Chapter I
Introduction

The estimation of velocity in structurally complex areas is of critical importance to imaging the earth with seismic reflection techniques. In such areas, however, conventional velocity estimation techniques often yield misleading or absurd results because they are based on a stratified medium assumption and associate the normal moveout time with only the root-mean-square (RMS) velocity at a given midpoint. If the medium velocity varies laterally, the normal moveout time is not only a function of the velocity at a particular midpoint but also of the velocity beneath all midpoints through which the recorded energy travels. Thus, if the velocity changes laterally with wavelengths on the order of a few cable-lengths any velocity estimation of the medium will fail unless these variations are taken into account.

The departure from lateral velocity homogeneity can be separated into two categories as shown in Figure 1.1. The first of these (Figure 1.1a) is the case wherein the structure of the medium invalidates the common-reflection-point assumption. In this case the media velocity is constant and the reflecting subsurface deviates substantially from being flat. The second category (Figure 1.1b) is the situation where the structure is nearly flat, but the velocity of the overlying media varies laterally.

Doherty and Claerbout (1976) showed how the first problem could be approximately compensated for by first migrating common-offset sections (fixed source-receiver separation) with an approximate velocity function and then estimating velocity from the subsequent common-midpoint gathers. More recently Judson, et al. (1978) also showed how a partial migration of common-offset sections can minimize structural effects prior to velocity estimation. The purpose of this thesis is to develop a method of handling the second problem of lateral velocity variations. The problem will be limited to lateral variations only in the plane of the seismic section, thus assuming a two-dimensional medium.

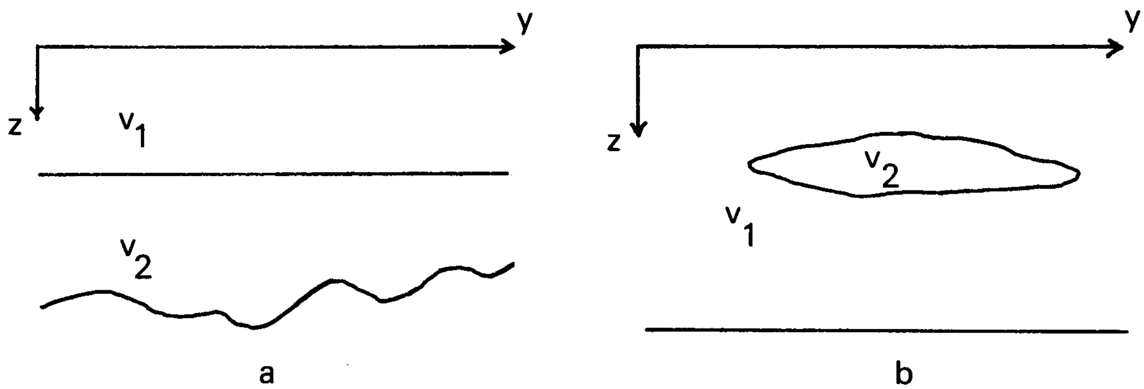

Figure 1.1. Two causes of problems for conventional velocity estimation (adapted from Doherty and Claerbout, 1976). The y and z axes correspond to midpoint and depth respectively. On the left the problem is that the underlying reflectors are not flat. Consequently, the common-depth-point assumption used in conventional velocity estimation is invalid. On the right, the reflecting interface is flat; however, some of the energy to it travels through a local velocity inhomogeneity. In reality, both problems are usually encountered together.

There exist many approaches to this problem. Perhaps the most accurate is a tomographic method (e.g. Kak, 1979; Dines, K.A. and Lytle, R.J., 1979) whereby the traveltimes to a given event for each offset and midpoint are used in the inversion. However, tomographic or statics-type methods suffer from resolution limitations for velocity variations greater than a cable-length (e.g. Wiggens, et al., 1976). The method proposed in this thesis is considerably simpler than tomographic methods in that it relates the true vertical RMS velocity and its lateral derivatives to the conventional velocity estimate to a given event, thereby yielding the true velocity by solving a linear system of equations.

