz$$

is the vertical phase time and x is the half-offset as a function of depth as shown in Figure 4.4. In order to make the integral tractable we again assume a straight raypath and change the integration variable from z to x using $x = z \tan\theta$, giving

$$t_p \;=\; \frac{R}{\tan\theta}\int_0^h \left[2 + \frac{\frac{x^2}{2}M''}{R^2} - \frac{x^2(M')^2}{4R^4}\right] dx$$

Performing the integration yields

$$t_p \;=\; \frac{R}{\tan\theta}\left[2h + \frac{h^3}{6R^2}\left(M'' - \frac{(M')^2}{2R^2}\right)\right] . \qquad (4.25)$$

In order to get an expression completely in the space-time domain

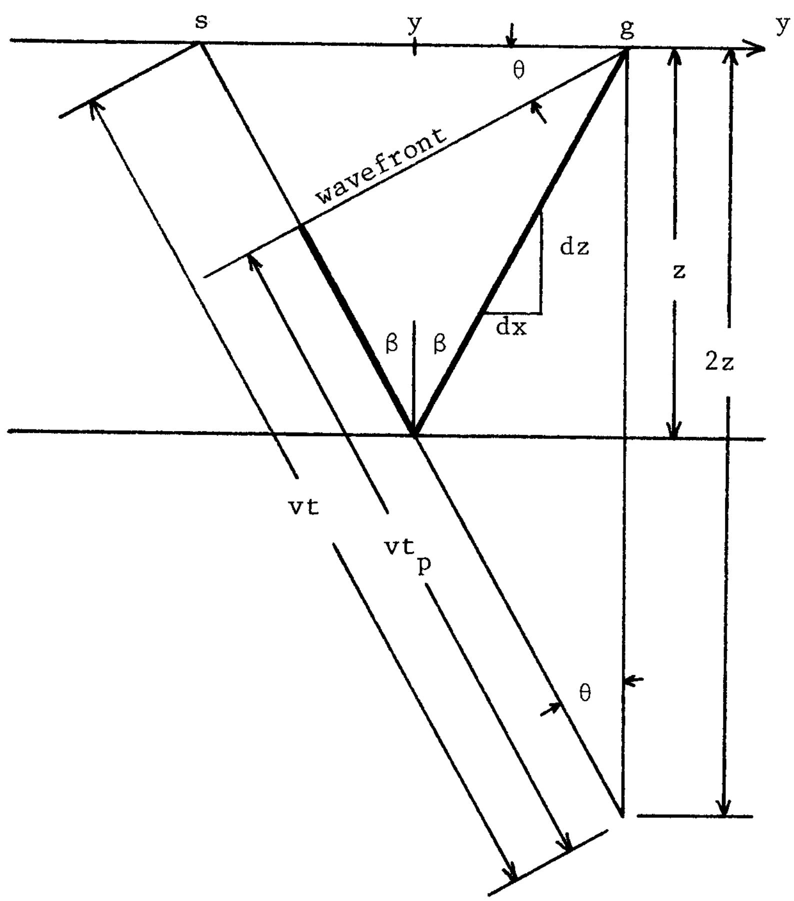

Figure 4.4. Relation between vertical phase and group time. Taken from Schultz and Claerbout (1978). The phase time, t_p, is given by the traveltime of the wavefront, which is shown at t=0, to travel down to the reflector and return to the surface location g. The group time, t, is the traveltime for a spherical spreading source which originates at the surface at the location s to reflect off the interface and return to the location g. The angle the wavefront makes with the surface is θ and for the laterally varying case is approximated by $\sin\theta = pv(y)$.