The basic premise of the lateral derivative technique is to characterize the slowness (= 1/velocity) in the vicinity of every midpoint in a Taylor-series expansion in the midpoint direction, an idea inspired by the results presented by Pollet (1974). Thus, the subsequent traveltime equations depend not only on the slowness at each midpoint, but also on the lateral derivatives of the slowness. From the lateral derivative traveltime equations a relation can be found between (1) a conventional hyperbolic velocity estimate and (2) the true velocity and its lateral derivatives. Approximating the derivatives as finite differences will then lead to a linear system of equations which is easily solved to obtain the velocity to a given event.

One of the key results from the development of the traveltime equations is that the most important lateral derivative is the second derivative of the slowness. This can be illustrated in a qualitative manner with the simple model shown in Figure 1.2a. The model consists of a flat reflector at a depth z with a thin low-velocity layer situated in between the surface and the interface. Consider the idealized traveltimes (with no diffractions) to the bottom reflector for two common-offset sections across the model shown in Figure 1.2b. Measuring the hyperbolic moveout between these two offsets would yield a velocity estimate like that shown as the bold line in Figure 1.2c. The light line shows the correct vertical RMS velocity to be a step function in the midpoint direction. The effect of the truncated bed is to produce a square-wave-shaped fluctuation. (The square-wave shape occurs because just two offsets are considered here; in reality the fluctuation is more nearly sinusoidal in shape.) Taking the difference between the true and the estimated vertical RMS velocities leaves the residual shown in Figure 1.2d.

Now consider taking the second lateral derivative of the true slowness shown in Figure 1.2c. Using a finite differencing spacing equal to half of the wavelength of the velocity perturbation, it is easily verified that the result would give the shape of the velocity perturbation in Figure 1.2d. Thus, at least in a qualitative sense, errors in the conventional velocity analysis are related to the second lateral derivative of the slowness.

The basic organization of the thesis is as follows. Chapter II begins with the derivation of the traveltime equations for a laterally varying medium using a simplified ray theory approach. From a slight