it is necessary to approximate the H which is in R. To do this we recognize that the phase time corresponds to the traveltime of a wavefront. With the straight ray approximation the wavefront is planar and its arrival can be simulated by stacking the event along a linear moveout trajectory of slope p

$$t' = t + 2ph . \qquad (4.26)$$

Equation (4.26) implies that

$$\partial_h = \partial_{h'} - 2i\omega p$$

For a slant stack $\partial_{h'} = 0$ and so

$$\frac{\partial_h}{2\omega} = -ip . \qquad (4.27)$$

Substituting equation (4.27) for H in (4.25) gives

$$t_p = \frac{(1 - p^2v^2)^{\frac{1}{2}}}{v} \left\{ 2z + \frac{h^2v^2z}{6(1 - p^2v^2)} \left[M'' - \frac{(M')^2}{2M(1 - p^2v^2)} \right] \right\}$$

where v is the velocity at the midpoint. Now recognizing that $(1 - p^2v^2)^{\frac{1}{2}} = \cos\theta$, where θ is the angle the wavefront makes with the horizontal and converting the phase time to the group time, t, with

$$t = \frac{t_p}{\cos^2\theta}$$

(Schultz and Claerbout, 1978) we obtain

$$t \cos\theta = \left\{ 2z + \frac{h^2}{6\cos^2\theta} \left[M'' - \frac{(M')^2}{2M\cos^2\theta} \right] \right\} \qquad (4.28)$$

We now make the following substitutions for the first two occurrences of $\cos\theta$ in equation (4.28):

$$\cos\theta = \left[1 - \left(\frac{2h}{vt}\right)^2 \right]^{\frac{1}{2}}$$

$$\frac{1}{\cos^2\theta} = (1 + \tan^2\theta) = \left(1 + \frac{h^2}{z^2}\right)$$

and approximate the last occurrence with 1. Squaring equation (4.28) and ignoring the square and cross terms of M'' and M'^2 gives

$$t^2 = 4\,(h^2 + z^2)\left\{M + \frac{h^2}{6}\left[M'' - \frac{(M')^2}{2M}\right]\right\}. \tag{4.29}$$

Equation (4.29) is the same as the traveltime equation derived in Chapter II using a ray theory approach except that the latter is written in terms of $w = M^{\frac{1}{2}}$. The number of approximations made may seem unwieldy, however, what we seek are only the first-order effects of the lateral velocity variations on the traveltime equations.

Consider now the effect of the Y^1 term of equation (4.20) on velocity estimation and imaging in laterally varying areas. The Y^1 part of equation (4.20) is

$$P_z = -i\omega\left[\frac{H}{(M_g+H^2)^{\frac{1}{2}}} - \frac{H}{(M_s+H^2)^{\frac{1}{2}}}\right] Y\,P\,. \tag{4.30}$$

where again $Y = \partial_y/2\omega$, $H = \partial_h/2\omega$, $M_g = 1/v^2(g)$, and $M_s = 1/v^2(s)$. Note that for $M_g = M_s$, the coefficient of the Y^1 term vanishes exactly. As with the Y^0 term, we will expand M_s and M_g around $M = M(y)$ using equations (4.21a,b) giving

$$P_z = \frac{1}{(M+H^2)^{\frac{1}{2}}}\left[\frac{H}{\left(1 + \dfrac{hM' + \frac{h^2}{2}M''}{M + H^2}\right)^{\frac{1}{2}}} - \frac{H}{\left(1 + \dfrac{-hM' + \frac{h^2}{2}M''}{M + H^2}\right)^{\frac{1}{2}}}\right] Y\,P\,. \tag{4.31}$$

Using the square-root approximation $(1+X)^{-\frac{1}{2}} \simeq 1 - X/2$, equation (4.31) becomes

$$P_z \simeq \frac{-i\omega h H M'}{(M+H^2)^{3/2}}\,Y\,P. \tag{4.32}$$

It is interesting to note that the coefficient of Y in equation (4.32) depends only on the first derivative of M, whereas the coefficient of the Y^0 term in equation (4.24) depends on M and M' as well.

To understand the physical significance of equation (4.32), assume $M \gg H^2$ and rewrite it using the appropriate expressions for Y:

$$P_z = \frac{-ihM'}{2M^{3/2}} \frac{\partial_h}{2\omega} P_y . \tag{4.33}$$

Equation (4.33) is like a shifting term in (y,z)-space, however, it also contains an offset derivative. To apply equation (4.33) to a common-offset section, we must again approximate H with some $\hat{H}$. Using the approximation given in equation (4.15) and writing M and M' in terms of v, we obtain the result

$$P_z = \frac{h^2 v'}{v^2 t} P_y, \tag{4.34}$$

where $v' = \partial v/\partial y$.

Equation (4.34) can be interpreted by considering different common-offset sections over a point scatterer in a medium where the velocity increases linearly in the midpoint direction. Figure 4.5 shows three different common-offset sections for such a model. Because of the lateral gradient, the midpoint location of the minimum traveltime is not directly above the point scatterer, nor is it in the same lateral position for the different common-offset sections. In fact, the minimum traveltime location moves in the direction of the velocity gradient as the offset is increased.

It is easily verified that the shift implied in equation (4.34) is in the correct direction. Again consider Figure 4.5. The velocity increases in the +y direction, so v' is positive, and thus so is the coefficient of P_y in equation (4.34). Denoting this coefficient as c, the solution to equation (4.34) is given by