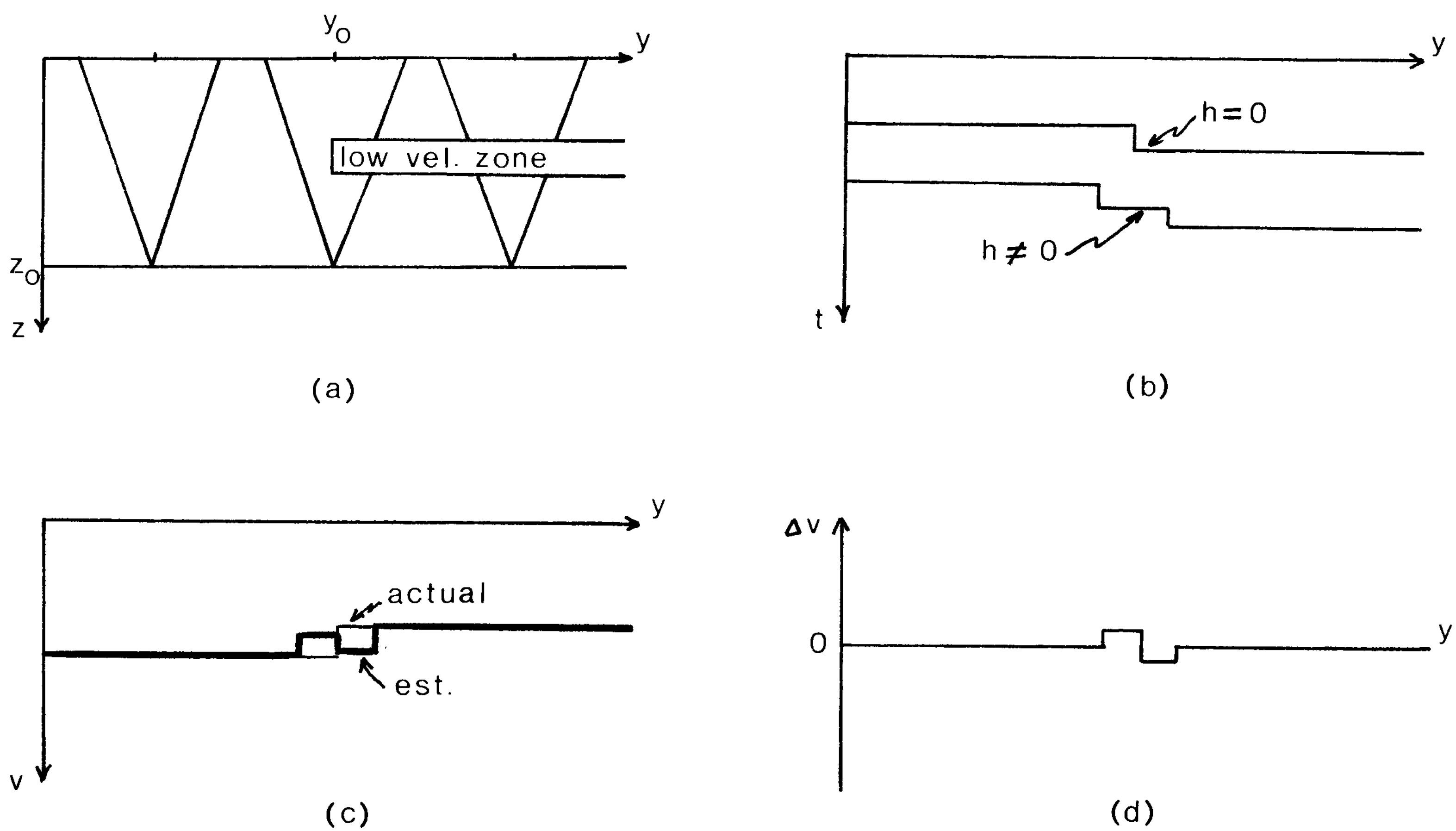

Figure 1.2. Relation of normal moveout travel time perturbations with the second lateral derivative of velocity. a) An earth model where there exists a low velocity layer which truncates at a midpoint y_o. b) Travel times to the interface at z_o for a zero and non-zero common offset (constant shot-receiver distance) section. c) The estimated (dark line) vertical RMS velocity to the reflector using the two common offset sections. The correct velocity is the step function shown as the light line. d) The difference between the correct and estimated vertical RMS velocities. The convolution of the correct RMS velocity to the interface at z_o (c) with a second derivative (1,-2,1) operator (of an appropriate length) is easily verified to be proportional to the velocity error in (d).

modification of the traveltime equation, a relation can be found between conventional slowness estimates and the true slowness and its second derivative. This relation forms the basis of the lateral derivative method (LDM) of velocity estimation. We will next examine the resolution of the lateral derivative method and how much lateral velocity variation is necessary before it becomes significant. Considered next is the implementation of the lateral derivative method and a discussion on the stability of the inversion. Under the assumptions used in deriving the traveltime equations, the effect of the first lateral derivative of velocity is negligible. As a last topic the validity of neglecting this term is considered, to see how large the term must be before becoming significant.

In Chapter III the lateral derivative method is applied to three synthetic datasets of increasing complexity, and the results are compared with those from a conventional velocity estimation procedure. The synthetics are designed to test how well the lateral derivative method works over areas with the following features: abrupt lateral discontinuities, lateral velocity variations at varying depths, and a regional gradient in velocity. The final section of Chapter III is devoted to the application of the lateral derivative method to field data. The data are marine data from the Grand Banks area and contain lateral velocity variations due to the seafloor topography in that region.

The ray theory approach is attractive for its simplicity; however, it does not help in going to higher orders of accuracy or in considering dipping or diffracting earth models. Thus, in the final chapter, the lateral velocity traveltime equations are rederived using a wave theory approach. This approach is much more useful, since it suggests how to preprocess data before velocity estimation to reduce the effects of dip and diffractions. A procedure is presented which can be used for areas where the medium is a combination of the models shown in Figures 1.1a and 1.1b.