$$P = P(cz + y)$$

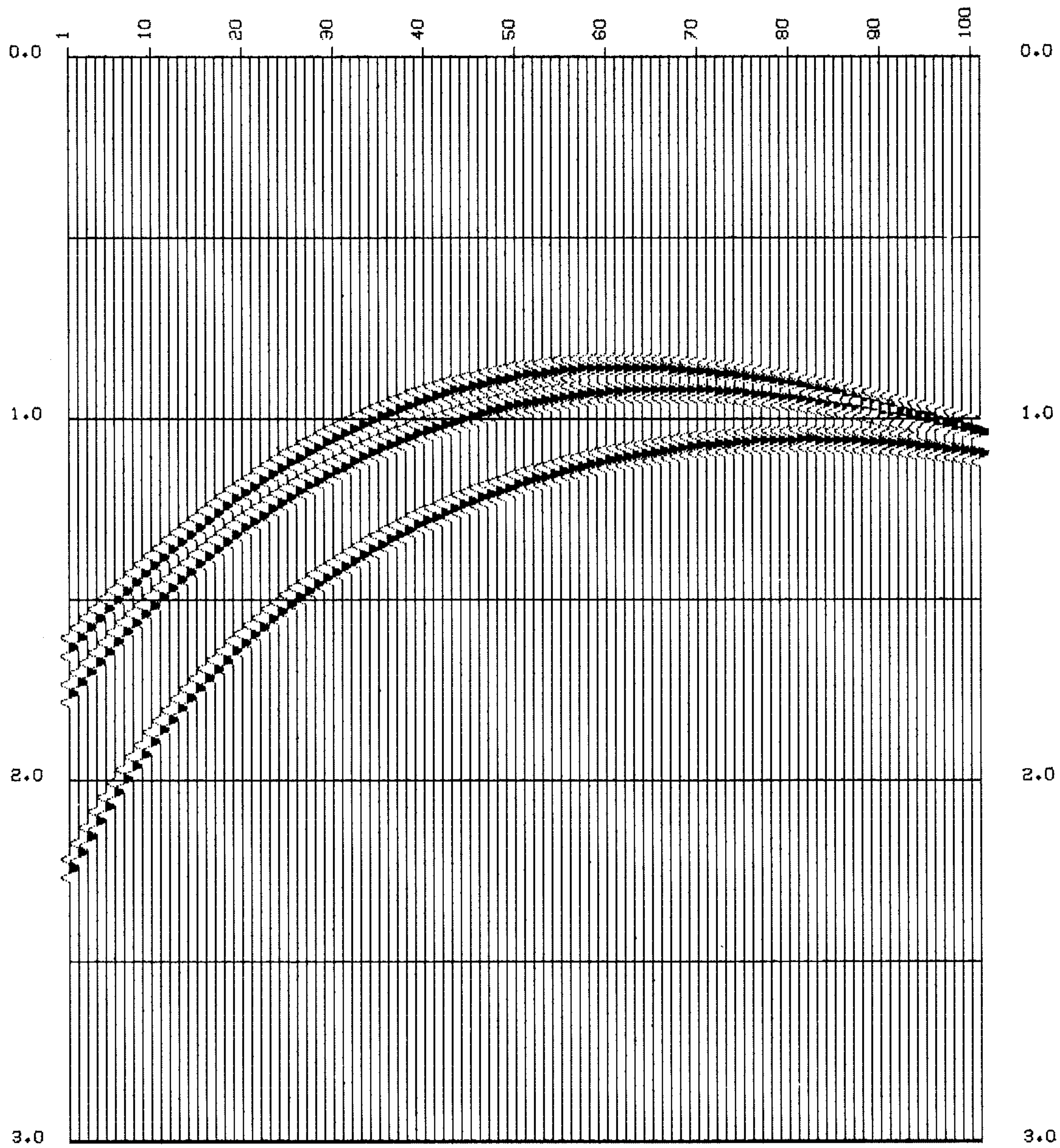

Figure 4.5. Three common-offset sections (h = 0, 2000, and 4000 ft.) across a point scatterer in a medium with a horizontal velocity gradient to illustrate the physical significance of equation (4.26). The velocity increases from left to right with a gradient of 1.0 ft/sec/ft. The point scatterer is located at midpoint #50 and at a depth of 5000 feet. The midpoint spacing is 100 feet and the velocity at midpoint #50 is 11300 ft/sec. Note that the minimum traveltime moves in the direction of the lateral gradient as the offset is increased.

Hence, as z increases, y must decrease for the argument to remain constant; which is in the direction of the zero-offset "hyperbola" top. Equation (4.34) can be used to shift the common-offset sections prior to velocity estimation and stack. This is a process, however, that would require at least two iterations since v is not known *a priori*.

The Y^2 term enters in a similar way as in the laterally invariant case. Copying this term from equation (4.20) we have

$$P_z = -i\omega \left[\frac{M_g}{\left(M_g + H^2\right)^{3/2}} + \frac{M_s}{\left(M_s + H^2\right)^{3/2}} \right] \frac{Y^2}{2} P \tag{4.35}$$

For the sake of simplicity, M_g and M_s in the denominator can be replaced with $M = M(y)$. Now expanding M_g and M_s in the numerator in second-order Taylor series [see equation (4.21)] yields

$$P_z = -i\omega \left[\frac{M + \frac{h^2}{2} M''}{(M + H^2)^{3/2}} \right] Y^2 P \tag{4.36}$$

The application of equation (4.36) represents a full migration in the midpoint dimension using all offsets. If we want to only do a complete migration on the common-midpoint stack, then we can perform a partial migration first on the non-zero offset sections as discussed in the previous section. The lateral derivative deviation operator is thus given by

$$P_z = -i\omega \left[1 - \frac{M + \frac{h^2}{2} M''}{(M + \hat{H}^2)^{3/2}} \right] Y^2 P \tag{4.37}$$

where $\hat{H}$ is given by equation (4.15).

In practice, in using equation (4.37) we would ignore the M" term since the deviation operator is not very sensitive to velocity. Thus, to implement this equation, we first make a crude velocity estimation and apply it to the non-zero common-offset sections. Regrouping the data into common-midpoint gathers the velocity is then reestimated with equation (4.29) or (2.7). With the estimate of w and its second derivative, the common-midpoint data can be stacked using equation (2.4). Finally, the stacked section is migrated with a lateral velocity migration algorithm.